DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

OPTIMAL SIZING OF RAW MATERIALS IN CARTON BOX INDUSTRIES

by
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October, 2023
İZMİR

OPTIMAL SIZING OF RAW MATERIALS IN CARTON BOX INDUSTRIES

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İlkim KARAMAN

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"OPTIMAL SIZING OF RAW MATERIALS IN CARTON BOX INDUSTRIES"** completed by **İLKİM KARAMAN** under supervision of **PROF.DR. ADİL BAYKASOĞLU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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OPTIMAL SIZING OF RAW MATERIALS IN CARTON BOX INDUSTRIES

ABSTRACT

In the literature, different research focused on stock selection and sizing problems to provide optimal cutting plans for different manufacturing industries. However, most of them don't combine optimization-based solution approaches with a meta-heuristic algorithm for solving a case-oriented cutting problem to balance the stock diversity and waste cost. Based on this motivation, this thesis addressed an integrated stock selection and sizing problem of a cardboard box manufacturing company to determine the optimal dimensions and the purchasing quantities of cardboards to be cut.

Firstly, a novel MINLP model of the present problem is developed. Then, a matheuristic algorithm that integrates a MILP into the SA algorithm is proposed due to the highly non-linear structure of the MINLP model. With the proposed matheuristic, the MINLP model can be easily reduced to a MILP model since some of the decision variables are taken as parameters that are supplied by the SA algorithm. After this reduction, the problem can be solved by the Gurobi MIP solver, and different solutions can be generated within reasonable computation times. The algorithm is tested on a real-life problem of a cardboard box manufacturing company in Manisa, Turkey, and a computational study is performed on different instances generated from the data provided by the company. Computational results have shown that the algorithm can provide minimum stock diversity and waste cost compared to the existing applications of the company as well as the results of some greedy search heuristics embedded in the MILP model.

Keywords: Stock selection and sizing problem, matheuristic, mixed-integer non-linear programming, simulated annealing, corrugated cardboard box production

KARTON KUTU ENDÜSTRİSİNDE OPTİMAL HAMMADDE BOYUTLANDIRMASI

ÖZ

Literatürde, farklı imalat endüstrilerinde optimum kesim planlarını sağlamak için stok seçim ve boyutlandırma problemlerine yönelik farklı araştırmalar bulunmaktadır. Ancak bu araştırmaların çoğu, stok çeşitliliğini ve fire maliyetini dengelemeyi amaçlayan bir gerçek hayat problemini çözmek için optimizasyona dayalı çözüm yaklaşımlarını bir metasezgisel algoritma ile birleştirmemektedir. Bu tezde, oluklu mukavva kutu üretimi yapan bir şirket için optimum hammadde boyutu ve satın alma miktarlarını belirlemeyi amaçlayan entegre stok seçim ve boyutlandırma problemi ele alınmıştır.

İlk olarak, mevcut problem için yeni bir karışık tam sayılı doğrusal olmayan programlama modeli geliştirilmiştir. Daha sonra, bu modelin doğrusal olmayan yapısından dolayı, karışık tam sayılı doğrusal programlama modelini benzetim tavlaması algoritması ile birleştiren bir matheuristik algoritma önerilmiştir. Önerilen doğrusal olmayan modeli karışık tamsayılı doğrusal programlama modeline indirgeyebilmek için karar değişkenlerinden bazıları benzetim tavlaması algoritması ile modele parametre olarak sağlanmıştır. Bu şekilde, problem Gurobi MIP çözücüsü ile kolayca çözülebilmiş ve kabul edilebilir süreler içerisinde farklı çözümler üretilebilmiştir. Geliştirilen algoritma, Manisa, Türkiye'de bulunan bir oluklu mukavva kutu imalatçısının gerçek hayat problemine uygulanmış ve firma tarafından sağlanan verilerden türetilmiş farklı örnekler ile test edilmiştir. Test sonuçları algoritmanın, şirketin mevcut uygulamalarına ve karışık tam sayılı programlama modeli ile birleştirilmiş farklı açgözlü sezgisellere kıyaslandığında minimum stok çeşitliliği ve fire maliyeti sağlayabildiğini göstermiştir.

Anahtar kelimeler: Stok seçim ve boyutlandırma problemi, matheuristik, karışık tam sayılı doğrusal olmayan programlama, benzetim tavlaması, oluklu mukavva kutu üretimi

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ABBREVIATIONS

MINLP	: Mixed-Integer Non-Linear Programming
MILP	: Mixed-Integer Linear Programming
SA	: Simulated Annealing
MIP	: Mixed-Integer Programming
DOE	: Design of Experiment
LP	: Linear Programming
ERP	: Enterprise Resource Planning
NP	: Non-Deterministic Polynomial-Time
INLP	: Integer Non-Linear Programming
ILP	: Integer Linear Programming
GHA	: Genetic Hybrid Algorithm
GRASP	: Greedy Randomized Adaptive Search Procedure
IP	: Integer Programming
PSO	: Particle Swarm Optimization
MOQ	: Minimum Order Quantity
CPU	: Central Processing Unit
S/N	: Signal/Noise
ANOVA	: Analysis of Variance
ISSS	: Integrated Stock Selection and Sizing

CHAPTER ONE
INTRODUCTION

1.1 Background and Research Motivations

In recent years, integrated stock selection and sizing problem has been a very popular and challenging problem in different manufacturing industries for both researchers and practitioners. Selecting the raw sheet materials (i.e., corrugated boards) and detecting their dimensions properly is the key concern of the decision-makers in the corrugated cardboard box production industries and these decisions affect the profitability of the business significantly (Mosallaeipour, Mahmoodirad, Niroomand, & Vizvari, 2018). The corrugated cardboard box manufacturers desire to provide maximum efficiency from the corrugated boards used in the production, since their waste may cause substantial financial losses because they usually need to procure corrugated boards of different sizes from their suppliers. In the presence of various limitations, the stock variety of corrugated cardboard for box manufacturers is not infinite. Within the limitation of technical or financial constraints and the minimum order quantities, the number of stock variants for the purchased corrugated boards is not as large as the number of different sizes of corrugated cardboard boxes. However, the increase in the stock variance could decrease the relevant waste cost that arises during the production of corrugated cardboard boxes. This points to the importance of solving the problem of simultaneously selecting and sizing the procured corrugated boards optimally. The total waste and purchasing costs as well as the stock variety can be reduced by generating effective and efficient cutting and procurement plans consisting of an optimal number of corrugated boards with their optimal sizes/dimensions.

Commercial suppliers/practitioners continue to develop many software tools to generate effective and efficient cutting and production plans during the last decade (Goulimis & Simone, 2020). However, most of them were not developed based on an optimization-based solution method or didn't include any mathematical programming-based approach or an artificial intelligence algorithm in their background. They mainly depend on problem-specific basic heuristic algorithms or greedy search procedures. In

this context, the development of new generation cutting stock, selection and sizing tools, or software based on the hybridization of mathematical programming approaches and artificial intelligence algorithms is necessary. The appropriate solution to the integrated stock selection and sizing problem generally needs to develop highly non-linear and non-convex mixed-integer programming models as seen in the recent studies developed by several researchers. In addition, these researchers also needed to develop additional approximation techniques or multi-stage models or solution methods to cope with the highly non-linear model structures (or non-convexity) while producing applicable solutions. However, when these techniques are applied, the model dimension will generally increase dramatically due to the definition of additional auxiliary variables and constraint functions. Fortunately, the matheuristic-based solution approaches can be very useful in overcoming the non-convex or non-linear forms of the developed mathematical programming models.

Based on these motivations, a novel MINLP model is developed to solve a real-world integrated stock selection and sizing problem of a corrugated cardboard box manufacturing company in Manisa, Turkey. However, since the proposed MINLP model has a highly non-convex and non-linear form, it couldn't be linearized or solved by any non-linear programming or MIP solver. Due to that reason, a matheuristic-based solution approach that incorporates a MILP into the SA algorithm is proposed and implemented in Python 3.8.8 in integration with the Gurobi MIP solver. With the help of the proposed matheuristic-based solution approach, the developed MINLP model can be reduced to a MILP model with a fully linear objective and constraint functions. Because the SA algorithm provides the dimensions of the purchased corrugated boards as parameters, which are normally decision variables in the model. After this reduction, the obtained MILP model can be easily solved by the Gurobi MIP solver and alternative solutions can be generated within reasonable computing times. The computational analysis has also demonstrated that superior results can be achieved within acceptable run times by using the matheuristic-based solution approach for large-scale or realistic-sized benchmark instances.

1.2 Contributions of the Thesis

A novel MINLP model is first developed in this thesis to solve a real-world integrated stock selection and sizing problem of a carton box manufacturing company in Turkey. However, since the proposed MINLP model has a highly non-convex and non-linearizable form, it couldn't be linearized or solved by any non-linear programming or MIP solver of an optimization suite. For that reason, a matheuristic-based solution approach that incorporates a MILP into the SA algorithm is also proposed and implemented in Python 3.8.8 modeling and development environment in integration with the Gurobi MIP solver. With the help of the proposed matheuristic-based solution approach, the developed MINLP model can be reduced to a MILP model with a fully linear objective and constraint functions since the dimension values of the purchased corrugated boards are provided by the SA algorithm. In other words, the stock sizing part of the present problem is extinguished and reduced to a stock selection problem only by using the proposed matheuristic-based solution approach. Hence, the resulting MILP model can be easily solved by the Gurobi MIP solver and many solution alternatives can be evaluated at each iteration within reasonable computing times.

With a thorough search of the relevant literature, no similar research has been found. There is no similar study in the related literature that develops such a matheuristic-based solution approach to overcome the highly non-convex and non-linear form of an integrated stock selection and sizing problem of a corrugated cardboard box manufacturing company. The computational analysis has also demonstrated that superior results can be achieved within acceptable run times by using such a matheuristic-based solution approach even for large-scale or realistic-sized benchmark instances.

The proposed approach can be used to solve different stock selecting and sizing problems by modifying the mathematical model and the metaheuristic part. By modeling case-specific constraints the model can be used for different cases in the industry. Also, developing different solution generation methods, or using various meta-heuristic algorithms can increase the efficiency of the problem-solving procedure.

These modification alternatives show the applicability and efficacy of the proposed solution approach.

1.3 Outline of the Thesis

This thesis consists of 7 chapters and the first chapter is the introduction part of this thesis. In this chapter, background, and research motivations are presented. Also, the contributions and outline of this thesis are given.

In Chapter 2, a brief literature review is presented about cutting stock, stock selecting, sizing, and assortment problems. Also, a summary table for the review is given in this chapter.

In Chapter 3, the proposed MINLP model of integrated stock selection and sizing problem for the real-life application is explained. Before defining the model, the company is introduced briefly. After that, the objective function, parameters, decision variables, and constraints of the manufacturing company are defined and detailed.

In Chapter 4, the SA-based matheuristic solution approach is introduced. Firstly, the SA algorithm is defined. Afterward, the proposed approach is presented. The details of solution representation, initialization, and neighborhood generation strategies are explained.

In Chapter 5, the proposed matheuristic approach is illustrated based on a real-life application from the corrugated cardboard box manufacturing company. The approach is used to minimize the related operational cost of the company while meeting the demands of towel radiator boxes. Also, the parameter setting is presented, for the fine-tuning process of the matheuristic approach through Taguchi's DOE approach.

In Chapter 6, the computational study is conducted. The proposed algorithm is tested on different instances, that are generated from the data provided by the company. The results are compared with the existing cutting and purchasing plan of

the company. Also, for comparison, the results were generated with greedy heuristic search procedures that combined with the MILP model are used.

Chapter 7 presents the summary of the thesis and future research directions.

CHAPTER TWO
LITERATURE REVIEW

This section presents the literature review on different optimization problems, i.e., cutting stock, stock selecting, sizing, assortment problems, and the integrated versions of them with production planning, scheduling, and lot sizing problems.

In the corrugated box production industry, the multi-stage cutting stock problem of two or more dimensions was first discussed by Gilmore & Gomory (1965) by involving an auxiliary sequencing problem for the slitting patterns. They proposed linear programming formulations of the stated problems and showed how the column generation algorithms can be practically applied to solve the higher dimensional cutting stock problems.

Bookbinder & Higginson (1986) developed a simulation model for a corrugated cardboard box production facility, that includes different phases of production. The main goal of their model was to analyze the relationship between various levels of trim waste and the service level, which mainly depends on the customers' due dates. In the model, they used different scheduling rules. They generated customer orders via different probability distributions, and after that, they used the model for scheduling. They evaluated the trade-off through their simulation model.

Christofides & Hadjiconstantinou (1993) tried to maximize the value of pieces cut for the two-dimensional cutting problem that requires cutting several small rectangular pieces from a rectangular stock plate, by using a tree-search algorithm.

Savsar & Cogun (1994) also developed a linear programming model for a production line in a corrugated box factory. They tried to combine different problems with their model, i.e., cutting stock, storage space, and machine loading problems. Their objective is to minimize the waste resulting from cutting the paper rolls into cardboard, while the utilization of the limited storage capacity and machine utilization

are considered constraints. They tested their model by using the actual data provided by the factory.

Harjunkoski, Westerlund, Isaksson, & Skiffar (1996) analyzed trim loss problems. They developed a general integer non-linear programming model, and formulated and solved it as an integer linear programming or mixed integer linear programming model. They compared the efficiency of these different approaches. Their objective function minimizes the trim loss and the time for cutting to produce a set of paper rolls.

Wäscher & Gau (1996) developed several heuristic methods to solve the standard one-dimensional cutting stock problem. They compared these methods concerning solution quality and computing time by solving 4000 randomly generated test problems.

Lai & Chan (1997) developed a simulated annealing searching technique to solve non-guillotine, two- or three-dimensional cutting stock problems. Their algorithm tries to minimize the amount of trim loss by considering the possibility of placing different sizes of small rectangles on a larger rectangle. The algorithm is tested by using randomly generated data and real data from a printing company.

Westerlund, Isoksson, & Harjunkoski (1998) developed a MILP model for the two-dimensional trim-loss problem. They tried to minimize the total cost for waste and cutting time to produce a set of paper rolls from raw materials by considering the width and length of the raw materials and the lengths of the products as variables.

Östermark (1999) developed a genetic hybrid algorithm to solve the trim loss problem which is designed for nonconvex mixed integer nonlinear programming problems. He solved the trim loss minimization problem of a Finnish paper-converting mill.

Bolat (2000) developed a linear programming model that maximizes the total length of converted paper boards in one shift of a corrugated box production factory. The

model has been developed to solve the cutting stock problem combined with buffer utilization and machine loading problems encountered in the production of corrugated boxes. He also considered set-ups in the converting machines and the operating characteristics of the roller conveyors in the buffer area. He developed a decomposition algorithm that applies successive LP relaxations and tested the model with the real data obtained from the factory.

Alvarez-Valdés, Parajón, & Tamarit (2002) developed heuristic algorithms for the two-dimensional cutting problem, which tries to maximize the value of pieces that are the resultant of cutting large single rectangular stock sheet into a set of small rectangular pieces of given sizes. They developed a greedy randomized adaptive search procedure and tabu search algorithm that produces high-quality results in short computing times.

Holthaus (2003) considered the one-dimensional assortment problem which includes the determination of the number of different sizes of standard lengths to be maintained as inventory and to be used to fulfill a set of cutting orders. He developed an integer linear programming model to determine which types of standard lengths should be stocked to minimize the total material cost. His paper evaluates the possible savings in the total material cost which can be realized by using an assortment with two or more types of stock lengths, compared to an assortment with a single type of stock lengths.

Umetani, Yagiura, & Ibaraki (2003) developed a metaheuristic for a one-dimensional cutting stock problem in which the number of different cutting patterns is constrained within a given bound. Their iterated local search algorithm with adaptive pattern generation aims to minimize the number of different cutting patterns. They conducted computational experiments for random instances and some instances taken from real applications in a chemical fiber company. They observed that the proposed algorithm provides comparable solutions to other existing heuristic approaches.

Correia, Oliveira, & Ferreira (2004) studied on minimization of trim loss that occurs when cutting paper reels. They developed a solution process that determines width combinations while satisfying an order set of reels and sheets that must be cut from master reels. Because of the problem's combinatorial nature, a solution method based on the enumeration of the cutting patterns has been used. The patterns that satisfy most of the constraints are selected and used as columns in a linear programming formulation that determines the approximate weight of paper to produce each pattern.

Tiwari & Chakraborti (2006) studied optimizing the layout of rectangular parts placed on a rectangular sheet to cut out various parts. They investigated cutting problems in which guillotine cutting is required, and the ones in which guillotine cutting is not essential where the sheets to be cut can be laid on top of one another or side by side and one single cut can be made. They tried to minimize the total required number of cuts and the length of the main sheet. They used a tree-encoded multi-objective genetic algorithm to solve both guillotine and non-guillotine cutting cases and obtained globally optimum solutions.

Alvarez-Valdes, Parreño, & Tamarit (2007) developed a heuristic algorithm based on the tabu search algorithm for a non-guillotine two-dimensional cutting stock problem. Their objective is to maximize the total value of the pieces cut by cutting rectangular pieces from a large stock rectangle. They developed two moves, based on the reduction and insertion of blocks of pieces, and the moves match the empty rectangles with the pieces that will be cut. They tested the algorithm through test instances from the literature and proved the efficiency of the algorithm.

Arbib & Marinelli (2007) developed a heuristic procedure to solve assortment, and trim loss minimization for glass cutting problem in an Italian company that produces parts for the automotive industry. The problem can be considered as an assortment problem since the feasible cutting patterns in the case consist of only one part type. In the plant, a two-phased glass cutting is organized: first the main rectangular glass sheet is obtained from a ribbon of flat glass, then the required small rectangular pieces (products) are cutting from this large-sized glass sheet according to demand. So, the

main problems are assortment of the large sheets to fulfill product demands and minimizing the total trim loss. The heuristic algorithm they developed is based on the p-median problem and considers additional constraints that are relevant to shop floor requirements. A computational study was conducted on real instances provided by the plant, and the tests showed that correct and consistent decisions can be made with a very small computational effort thanks to the developed method.

Gasimov, Sipahioglu, & Sarac (2007) studied a 1.5-dimensional cutting stock and assortment problem in the production of corrugated boxes, which includes the determination of the number of different widths of roll stocks to be maintained as inventory and the determination of how these roll stocks should be cut by choosing the optimal cutting pattern combinations. They proposed a multi-objective mixed integer linear programming model to minimize trim loss and inventory cost simultaneously.

Rodríguez & Vecchietti (2007) developed a two-step model to solve the production planning and cutting stock problems of the corrugated board boxes production company. First, a cutting pattern generation algorithm is executed, and the solutions generated by this algorithm are used as input for the developed MILP model. Their objective was to minimize the trim loss costs. By developing this two-step algorithm, they were able to avoid the usage of additional linearization techniques to cope with the non-convex non-linear constraints. They also linked their algorithm to the company's ERP system to optimize its production process.

Chauhan, Martel, & D'Amour (2008) considered the assortment problem as roll assortment optimization. They developed a binary non-linear programming model to decide the parent roll assortment and assignments to finished products based on demand processes, desired service levels, trim loss, and inventory holding costs. They also developed two solution methods: a branch and price algorithm based on the column generation method and a fast-pricing heuristic, and a marginal cost heuristic.

Rodríguez & Vecchietti (2008) also handled the assignment of paper width to cut patterns according to the available paper stock in addition to the trim loss problem.

They developed a non-convex MINLP model formulation and applied some transformation and linearization techniques to solve the developed model. They concluded that these techniques couldn't be applied to real-world enterprise optimization problems, because the computational complexity and run time are increasing with the problem size. Therefore, they developed a two-stage model again for eliminating original non-convexities and obtaining a more robust and faster problem-solving. They tested the model with the data provided by the factory and again, they linked their model with the company's ERP system.

Arbib & Marinelli (2009) considered the assortment problem as the p-median problem. Their model chooses a set of resource types and assigns these resources to products to fulfill a demand vector at a minimum cost. They also developed a branch-and-price scheme that can either be used to find optimal solutions or tuned by choosing columns in a suitable class to get approximate solutions.

Gramani, França, & Arenales (2009) developed a mixed-integer mathematical model with a Lagrangian relaxation-based heuristic solution algorithm to solve the combined lot-sizing and cutting stock problem. They used a decomposition heuristic to compare the results of the Lagrangian heuristic. The decomposition heuristic first solves the lot-sizing problem, and then for each planning period, it solves the cutting stock problem. They have demonstrated that the Lagrangian heuristic can give lower-cost production plans.

Matsumoto, Miwa, & Ibaraki (2009) developed a multi-objective integer programming model and a heuristic algorithm to solve the problem of scheduling a given set of orders on the corrugator machine. They tried to minimize the total length of roll papers, total paper loss and the number of machine stops. For the solution, they used integer programming to assign all orders to standard breadths, then used the algorithm for sequencing the orders assigned to each standard breadth. By the assignment, they determined the total length of roll papers and the total loss of papers caused by the differences between the standard breadths and real breadths of the orders. The sequence of orders for each standard breadth is closely related to the

production efficiency measured by the number of machine stops incurred during production. They tested their algorithm on real-world problems.

Tombus, Ertek, Atay, Kokten, & Cilacı Tombus (2010) developed a network flow model to solve the problem of selecting the optimal set of roll types for use in production in the packaging industry. They considered the problem as the shortest path problem. They also demonstrated the trade-off between inventory cost and the cost of waste with a graphical demonstration.

Kasimbeyli, Sarac, & Kasimbeyli (2011) considered a one-dimensional cutting stock and assortment problem. The purpose of their study is to develop a mathematical model without the use of cutting patterns as model parameters. They proposed a two-objective linear integer programming model in the form of simultaneous minimization of two contradicting objectives related to the total trim loss amount and the total number of different lengths of stock rolls to be maintained as inventory. They also suggested a heuristic algorithm for solving the presented model.

Rodríguez & Vecchietti (2013) developed two MINLP models that integrate the cutting plan and scheduling problems to minimize trim loss cost in corrugated board box production. They underlined that the integration of scheduling and determining the cutting plan is necessary due to cutting decisions affect some scheduling variables such as setup times and pattern processing. They used a disjunctive approach to transform their formulations into MILP models. The main difference between the models is that one considers immediate precedence relationships for sequencing while the other considers general precedence variables. They tested the models by solving three examples and compared the results in their study.

Kim, Kim & Cho (2014) studied the two-stage two-dimensional cutting stock problem for a paper mill company. They developed a mixed integer linear programming model that represents the constraints that the factory faces and proposed a knapsack-based heuristic algorithm to solve the problem. They tested their model and algorithm by using the test instances generated from the real-world data provided

by the paper mill company. The results indicate that the heuristic algorithm can generate effective solutions that near the optimal solutions provided by the model within a short period, while the model fails to give optimal solutions for large test instances within a given time.

Ayadi, Masmoudi, Ben Ameura, & Masmoudi (2017), proposed an approach based on particle swarm optimization, for the two-dimensional non-guillotine non-oriented cutting stock problem. The algorithm cuts small rectangular pieces of predetermined sizes from large, finite rectangular plates. The algorithm fulfills customer orders by generating cutting patterns and minimizes the unused area. The algorithm was tested on different test instances and the results show that the algorithm performs well, minimizes the trim loss better, between 2.6% and 7.8% for all instances.

Mosallaeipour, Nazerian, & Ghadirinejad (2017) proposed a two-phase optimization approach for the integrated material selection and cutting problem in the corrugated box production industry. The proposed method reduces the problem size, so that, it is more efficient to use for real-life applications. The approach minimizes waste and production costs while maximizing material selection efficiency. It has been tested through the data of the factory and compared to the company's traditional method.

Mosallaeipour, Mahmoodirad, Niroomand, & Vizvari (2018) developed a fuzzy multi-objective mixed integer mathematical programming model to minimize raw material waste, cost, and product surplus. To show market uncertainty, they considered some of the parameters as fuzzy numbers, i.e., product demand, and raw material prices. They tried to determine the dimensions, quantities, and suppliers of the purchased raw materials. They applied a possibilistic approach to convert the fuzzy model to its crisp equivalent form and tested their approach through a case study.

Campello, Ghidini, Ayres & Oliveira (2019) developed a multi-objective linear programming model that combines lot sizing and cutting stock problems. They combined the objective functions that minimize production cost, inventory cost, and

machine setups for lot sizing problem, and material waste and inventory cost of pieces for cutting stock problem. They solved their model by using the weighting and e-constraint method with different test instances and analyzed the Pareto-optimal solutions.

Mellouli, Mellouli, & Masmoudi (2019) discussed the two-dimensional cutting stock problem by considering two objective functions: minimizing trim loss (material waste) and minimizing the number of machine setups. Due to the NP-hard nature of the cutting stock problems they proposed an approximate method. The method combines a linear programming model with a genetic algorithm to determine the optimal Pareto front of these two objectives. The method has been tested through the test instances from the literature.

Niroomand, Mahmoodirad, & Mosallaeipour (2019) developed a fuzzy multi-objective mixed integer linear programming model. The model minimizes waste remaining from raw sheets, purchasing cost of raw sheets under discount policy, and surplus quantity of carton boxes by determining the size and amount of the raw sheets, and suppliers. The product demands and raw material prices are considered fuzzy numbers. The fuzzy problem was converted to a crisp form by using a necessity-based chance-constrained modeling approach. The model is implanted in a real-life case by gathering data from a box production company. The solution approach is compared with multi-objective optimization methods in the literature and the study shows that it reveals better performance and can be used to assist the company in decision-making for the procurement process.

Ferreira, Camargo, & De Araujo (2020) proposed a mixed-integer mathematical programming model to minimize the number of reels used, the partial consumption of reels, the unusable leftovers, and the number of production stoppages by deciding how to allocate reels to the corrugator. They generated instances based on real-world problem data to test their model.

Cerqueira, Aguiar & Marques (2021) proposed a modified constructive greedy search heuristic for the one-dimensional cutting stock problem to minimize the number of cut items. Instead of sorting the item sizes in decreasing order, their algorithm first sorts the pair or odd lengths in decreasing order, which is the more frequent in the problem. Computational tests show that the proposed algorithm performs better and generates fewer cut objects than the greedy heuristic, and the other heuristics that are used for comparison.

Christofoletti, De Araujo & Cherri (2021) deal with the combined lot-sizing and three-dimensional cutting stock problem in their study. They developed a mixed-integer programming model to reduce costs and waste. For computational tests, they collected data from a mattress factory and compared the solutions that the factory has adopted, and the solution proposed by the model. Also, they tested the model with randomly generated data, and the model performs well and reduces waste and costs by increasing the cutting pattern number and possible cutting combinations.

Tang, Liu, Ding, & Wan (2021) developed a mathematical model to solve the solid wood board cutting stock problem, which maximizes the board utilization rate. They proposed an ant colony-immune genetic algorithm to solve this model. The solutions of the ant colony algorithm are used as the initial population of the immune genetic algorithm, and the optimal solution is obtained using the immune genetic algorithm. To test the proposed algorithm, different experiments are designed and compared with the ant colony algorithm, genetic algorithm, grey wolf optimizer, and polar bear optimization. The results show that the proposed algorithm has better optimization ability and robustness.

Finally, the previously mentioned studies are summarized in Table 2.1 according to the problem characteristics, objective functions, and solution methodologies.

Table 2.1 Summary of the literature review

Article	Problem Description	Objective(s)	Solution Methodology
Gilmore & Gomory (1965)	Multi-stage cutting stock problem	Minimize paper waste and maximize machine utilization	LP model and column generation algorithm
Bookbinder & Higginson (1986)	Cutting stock problems with due dates	Maximize customer service level and minimize trim waste	Discrete event simulation model
Christofides & Hadjiconstantinou (1993)	Two-dimensional cutting stock problem	Maximize the value of pieces cut	Tree-search algorithm
Savsar & Cogun (1994)	Cutting stock integrated with machine loading and storage space problems	Minimize total trim loss	LP model
Harjunkoski et al. (1996)	1.5-dimensional cutting stock problem	Minimize the total cost for trim loss and cutting time	INLP, ILP, MILP
Wäscher & Gau (1996)	One-dimensional cutting stock problem	Minimize the total number of standard lengths to be cut	Heuristic approaches
Lai & Chan (1997)	Two- and three-dimensional cutting stock problem	Minimize the amount of trim loss	SA algorithm
Westerlund et al. (1998)	Two-dimensional cutting stock problem	Minimize the total cost for trim loss and cutting time	MILP
Östermark (1998)	One-dimensional cutting stock problem	Minimize the trim loss	GHA
Bolat (2000)	Combined cutting stock problem with buffer utilization and machine loading problems	Maximize the total length of corrugated papers that used to convert to final products in one shift	Decomposition algorithm for LP relaxations
Alvarez-Valdés et al. (2002)	Two-dimensional cutting stock problem	Maximize the total value of the pieces cut	Tabu search algorithm and GRASP algorithm
Holthaus (2003)	One-dimensional assortment problem	Minimize total material cost	ILP
Umetani et al. (2003)	One-dimensional cutting stock problem	Minimize the number of different cutting patterns	Iterated local search algorithm with adaptive pattern generation
Correia et al. (2004)	Production planning and cutting optimization of reels and sheets	Minimize waste while satisfying production order	Three-stage solution approach that includes LP and heuristics

Table 2.1 Continues

Article	Problem Description	Objective(s)	Solution Methodology
Tiwari & Chakraborti (2006)	Two-dimensional cutting stock problem	Minimize the length of the main sheet and the total number of cuts required	Multi-objective genetic algorithm
Alvarez-Valdés et al. (2007)	Two-dimensional non-guillotine cutting problem	Maximize the total value of the pieces cut	Tabu search algorithm
Arbib & Marinelli (2007)	Cutting stock and assortment problem	Minimize total trim loss	P-median model-based heuristic algorithm
Gasimov et al. (2007)	1.5-dimensional cutting stock and assortment problem	Minimize total trim loss and inventory cost	Multi-objective MILP model
Rodríguez & Vecchietti (2007)	Production planning and cutting stock problem	Minimize the trim loss cost	A two-step solution strategy including a MILP model
Chauhan et al. (2008)	1.5-dimensional assortment problem	Minimize inventory holding and trim loss cost	INLP
Rodríguez & Vecchietti (2008)	Non-convex cutting stock and assignment problem	Minimize the weight and cost of the trim loss, changing pattern, and production costs	MINLP model, linearization techniques
Arbib & Marinelli (2009)	Two-dimensional cutting stock and assortment problem	Minimize total cost of used resources	IP
Gramani et al. (2009)	Combined lot-sizing and cutting stock problem	Minimize the total cost of production, inventory, setup, trim loss	Mixed-integer mathematical model and Lagrangian relaxation-based heuristic
Matsumoto et al. (2009)	Scheduling problem in corrugated paper production	Minimizing the total length of roll papers, total loss of papers, and the number of machine stops	IP and heuristic sequencing algorithms
Tombus et al. (2010)	Two-dimensional assortment problem	Minimize total setup and waste cost	LP
Kasimbeyli et al. (2011)	One-dimensional assortment problem	Minimize total trim loss amount and total number of roll types	Multi-objective ILP
Rodríguez & Vecchietti (2013)	Integrated cutting stock and scheduling problem	Minimize total trim loss	MINLP and a disjunctive linearization technique

Table 2.1 Continues

Article	Problem Description	Objective(s)	Solution Methodology
Kim et al. (2014)	Two-stage two-dimensional cutting stock problem	Minimize over- and underproduction and the waste weights, the number of patterns, and auxiliary rolls	MILP model and multiple-choice knapsack-based heuristic algorithm
Ayadi et al. (2017)	Two-dimensional non-guillotine non-oriented cutting stock problem	Minimize total trim loss	PSO-based metaheuristic approach
Mosallaeipour et al. (2017)	Integrated material selection and cutting stock problem	Minimize production and waste costs, maximize material selection efficiency	A two-phase optimization approach
Mosallaeipour et al. (2018)	Material selection, sizing, and supplier selection problem	Minimize raw material waste, cost, and product surplus	MILP model and a fuzzy possibilistic multi-criteria approach
Campello et al. (2019)	Integrated lot sizing and cutting stock problem	Minimize material waste and storage costs	Multi-objective LP model
Mellouli et al. (2019)	Two-dimensional cutting stock problem	Minimize trim loss and the number of machine setups	Genetic algorithm combined with an LP model
Niroomand et al. (2019)	Integrated material and supplier selection problem	Minimize purchasing and waste costs, and quantity of surplus	Fuzzy multi-objective MILP model with a chance-constrained transformation method
Ferreira et al. (2020)	Reel allocation problem	Minimize unusable leftovers, the number of partially used reels, and the number of machine stoppages	Mixed-integer mathematical programming model
Cerqueira et al. (2021)	One-dimensional cutting stock problem	Minimize the number of cut items	Modified constructive greedy search heuristic
Christofoletti et al. (2021)	Integrated lot-sizing and cutting stock problem	Minimize the cost of purchasing, holding, setup, and cutting	Mixed-integer mathematical programming model
Tang et al. (2021)	One-dimensional cutting stock problem	Maximize the board utilization rate	LP model and ant colony-immune genetic algorithm
The present paper	A case-oriented integrated stock selection and sizing problem	Minimize waste and purchasing costs	A matheuristic approach combines a MILP model and the SA algorithm

CHAPTER THREE
PROBLEM DESCRIPTION

Many enterprises that produce corrugated cardboard boxes do not have an entire production line, starting from cutting the raw sheets from reels to producing the final products. Hence, the corrugated box manufacturers must supply their raw materials, i.e., corrugated cardboard, from their suppliers. This emerges the stock selection and sizing problem, since the manufacturers have to obtain maximum efficiency from the raw materials they use, because the waste during the manufacturing process may cause significant financial losses. The manufacturers have to determine the sizes and the purchasing quantities of the corrugated cardboards optimally, to minimize the costs that occur during raw material supply. Even if the manufacturers can produce different sizes of corrugated boxes, the corrugated cardboards they stock must be limited due to different constraints, such as high costs, limited storage, the shelf life of the sheets, etc. In other words, the raw material stock cannot be infinite or equal to the product number. For this reason, an integrated stock selection and sizing problem will arise to produce requested corrugated boxes by using a limited number of corrugated cardboards.

In this thesis, the integrated stock selecting and sizing problem of a corrugated box manufacturing company in Manisa, Turkey is examined and defined as a MINLP model. The real-life constraint of the factory is considered in this model. Because of that, firstly, a brief company introduction is given to explain the production process (from http://www.ekambalaj.com.tr/index.html), and after that, the MINLP model is defined. The nomenclature of the model is defined, and the objective function and the constraint are explained thoroughly.

3.1 Company Introduction

In this thesis, the production process of Ek Ambalaj Label and Box Industry and Trade Inc. is examined. The company was established in Izmir in 1999 to produce corrugated cardboard boxes. Due to the increased business volume, it was moved to

Turgutlu Organized Industrial Zone, Manisa in 2012. After that, the company started to produce adhesive labels.



Figure 3.1 Exterior of the company

The company produces corrugated cardboard boxes in many shapes and sizes for various products and industries, i.e., the fast food industry, cleaning products, glass and ceramic products, agriculture and nourishment products, and textile products. Corrugated cardboards are used as raw material in corrugated box production and are formed of several layers. The type, weight, and other features of the outer, inner, and intermediate papers that will form the corrugated cardboard are of great importance because they are directly related to the box's performance. The success of the produced packaging depends on the supply and the quality of the paper required by the market and its proper use.



Figure 3.2 Some of the corrugated boxes produced by the company

The general manufacturing processes of the company is summarized in Figure 3.3

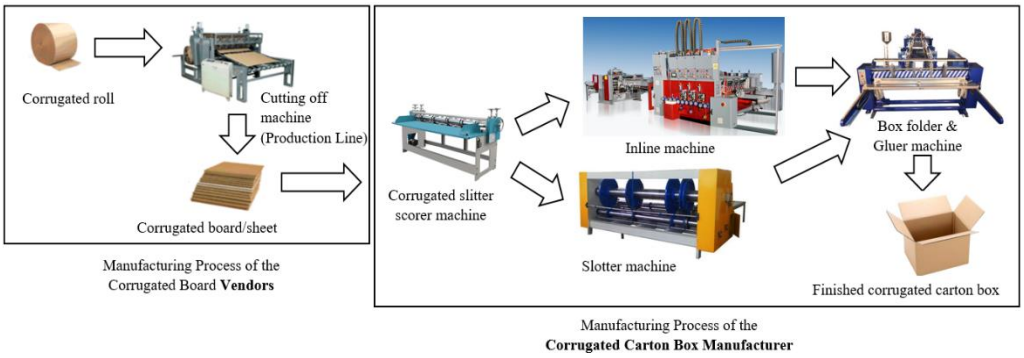


Figure 3.3 General production processes of the company

The main production flow is as follows: First, the logos or any other patterns required are printed on the corrugated cardboards in the printer machine. After printing, the slitter machine cuts the cardboards into the proper dimensions to produce the requested product and also cuts for any other purposes (for handle, vent, etc.) that are requested. After that, the fold lines and the flaps that allow folding are formed. From this point, the operation to be done depends on the product. Some products can be folded automatically and glued into the final product, on the other hand, some of them can be left unglued and unfolded to let the folding be done manually (for example for pizza boxes). After all of these steps, the final product can be sent to the packaging area, to be packed and sent to the customers.

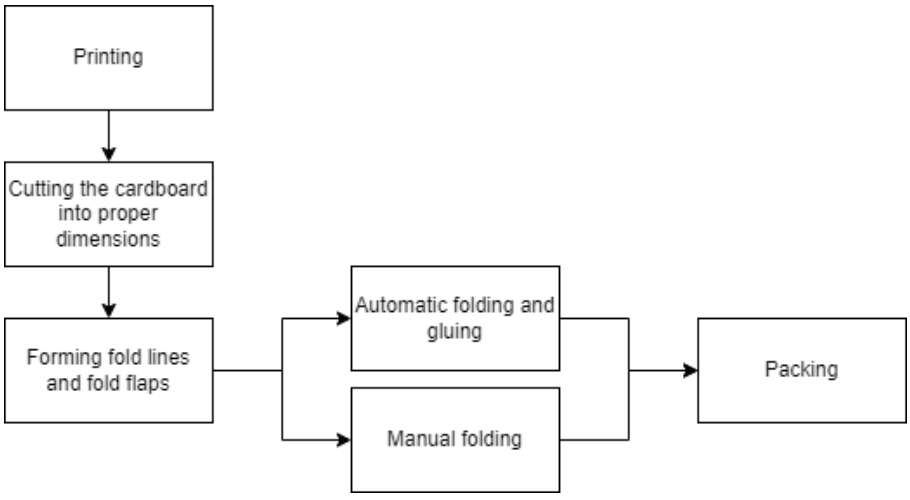


Figure 3.4 The main manufacturing process of the company

In this thesis, the towel radiator box production process of the company is considered. The company makes production for one of the biggest customers'

production on a specific line and does not produce any other customer's products, due to the high volume of production. All of the constraints are modeled according to the company's production process and limitations. The company has 236 different cardboard dimensions (raw material) to produce corrugated boxes to meet customer demands. The customer requests are forwarded to the company every week, and an average of 30 to 40 different raw materials are generally required to produce these demands. An illustration of towel radiators with different sizes and the relevant product (towel radiator boxes) of the manufacturing company is given in Figure 3.5.



Figure 3.5 Illustration of towel radiators with different sizes and the related corrugated boxes

The company determines the purchased corrugated cardboard dimensions heuristically according to the demands, with the experience gained over the years. But also, it's known by the management of the company that better purchasing plans can be implemented to minimize waste, purchasing, and storage costs of the raw materials, and the stock variety can be reduced. In order to present a better purchasing plan to the company, a MINLP model is developed, and presented in this thesis. The model provides optimal stock selection and sizing of the right number of corrugated cardboards to be purchased.

3.2 The Developed MINLP Model

The model determines the quantity, width, and length of the corrugated cardboards that will be purchased and matches the corrugated cardboards with corrugated boxes, i.e. decides which product will be produced by which raw material. The developed

MINLP model is formulated through Equations (3.1) - (3.20), by using the nomenclature defined in Table 3.1.

Table 3.1 Mathematical nomenclature including indices, sets, parameters, and decision variables

Indices &Sets	
$i \in I$	Set of finished corrugated cardboard boxes (i.e., products)
$j \in J$	Set of corrugated cardboards (i.e., raw sheets)
Parameters	
DE_i	Weekly demand of the corrugated box *i*
WP_i	Width of the corrugated box *i*
LP_i	Length of the corrugated box *i*
HP_i	Height of the corrugated box *i*
OW_i	Minimum width of the corrugated cardboard required to produce corrugated box *i* with zero waste
OL_i	Minimum length of the corrugated cardboard required to produce corrugated box *i* with zero waste
OC_i	Production cost of the corrugated box *i* with zero-waste
MW	Maximum width of any corrugated cardboard that is pre-specified
ML	Maximum length of any corrugated cardboard that is pre-specified
MTA	Minimum total area for each size of the purchased corrugated cardboards
c	Unit purchasing cost of corrugated cardboard per square meter
FA	A constant value for fold allowance in corrugated box production
AM	A constant value for adhesive margin for gluing or stapling in corrugated box production
M	An arbitrarily chosen big positive number
Decision variables	
WR_j	Width of the corrugated cardboard *j*
LR_j	Length of the corrugated cardboard *j*
PC_{ij}	Unit actual production cost of the corrugated box *i* in case of using the corrugated cardboard *j*
OB_{ij}	Quantity of the corrugated box *i* that can be obtained from the usage of one unit of the corrugated cardboard *j*
WC_{ij}	Unit waste cost of the corrugated box *i* in case of using the corrugated cardboard *j*
U_{ij}	1, if the corrugated cardboard *j* is used to produce the corrugated box *i*; 0 otherwise
PUR_j	Purchasing order quantity of the corrugated cardboard *j*
Q_{ij}	The required quantity of corrugated cardboard *j* to produce the corrugated box *i*
Auxiliary binary variables for the IF-THEN constraints	
α_j	Auxiliary binary variable for purchasing quantity of the corrugated cardboards
β_{ij}	Auxiliary binary variable for combining two corrugated cardboards for the large-sized corrugated boxes
δ_{ij}	Auxiliary binary variable for a logical if-then constraint related to the usage of corrugated cardboards in production

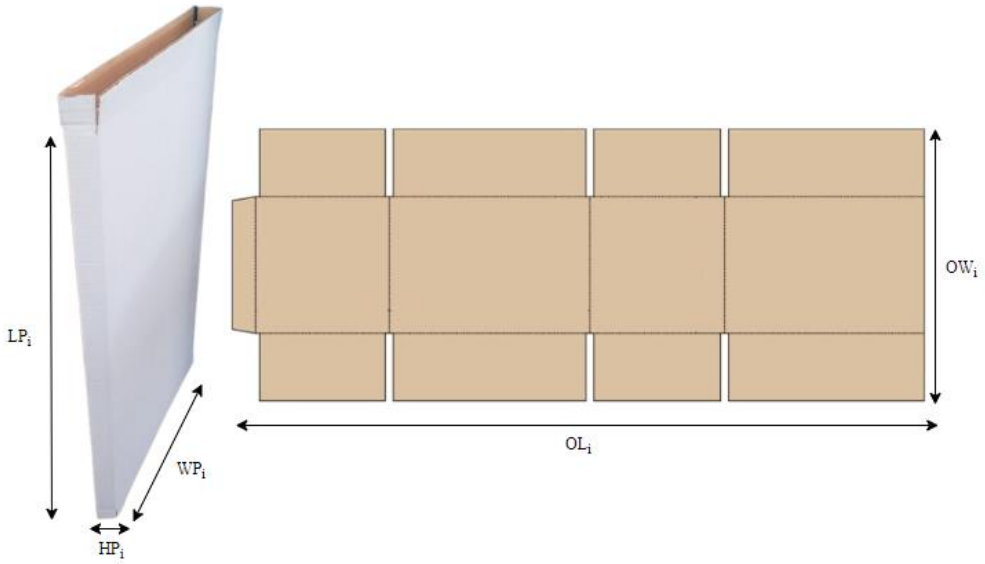


Figure 3.6 Corrugated box dimensions with corrugated cardboard dimensions for zero waste

The parameters related to the corrugated box dimensions are illustrated in Figure 3.6. In order to summarize the inputs, outputs, and the objective of the developed model, Figure 3.7 is given.

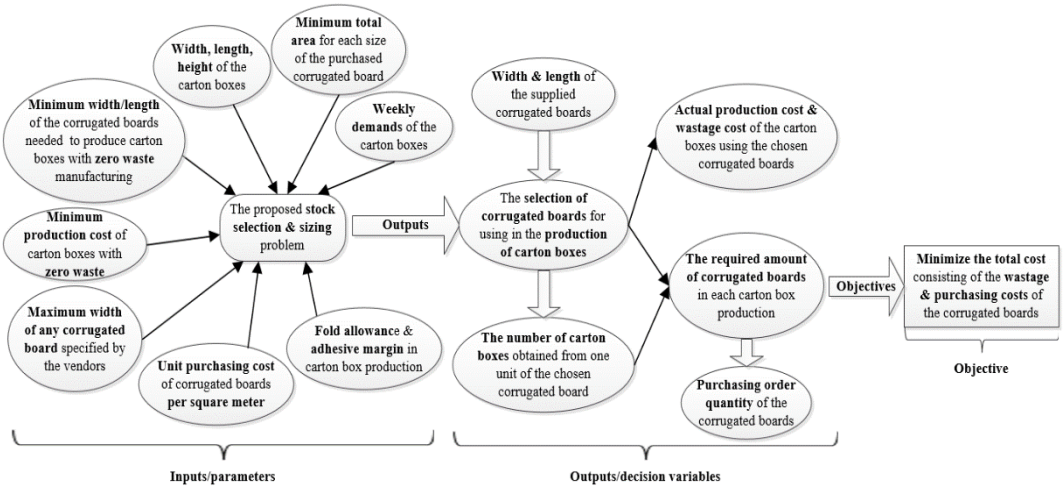


Figure 3.7 Summary of the inputs, outputs, and objective of the proposed problem

After the company receives the customer demands of corrugated boxes (i.e., DE_i) with different sizes (i.e., WP_i, LP_i, and HP_i), it can calculate the width and length (OW_i and OL_i) of corrugated cardboards that required to produce the products with zero waste. For this calculation, Equations (3.1) and (3.2) can be used. These equations calculate the width and length of the corrugated cardboards by using the fold allowance

(i.e., FA), and the adhesive margin (i.e., AM) for gluing or stapling in addition to the widths, lengths, and heights of the corrugated boxes. Equation (3.3) can be used to calculate the production cost of the corrugated boxes with zero waste (i.e., OC_i), by calculating the area of the corrugated cardboards that will be used and multiplying it by its unit purchasing cost per square meter (i.e., c).

The suppliers of the company limit the dimensions of the corrugated cardboards (i.e., MW and ML). Also, there are price agreements according to the MOQ (in this case minimum total area (i.e., MTA)) between the vendors and the manufacturing company, that must be considered for purchasing. The company must consider these constraints to determine optimal sizes (i.e., WR_j and LR_j) of the purchased corrugated cardboards.

The waste cost (i.e., WC_{ij}) of the corrugated cardboards can be minimized by selecting the corrugated cardboard with optimal dimensions (i.e., U_{ij}). This selection also minimizes the production cost (i.e., PC_{ij}). After this selection, the number of corrugated boxes that can be obtained from using one unit of the chosen corrugated cardboard (i.e., OB_{ij}) can be determined.

If the minimum width of a corrugated cardboard required to produce a corrugated box with zero waste is greater than the selected corrugated cardboard ($OW_i > MW$), then the number of corrugated boxes that can be obtained from that cardboard will be zero (i.e., $OB_{ij} = 0$). If a corrugated box cannot be obtained from any cardboard, then two pieces of selected cardboard must be combined to produce that corrugated box. Also, the minimum length of corrugated cardboard required to produce a corrugated box with zero waste is greater than the length of the selected cardboard ($LR_j < OL_i$), and that cardboard cannot be used to produce the corrugated box. The required quantity of the cardboard (i.e., Q_{ij}) can be calculated by using the demand and the number of corrugated boxes that can be produced by the selected cardboard. The company also must decide on the purchasing quantities of the corrugated cardboards (i.e., PUR_j) considering the required quantities to make production.

In the mathematical model, the minimum width, length, and production cost of the corrugated cardboards required to produce corrugated boxes with zero waste are calculated by Equations (3.1) - (3.3). As mentioned before, the fold allowance and the adhesive margin are used to calculate minimum width and lengths. To calculate the zero-waste production cost of a corrugated box, the area of the cardboard is calculated and multiplied by the unit purchasing cost in Equation (3.3). It should be noted here that since the corrugated box dimensions are expressed in millimeters instead of meters, the resulting cost value in Equation (3.3) is divided by a conversion constant, i.e., 1000000. Similarly, this measurement conversion is also realized for the objective function in Equation (3.4), and some constraints such as in Equations (3.5), (3.15), and (3.15.b).

$$OW_i = 2 \times \big((WP_i + FA) + (HP_i + FA)\big) + AM \quad \forall i \in I \tag{3.1}$$

$$OL_i = 2 \times (HP_i + FA) + (LP_i + FA) \quad \forall i \in I \tag{3.2}$$

$$OC_i = \frac{OW_i \times OL_i \times c}{1000000} \quad \forall i \in I \tag{3.3}$$

The objective function and the constraints of the model are formulated below.

$$Minimize \sum_{i \in I} \sum_{j \in J} WC_{ij} \times Q_{ij} + \sum_{j \in J} PUR_j \times c \times \left(\frac{WR_j \times LR_j}{1000000}\right) \tag{3.4}$$

Subject to

$$\begin{cases} PC_{ij} = M \quad \forall i \in I, j \in J \,|\, OW_i \le MW \;\&\; OB_{ij} = 0 \\ PC_{ij} = {WR_j \times LR_j \times c}\big/{\left(1000000 \times OB_{ij}\right)} \quad \forall i \in I, j \in J \;\Big|\; \begin{matrix} OW_i \le MW \\ OB_{ij} \ge 1 \end{matrix} \\ PC_{ij} = {2 \times WR_j \times LR_j \times c}\big/{1000000} \quad \forall i \in I, j \in J \,|\, OW_i > MW \end{cases} \tag{3.5}$$

$$WC_{ij} = PC_{ij} - OC_i \quad \forall i \in I, j \in J \tag{3.6}$$

$$\sum_{j \in J} U_{ij} \ge 1 \quad \forall i \in I \tag{3.7}$$

$$OB_{ij} = int\left\lfloor \frac{WR_j}{OW_i} \right\rfloor \times int\left\lfloor \frac{LR_j}{OL_i} \right\rfloor \quad \forall i \in I, j \in J \tag{3.8}$$

$$Q_{ij} = U_{ij} \times int\left[\frac{DE_i}{OB_{ij}}\right] \quad \forall i \in I, j \in J \,|OW_i \leq MW \;\&\; OB_{ij} \geq 1 \tag{3.9}$$

$$Q_{ij} \geq 2 \times U_{ij} \times DE_i \quad \forall i \in I, j \in J \,|OW_i > MW \tag{3.10}$$

$$If\ U_{ij} > 0, Then\ \left(2 \times U_{ij} \times WR_j\right) \geq OW_i \quad \forall i \in I, j \in J \,|OW_i > MW \tag{3.11}$$

$$U_{ij} \leq \left(1 - \beta_{ij}\right) \times M \quad \forall i \in I, j \in J \,|OW_i > MW \tag{3.11.a}$$

$$OW_i - \left(2 \times U_{ij} \times WR_j\right) \leq \beta_{ij} \times M \quad \forall i \in I, j \in J \,|OW_i > MW \tag{3.11.b}$$

$$U_{ij} \leq 0 \quad \forall i \in I, j \in J \,|OW_i \leq MW \;\&\; OW_i > WR_j \tag{3.12}$$

$$Q_{ij} \leq U_{ij} \times M \quad \forall i \in I, j \in J \tag{3.13}$$

$$PUR_j \geq \sum_{i \in I} Q_{ij} \quad j \in J \tag{3.14}$$

$$If\ PUR_j > 0,\ Then\ PUR_j \times \left(\frac{WR_j \times LR_j}{1000000}\right) \geq MTA \quad \forall j \in J \tag{3.15}$$

$$PUR_j \leq \alpha_j \times M \quad \forall j \in J \tag{3.15.a}$$

$$MTA - PUR_j \times \left(\frac{WR_j \times LR_j}{1000000}\right) \leq \left(1 - \alpha_j\right) \times M \quad \forall j \in J \tag{3.15.b}$$

$$PUR_j \leq M \times \sum_{i \in I} U_{ij} \quad \forall j \in J \tag{3.16}$$

$$U_{ij} \times OL_i \leq LR_j \quad \forall i \in I, j \in J \tag{3.17}$$

$$WR_j, LR_j, PC_{ij}, WC_{ij} \geq 0 \quad \forall i \in I, j \in J \tag{3.18}$$

$$OB_{ij}, PUR_j, Q_{ij} \geq 0\ and\ integer \quad \forall i \in I, j \in J \tag{3.19}$$

$$U_{ij}, \alpha_j, \beta_{ij}, \delta_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \tag{3.20}$$

The objective function, Equation (3.4), minimizes the total cost that combines the purchasing and waste costs of the corrugated cardboards. The unit waste cost is calculated by Equation (3.6), by subtracting the zero waste production cost (OC_i) from the actual production cost (PC_{ij}). The total waste cost in the objective function is calculated by multiplying this unit waste cost value (i.e., WC_{ij}) with the required quantity of chosen corrugated cardboard (i.e., Q_{ij}). The total purchasing cost in the objective function is calculated by multiplying the purchasing quantity of the cardboard (i.e., PUR_j) by the unit purchasing cost (i.e., c) and the dimensions of the corrugated cardboard (i.e., WR_j and LR_j). The multiplication of the dimensions, the purchasing quantity of corrugated cardboard, and unit production cost, which are

decision variables, present the highly non-linear non-convex structure of the objective function. Unfortunately, the multiplication of two continuous variables cannot be linearized exactly or directly without any approximation due to the non-convex area calculation for the purchased two-dimensional corrugated cardboard (Konak, Kulturel-Konak, Norman, & Smith, 2006; Subulan, Varol, & Baykasoğlu, 2023). Also, the non-linear constraints (i.e., (3.5), (3.8), (3.9), (3.11.b), (3.12), (3.15.b)) makes it much more difficult to solve the proposed MINLP model.

Equation (3.5) calculates the unit actual production cost of a corrugated box by using specified corrugated cardboard (i.e., PC_{ij}). This constraint consists of three different conditions. According to the first part, if a corrugated box cannot be produced by a specified cardboard even if the width required to produce a box with zero waste is less than or equal to the pre-defined maximum width, then the actual production cost will be equal to a very big positive value ($OW_i \leq MW \;\&\; OB_{ij} = 0$). If the width required to produce a box with zero waste is less than or equal to the pre-defined maximum width, and the requested box can be produced by using the specified cardboard ($OB_{ij} \geq 1$), the actual production cost can be calculated by the second part of the constraint. Finally, if the width required to produce a box with zero waste is greater than the pre-defined maximum width ($OW_i > MW$), which means that if a corrugated box cannot be produced only one corrugated cardboard, then two cardboards must be combined, and the actual production cost is multiplied by two and also not divided by the number of corrugated boxes that can be obtained from using one unit of the chosen corrugated cardboard. Again, due to the multiplication of two continuous decision variables (i.e., WR_j and LR_j) as well as the division by a positive integer variable (i.e., OB_{ij}), this constraint set is also highly non-linear.

As mentioned earlier, Equation (3.6) calculates the unit waste cost that occurs in the production of a corrugated box. Equation (3.7) ensures that at least one type of corrugated cardboard is used to produce each corrugated box. By using this constraint, the production of each requested corrugated box is guaranteed. Equation (3.8) is used to calculate the number of a specified corrugated box that can be obtained from a cardboard. To calculate this, the width and length of cardboards are divided by the zero

waste dimensions and multiplied. Since the dimensions of the cardboards are decision variables, this constraint is also non-linear. Furthermore, the *int* function is utilized to round the largest integer values that are not greater than the $\frac{WR_j}{OW_i}$ and $\frac{LR_j}{OL_i}$ values, respectively.

Equation (3.9) calculates the required quantity of cardboards to produce a specified corrugated box (i.e., Q_{ij}). To do this, the demand of the specified corrugated box is divided by the number of corrugated boxes that can be obtained by using the chosen corrugated cardboard. This constraint can be used only if the width required to produce a box with zero waste is less than or equal to the pre-defined maximum width, and the requested box can be produced by using the specified cardboard. If not, the required quantity of cardboards can be calculated by Equation (3.10), by doubling the demand of corrugated boxes. In this case, to produce the specified product, two corrugated cardboards are combined. It is assumed that the same two cardboards must be used for the combining, and this process is formulated by Equation (3.11). Since the constraint consists of an if-then statement, it is also formulated by Equation (3.11.a) and (3.11.b). The constraint presents that if two corrugated cardboards of the same size are chosen to combine, the sum of their widths must be greater than the width of the corrugated box to make production with zero waste. If not, they cannot be used in the production of that corrugated box. Due to the multiplication of two decision variables, this constraint is also non-linear.

Equation (3.12) ensures that a cardboard cannot be used for production of a corrugated box that has a larger width than the cardboard's (i.e., $OW_i > WR_j$)., even if the width required to produce a box with zero waste is less than or equal to the pre-defined maximum width (i.e., $OW_i \leq MW$). This constraint has also a non-linear form because of using a decision variable (i.e., WR_j) in its conditional side. Again, this constraint can be formulated as an if-then constraint, as in Equation (3.21), (3.21.a), and (3.21.b). However, this linearization operation has no substantial impact on the solution of the proposed MINLP model.

$$If\ OW_i > WR_j,\ \ Then\ U_{ij} \leq 0 \quad \forall i \in I, j \in J\ |OW_i \leq MW \tag{3.21}$$

$$U_{ij} \leq \delta_{ij} \times M \quad \forall i \in I, j \in J \,|OW_i \leq MW \tag{3.21.a}$$

$$OW_i - WR_j \leq (1 - \delta_{ij}) \times M \quad \forall i \in I, j \in J \,|OW_i \leq MW \tag{3.21.b}$$

Equation (3.13) ensures that if a cardboard will not be used in the production of a specified corrugated box, then the required quantity of the cardboard for that product will be zero. Equation (3.14) ensures the purchasing quantity of a cardboard will be equal to or greater than the required cardboard quantity to make production, in order to meet production demand. Equation (3.15) presents the MOQ agreement between the suppliers and the production company. If a cardboard will be purchased, the total purchased area cannot be less than the agreed area (i.e., MTA, in square meters). Again, the constraint consists of an if-then statement, it is formulated by the Equations (3.15.a) and (3.15.b).

Equation (3.16) ensures that if a cardboard is not used in the production of any of the products, then the purchasing quantity must be equal to zero. Equation (3.17) ensures that a cardboard can be used in the production of a corrugated box only if the length of the cardboard is equal to or greater than the required length to make production with zero waste. Equation (3.18) defines the lengths and widths of the corrugated cardboard, the unit actual production, and the waste costs of the corrugated boxes as continuous decision variables. Equation (3.19) defines the quantity of obtained corrugated boxes from cardboard, the purchasing quantity of the corrugated cardboard, and required cardboard quantities as positive integer variables. Finally, Equation (3.20) defines the binary decision variables, that is whether a cardboard is used to produce a specified corrugated box, and the auxiliary variables.

It should be noted that the conditions can be added to the constraints, that ensure the cardboard length cannot exceed the pre-defined maximum cardboard length (i.e., $LR_j \leq ML$). But in this real-world case, the products that have longer length than ML have an exceptional production process, which has not been considered. So, in order to not increase the model size, this constraint is also disregarded.

CHAPTER FOUR
THE MATHEURISTIC-BASED SOLUTION APPROACH

As stated in the previous section of this thesis, the developed mathematical model involves non-linear objective function and constraints, and this characteristic of the model makes it nearly impossible to obtain a global optimal solution by using any optimization solvers. Also, due to the type of non-linearity, i.e. multiplication of two continuous decision variables, etc., it's not possible to convert some of the constraints and the second part of the objective function to their linear form without using approximation methods. On the other hand, some of the constraints and the first part of the objective function can be linearized by adding some auxiliary variables and extra constraints, but this increases the model dimension significantly, so providing solutions will cost much more computational time.

All of these reasons above make it necessary to develop an efficient solution approach to provide better solutions for the problem. It's decided to develop a matheuristic solution approach instead of heuristic and metaheuristic approaches, to improve the local search process by solving the subproblem of stock selection a MILP model is integrated into a metaheuristic algorithm, i.e., SA algorithm.

4.1 Matheuristic Algorithms

A matheuristic is the hybridization of mathematical programming with metaheuristics (Fischetti & Fischetti, 2016). It also defined as heuristic algorithms made by the interoperation of metaheuristics and mathematical programming techniques by Boschetti, Maniezzo, Roffilli, & Röhler (2009). Both mathematical modelling and metaheuristic parts have different advantages and disadvantages, and by combining them, it's aimed to make use of the advantages of them. While mathematical modeling can provide a global optimal solution, it's not applicable to every problem. If a problem involves non-linearity and non-convexity, stochastic search techniques will be needed. Also, even if a problem is solvable by exact solution methods, a global optimal solution may not be provided within reasonable computational time. This scenario is valid especially for combinatorial problems, such

as assignment, scheduling, cutting, routing problems, because, with the increased problem size, the search space will be expanded. On the other hand, metaheuristics can be applied to different problems, because they are not problem-specific, but they don't ensure the global optimality of the solution found.

Dumitrescu & Stützle (2009) categorized different combinations of the exact solution methods and metaheuristics:

- The usage of exact algorithms to explore large neighborhoods in metaheuristics,
- Enhancing metaheuristics by solving some subproblems exactly,
- The usage of branch-and-bound techniques to enhance constructive metaheuristics,
- The exploitation of the structure of good solutions identified by metaheuristics, and
- The exploitation of information from relaxations in metaheuristics.

Recently, the usage of matheuristics has become common, and different algorithms have been applied by researches. Lots of papers are examined for the different surveys proposed in the literature about matheuristic algorithms (Archetti & Speranza, 2014; Ball, 2011; Boschetti, Letchford, & Maniezzo, 2023; Doerner & Schmid, 2010; Puchinger & Raidl, 2005).

In this thesis, the combination is used to solve the subproblem of stock selection. By solving the stock selection problem with MILP, the local search process of metaheuristic is improved. The SA algorithm is used since it's not a population-based algorithm. So, the solutions can be obtained in shorter computational times as against the population-based algorithms. The developed matheuristic-based solution approach is explained in detail in Chapter 4.3.

4.2 Simulated Annealing Algorithm

In the SA algorithm that was first introduced by Kirkpatrick, Gelatt, & Vecchi (1983), the analogy of the annealing procedure of metals (i.e., heating, and slow cooling of metals) is considered to guide the stochastic search for reaching an optimal

point. In this study, the algorithm is stated as an adaptation of the Metropolis algorithm developed by Metropolis, Rosenbluth, Rosenbluth, Teller, & Teller (1953). It adopts an iterative movement according to a variable temperature scheme that mimics the annealing transaction of the metals (Eren, Küçükdemiral, & Üstoğlu, 2017; Lee, Graham, Jackson, Galindo, & Adjiman, 2019).

The first step of the algorithm is to generate a random initial solution for the problem and calculate its objective value, i.e., fitness value. After the proper generation of the initial solution, the annealing procedure starts. Until reaching the final temperature of the process, the algorithm searches for better solutions, i.e., solutions that have less or greater objective values according to the problem objective, by generating neighborhood. Neighborhood generation is one of the key steps of the algorithm that must be designed well enough to generate better-quality solutions. In the annealing procedure, if the generated neighbor solution has a better fitness value than the current solution, it's accepted as the new solution directly. The SA employs an intensification scheme in this manner to search the neighbors for the initial solution efficiently (Baykasoğlu & Gindy, 2001). As a diversification scheme, the algorithm also accepts worse solutions if a uniformly distributed random number U is less than the acceptance probability (Equation (4.1)), where Δ is the difference between the current and neighbor solution's objective values. It means that to search all the solution space and not stick with a local optimum solution, the SA algorithm accepts worse solutions with a probability. These intensification and diversification schemes help the SA algorithm to converge global optimal solutions for many complex engineering problems in case of slow temperature reduction (Tsang & Wiese, 2015). The flow chart of the algorithm is given in Figure 4.1.

$$\left(U < e^{-\Delta/T_{el}}\right) \tag{4.1}$$

The algorithm's performance mainly depends on parameters such as the initial temperature (T_0), cooling rate (q), the length of the Markov chain (LMC), and the minimum number of accepted solutions (MNM_{min}) in the inner loop as well as the

maximum number of iterations at the outer loop (el_{max}). All of these parameters must be selected properly to increase the efficiency of the algorithm.

One other aspect of the algorithm is its convergence mechanism. As it happens in the annealing process, the high temperatures cause big changes in the objective function, while at the low temperatures it seems to be stable, and converges one value. Since it is not a population-based algorithm, it can find a solution faster than particle swarm optimization, genetic algorithm, tabu search algortihm, etc., but it may cause low quality search of the solution space. Also, it's generally applied for the problems that has discrete search space.

The SA algorithm is applied to different optimization problems in the literature, and the popularity of the algorithm enables the creation of different variants of it. Also, the cutting stock, stock selection, and sizing problems have been solved via the SA algorithm by many researchers (Chen, Hart, & Tham, 1996; Faina, 1999; Jahromi, Tavakkoli-Moghaddam, Makui, & Shamsi, 2012; Lai & Chan, 1997; Leung, Yung, & Troutt, 2001; Li, Zheng, & Dai, 2007; Sanchez, Vargas, Santos, Mendoza, & Moctezuma, 2018). Also, Baykasoğlu, Subulan, & Hamzadayı (2022), Kulturel-Konak (2019), Kulturel-Konak & Konak (2015), Meira et al. (2020), Ozkan (2021), Sanchez et al. (2018), and Wang, Zuo, Liu, Zhao, & Li (2015) developed matheuristic algorithms in their studies and provided satisfactory solutions by implementing the SA algorithm.

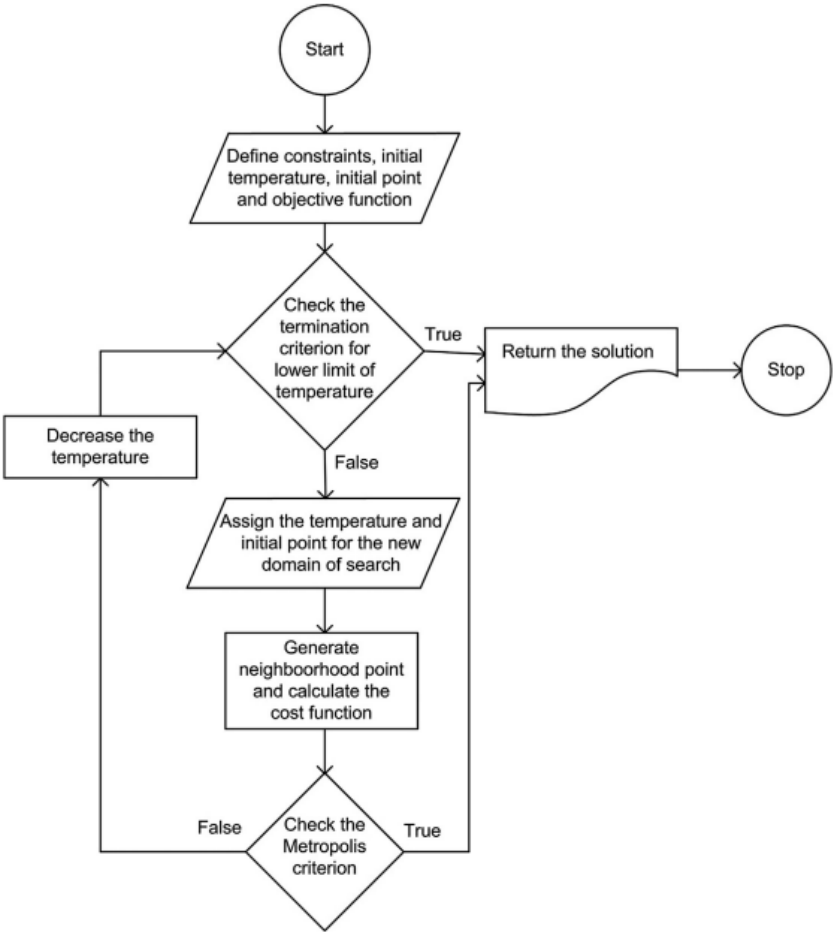


Figure 4.1 The flow chart of the SA algorithm (Eren, Küçükdemiral, & Üstoğlu, 2017)

4.3 The Developed Matheuristic Solution Approach

As explained in Chapter 3, the multiplication of two continuous decision variables cannot be linearized, unlike the other non-linearity constraints of the MINLP model. It means that, if the decision variables of corrugated cardboard dimensions can be considered as parameters, then the MINLP model can be reduced to a MILP model and can be solved by any MIP solver. For that reason, in the developed solution approach, the corrugated cardboard dimensions are provided by the SA algorithm and the selection is conducted by the MILP model that is embedded in the algorithm.

The proposed SA-based matheuristic algorithm is coded in Python 3.8.8 on an Intel Core i7 2.70GHz personal computer by using the callable library of the Gurobi MIP solver. A schematic depiction of the proposed SA-based matheuristic solution approach is given in Figure 4.2.

The SA algorithm provides the dimensions of the corrugated cardboards and sends the dimension to the MILP model as input. By that, the MILP can be solved optimally and sends the resulting objective function value back to the SA algorithm.

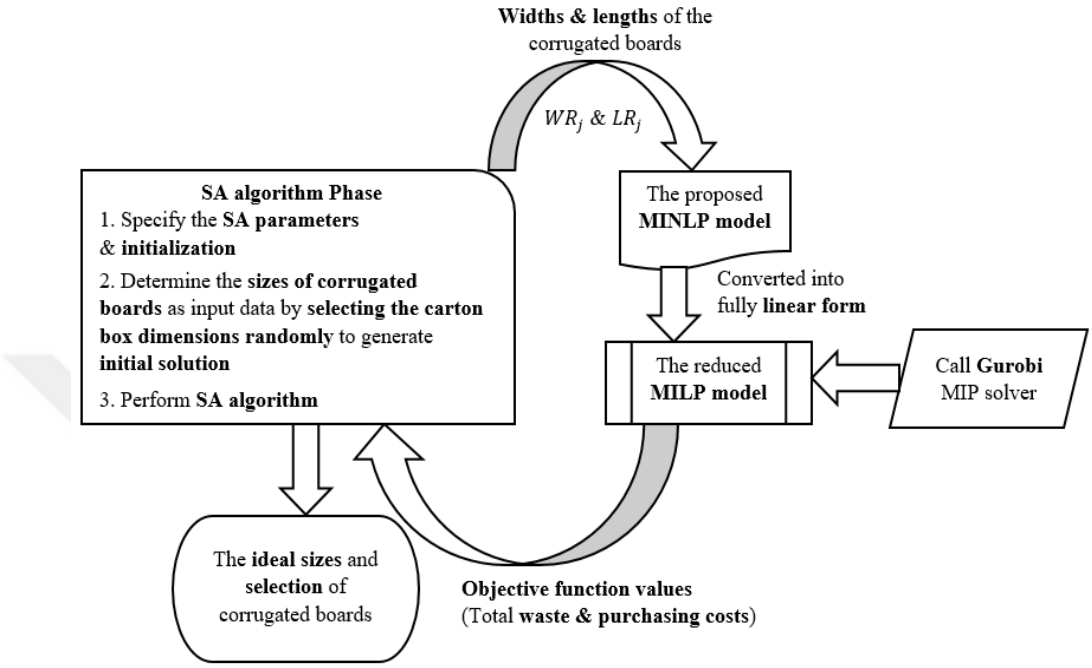


Figure 4.2 The summary of the proposed SA-based matheuristic solution approach

The terms of the proposed solution algorithm are given in Table 4.1. Also, the pseudo-code of it is presented in Figure 4.3.

Since a MILP model is solved in each iteration, it's decided to use a single solution-based metaheuristic technique, i.e., the SA algorithm, to reduce the computational effort, due to the fact that the usage of population-based metaheuristic algorithms may cause excessive computational effort (Baykasoğlu et al., 2022).

4.3.1 The Solution Representation

The solution representation of the proposed matheuristic algorithm is a combination of the matrix and permutation. The solution is composed of the permutation of I, the number of corrugated boxes, and the matrix of their minimum dimensions to produce them with zero waste (OW_i and OL_i). By using such a solution representation, the widths and lengths of the corrugated boards can be determined considering these

values as the solution set, because those values are the dimensions required to make zero waste production.

Table 4.1 The terms of the proposed solution approach

Terms	Definition
BestSolution(.)	The best solution found
CurrentSolution(.)	Current solution whose neighborhood is going to be searched
NeighborSolution(.)	The solution generated from the *CurrentSolution* by using the neighborhood generation strategy
BestCost	Fitness function value of *BestSolution*
CurrentCost	Fitness function value of C*urrentSolution*
NeighborCost	Fitness function value of *NeighborSolution*
P_{acc}	Probability of acceptance
U	A uniformly distributed random number between 0 and 1
T_0, T_{el}	Initial and current temperatures, respectively
q	Cooling rate
il	Inner loop counter
LMC	Length of the Markov chain
MNM	The accepted number of solutions
MNM_{min}	The minimum number of accepted solutions
el_{max}	The maximum number of iterations
el	Iteration counter
SP	Step size parameter: a division factor
NCB	The maximum number of corrugated boards of different sizes
StepSize	Step size used in the proposed neighborhood generation strategy

For instance, if the maximum number of different corrugated board sizes (NCB) is specified as 10 by the corrugated box manufacturer, an illustrative solution representation can be shown in Figure 4.4. In all of the solutions, the first column is fixed with a dummy corrugated board with maximum allowable width and length (MW and ML) values. This dummy corrugated board is not altered by the proposed neighbor generation strategies throughout the search process to generate always feasible solutions at every iteration. If this dummy corrugated board is not used, there may be a scenario that a corrugated box cannot be produced by using any of the corrugated cardboard in the solution, which causes infeasibility. In order to not to increase computational effort only feasible solutions are evaluated. The remaining columns consist of randomly chosen corrugated boxes with their minimum dimensions to produce them with zero waste.

1. **Initialization;** first specify the ***SA* parameters** (T_0, q, MNM_{min}, LMC, el_{max})
Then also specify the maximum number of corrugated boards of different sizes (*NCB*) and the **step size** by using a **division factor (*SP*)**. Calculate the **step size:** *StepSize = NCB / SP*
Initialize the iteration counter: $el = 0$ (Outer loop counter)
2. *CurrentSolution(.)*←Generate an **initial solution** for **widths and lengths** ($\boldsymbol{WR_j}$ & $\boldsymbol{LR_j}$) of the corrugated boards by **randomly selecting the minimum dimensions** ($\boldsymbol{OW_i}$ & $\boldsymbol{OL_i}$) of corrugated boxes with zero-waste manufacturing
3. *CurrentCost*←**Calculate the fitness function value** of *CurrentSolution(.)* by using the reduced **MILP model**
4. *Best solution (.)*←*CurrentSolution(.)*, *BestCost*←*CurrentCost*
5. **For** $el = 1$ to el_{max} **do**
6. $il = 0$ & $MNM = 0$
7. **While** $il < LMC$ & $MNM < MNM_{min}$ **do**
8. Apply the **proposed neighborhood generation strategy** to generate new dimensions for corrugated boards
9. *NeighborSolution(.)* ← *CurrentSolution(.)*
10. *NeighborCost* ← Calculate the fitness value of *NeighborSolution(.)* by using the reduced **MILP model**
11. **If** *CurrentCost > NeighborCost* **Then**
12. *CurrentCost*←*NeighborCost*, *CurrentSolution(.)*←*NeighborSolution(.)*
13. $MNM = MNM + 1$
14. **Else**
15. Δ← *NeighborCost - CurrentCost*
16. P_{acc}←exp(-Δ/T_{el}), U ← Create a random number $\in$[0,1]
17. **If** $U < P_{acc}$ **Then**
18. *CurrentSolution(.)*←*NeighborSolution(.)*, *CurrentCost*←*NeighborCost*
19. $MNM = MNM + 1$
20. **Else** reject the solution
20. **End if**
21. **End if**
22. $il = il + 1$
23. **End while**
24. $T_{el+1} \leftarrow T_{el} \times q$
25. **If** *BestCost > CurrentCost*
26. *BestCost*←*CurrentCost*, *BestSolution(.)*←*CurrentSolution(.)*
27. **End if**
28. $el = el + 1$
29. **End for** terminate the outer loop
30. Report the *BestCost* and *BestSolution*

Figure 4.3 The pseudo-code of the proposed solution approach

The maximum number of corrugated boards of different sizes ($NCB = 10$)

NCB index	0*	1	2	3	4	5	6	7	8	9
Width (WR_j, OW_i)	1750*	1250	1150	1350	1720	1050	1350	1110	1150	1150
Length (LR_j, OL_i)	2000*	1971	805	1905	1935	1571	1105	1925	1331	1705
Index of carton boxes	0*	15	5	22	28	3	17	27	10	12

* Dummy corrugated board with maximum allowable width and length.

Figure 4.4 Illustration of the solution representation for the corrugated board selection and sizing

The last row displays the indexes of these randomly chosen corrugated boxes among all of the corrugated boxes demanded by the customers. It is assumed that a total number of 30 corrugated boxes with different sizes is available. It should be also

noted that if the minimum dimensions of a corrugated box to produce them with zero waste are greater than the maximum allowable width and length values, the dimensions are not involved in the solution because they can cause infeasibility.

The rows between show the dimensions of the corrugated boxes that are required to make zero waste production. These dimensions are also will be selected as the dimensions of the purchased corrugated boards. These two rows can be considered as the matrix part of the solution representation and are sent to the MILP model as input data to calculate the objective function value. The first and the last row are the permutation part and are utilized to generate neighbor solutions.

4.3.2 The Proposed Neighborhood Generation Strategies

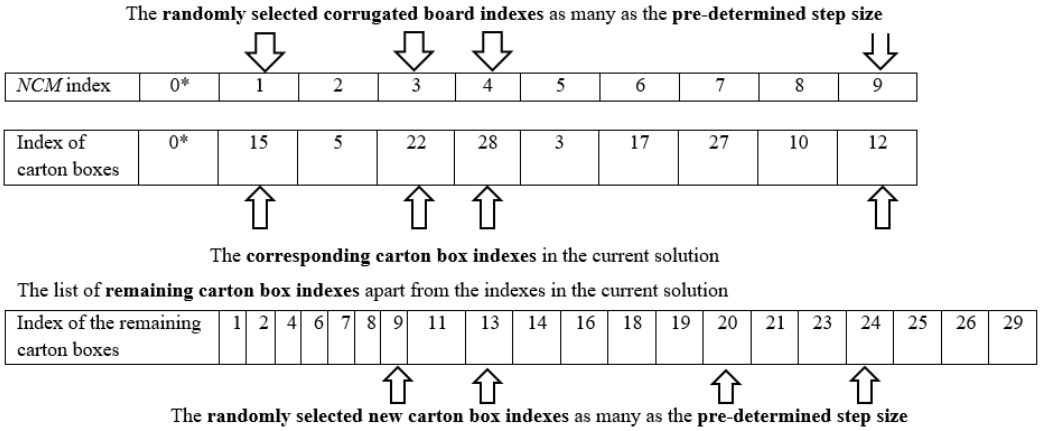
Two different neighborhood generation mechanisms, namely the random and greedy search-based strategies are generated for the proposed matheuristic algorithm and illustrated with examples in Figure 4.5 and 4.7 respectively.

In the proposed random neighborhood generation strategy, the step size is first calculated by Equation (4.2) by using the maximum number of corrugated boards of different sizes (*NCB*) and a division factor (*SP*), which are pre-specified in the initialization phase. The "*int*" function is applied to round the largest integer value that is not greater than the $NCB/_{SP}$ value.

$$StepSize = int\left\lfloor\frac{NCB}{SP}\right\rfloor \tag{4.2}$$

If the values of these parameters are set to 10 and 2.2 respectively, then the step size will be 4. It means that 4 boxes will be chosen from the solution to swap with the ones that are not in the solution. In this example, the following indexes [1, 3, 4, 9] of the corrugated boards and their corresponding indexes of the corrugated boxes [15, 22, 28, 12], are chosen randomly as depicted in Figure 4.5. So [15, 22, 28, 12] will be removed from the solution, and instead of them, another randomly selected set from the remaining boxes will be inserted, which is [9, 13, 20, 24]. As a result, a new

neighbor solution that includes the following indexes [9, 5, 13, 20, 3, 17, 27, 10, and 24] of the corrugated boxes is generated.



The neighbor solution generated from the current solution

NCM index	0*	**1**	2	**3**	**4**	5	6	7	8	**9**
Width (WR_j, OW_i)	1750*	**1150**	1150	**1150**	**1350**	1050	1350	1110	1150	**1350**
Length (LR_j, OL_i)	2000*	**905**	805	**1905**	**1605**	1571	1105	1925	1331	**1971**
Index of carton boxes	0*	**9**	5	**13**	**20**	3	17	27	10	**24**

Figure 4.5 Illustrative example of the random neighborhood generation strategy

The pseudo-code of the random neighborhood generation strategy is presented in Figure 4.6.

In the greedy search-based neighborhood generation strategy, a set of ineffective board dimensions is first determined by solving the reduced MILP with the exclusion of each corrugated box dimension available in the current solution. If the objective function value deteriorates or doesn't change by excluding any corrugated box dimension, it will be added to the set of ineffective or unused board dimensions. The specification of these ineffective board dimensions will increase the computational effort since it requires a repetitive solution of the MILP model. On the other hand, it seems more efficient than the random one since it can determine the indexes of corrugated boxes that will be removed from the current solution more intelligently. The strategy includes four main conditions, i.e., cases.

1. **Inputs:** *CurrentSolution(.)*, *NCB*, *SP*, *StepSize*, indexes of randomly chosen corrugated boxes in the current solution, the minimum widths/lengths **(OW_i** and **OL_i)** of these corrugated boxes in the *CurrentSolution(.)*
2. **Calculate** the $StepSize = \lfloor NCB/SP \rfloor$
3. **For** *k* =1 to *StepSize* **do**
4. **Select random indexes** of the corrugated boards of different sizes from the *CurrentSolution(.)*
5. **Find their corresponding indexes** *(CI)* of corrugated boxes in the *CurrentSolution(.)*
6. **End for**
7. **Find the list of** remaining corrugated box indexes, which are not included in the *CurrentSolution(.)*

The remaining indexes (*RI*) = *Set of all of the* corrugated *boxes – indexes of the* corrugated *boxes in the CurrentSolution(.)*

8. **For** *k* =1 to *StepSize* **do**
9. **Select a new random index** (*NI*) of the corrugated box from the list of the pre-specified *remaining indexes* (*RI*)
10. **End for**
11. **Remove the pre-found corresponding indexes** (*CI*) of corrugated boxes from the *CurrentSolution(.)*

The list of remaining indexes in the CurrentSolution(.) = Indexes of the corrugated *boxes in the CurrentSolution(.) – the pre-found corresponding indexes* (*CI*)

12. **Insert the pre-selected new indexes** (*NI*) of the randomly chosen corrugated boxes into the *list of remaining indexes in the CurrentSolution(.)*

Indexes of corrugated *boxes in the NeighborSolution = The list of remaining indexes in the CurrentSolution(.) + new random indexes* (*NI*) *of* corrugated *boxes selected from the list of the remaining indexes* (*RI*)

13. **For** *i* =1 to *NCB* **do**
14. *NeighborSolution(i)* = [$MW + OW_i$, $ML + OL_i$] where *i* is the index of the corrugated boxes in the *NeighborSolution(.)* and MW/ML correspond to the maximum allowable width/length of the **dummy corrugated board**
15. **End for**
16. **Output:** *NeighborSolution(.)*

Figure 4.6 The pseudo-code of the random neighborhood generation strategy

In the first case, if the number of cardboard boxes that are not in the solution is greater than the number of ineffective board dimensions, all the ineffective ones are removed from the solution, and randomly selected ones from the remaining cardboard boxes are inserted into the empty positions in the current solution. In other words, the step size is equal to the number of ineffective dimensions in this case.

In the second case, if the number of the corrugated box indexes that are not in the solution is less than the number of ineffective ones, a certain number of corrugated box indexes as many as the step size will be randomly selected from the ineffective ones and removed from the solution. Afterward, new corrugated box indexes as many as the step size are again randomly chosen from the list of remaining corrugated box indexes and inserted into the empty positions.

In the third case, if the step size is larger than the number of ineffective dimensions, some effective dimensions should also be removed from the solution. In addition to all the ineffective dimensions, a certain number of effective dimensions that is equal to the difference between the step size and the number of ineffective dimensions will also be removed from the current solution. Then, the randomly selected new indexes (as many as the step size) from the list of the remaining corrugated boxes will be inserted into the empty positions. An illustrative example is also given for the third case in Figure 4.7.

In the final case, if there is no ineffective dimension in the current solution, the previously mentioned neighbor strategy in Figure 4.6 should be applied to generate neighbor solutions.

The pseudo-code of the greedy search-based neighborhood generation strategy is presented in Figure 4.8 and It's considered that the step size equals 5, and the number of the ineffective dimensions equals 2 (i.e., 15^{th} and 28^{th} indexes of corrugated boxes). An effective dimension of the previous solution may become ineffective in the successive neighbor solution since the combination of board dimensions is changed in every neighbor solution. In other words, the other board dimensions in the neighbor solution may be utilized in the production of the demanded corrugated boxes instead of that board dimension which is specified as an effective one in the previous solution. Thus, the set of ineffective board dimensions is not constant throughout the search process, and it's aimed to find the best combination of board dimensions consisting of the most effective ones in the final solution.

Randomly selected effective board dimensions as many as the difference between step size –the number of ineffective ones

Index of carton boxes in the current solution	0*	15	5	22	28	3	17	27	10	12
Width (WR_j, OW_i)	1750*	1250	1150	1350	1720	1050	1350	1110	1150	1150
Length (LR_j, OL_i)	2000*	1971	805	1905	1935	1571	1105	1925	1331	1705

The **ineffective board dimensions** specified by solving the **reduced MILP model**

The list of **remaining carton box indexes** apart from the indexes in the current solution

Index of the remaining carton boxes	1	2	4	6	7	8	9	11	13	14	16	18	19	20	21	23	24	25	26	29

The **randomly selected new carton box indexes** as many as the **step size = 5**

The neighbor solution generated from the current solution

NCM index	0*	**1**	**2**	3	**4**	5	**6**	7	8	**9**
Width (WR_j, OW_i)	1750*	**1050**	**1150**	1350	**1350**	1050	**1410**	1110	1150	**1350**
Length (LR_j, OL_i)	2000*	**1971**	**1571**	1905	**1605**	1571	**1925**	1925	1331	**1865**
Index of carton boxes	0*	**4**	**11**	22	**20**	3	**25**	27	10	**26**

Figure 4.7 Illustrative example of the greedy search-based neighborhood generation strategy

4.3.3 The Fitness Function Evaluation

After the solution generation process the MILP model is utilized to compute the objective function values. As mentioned before, the decision variables of the cardboard dimensions are provided by the SA algorithm, thus they can be considered as a parameter. Also, the maximum number of corrugated boards of different sizes (i.e., *NCB*) which is a problem-specific parameter and generally pre-specified by the corrugated box manufacturer is used as input data in the proposed MINLP model for the set of corrugated boards with different dimensions ($j \in J$). So, the MINLP model can be converted to a MILP model and can be reformulated as in the following Equations (4.3) - (4.10). By linearizing the MINLP model, the stock selecting and sizing problem is reduced to a stock selection problem only.

$$Minimize \sum_{i \in I} \sum_{j \in NCB} WC_{ij} \times Q_{ij} + \sum_{j \in NCB} PUR_j \times c \times \left(\frac{\widehat{WR}_j \times \widehat{LR}_j}{100000} \right) \tag{4.3}$$

$$\begin{cases} \widehat{PC}_{ij} = M \quad \forall i \in I, j \in NCB \,|\, OW_i \leq MW \;\&\; \widehat{OB}_{ij} = 0 \\ \widehat{PC}_{ij} = \left. \widehat{WR}_j \times \widehat{LR}_j \times c \,\middle/ \left(1000000 \times OB_{ij}\right) \right. \forall i \in I, j \in NCB \,\middle|\, \begin{matrix} OW_i \leq MW \\ \widehat{OB}_{ij} \geq 1 \end{matrix} \\ \widehat{PC}_{ij} = \left. 2 \times \widehat{WR}_j \times \widehat{LR}_j \times c \,\middle/ 1000000 \right. \quad \forall i \in I, j \in NCB | OW_i > MW \end{cases} \tag{4.4}$$

$$\widehat{WC}_{ij} = \widehat{PC}_{ij} - OC_i \quad \forall i \in I, j \in NCB \tag{4.5}$$

$$\widehat{OB}_{ij} = int\left[\frac{\widehat{WR}_j}{OW_i}\right] \times int\left[\frac{\widehat{LR}_j}{OL_i}\right] \quad \forall i \in I, j \in NCB \tag{4.6}$$

$$OW_i - \left(2 \times U_{ij} \times \widehat{WR}_j\right) \leq \beta_{ij} \times M \quad \forall i \in I, j \in NCB \mid OW_i > MW \tag{4.7}$$

$$U_{ij} \leq 0 \quad \forall i \in I, j \in NCB \mid OW_i \leq MW \;\&\; OW_i > \widehat{WR}_j \tag{4.8}$$

$$MTA - PUR_j \times \left(\frac{\widehat{WR}_j \times \widehat{LR}_j}{1000000}\right) \leq \left(1 - \alpha_j\right) \times M \quad \forall j \in NCB \tag{4.9}$$

$$U_{ij} \times OL_i \leq \widehat{LR}_j \quad \forall i \in I, j \in NCB \tag{4.10}$$

1. **Inputs:** *CurrentSolution(.), RemainingSolution(.), NCB, SP, StepSize*, indexes of randomly chosen corrugated boxes from the current solution, indexes of randomly chosen corrugated boxes from the effective and ineffective corrugated board sizes, the minimum widths/lengths **(OW_i** and **OL_i)** of these corrugated boxes in the *CurrentSolution(.)*
2. **Calculate** the $\boldsymbol{StepSize} = \lfloor {}^{NCB}/_{SP} \rfloor$
3. **For** *i* = 1 to *NCB* **do**
4. **Select and remove the *i*th index (*IB*)** of the corrugated boxes from the *CurrentSolution(.)*
5. *The residual indexes* (*RIB*) = *Indexes of all the* corrugated *boxes in CurrentSolution(.) – the selected index* (*IB*)
6. **For** *i* = 1 to *NCB* – 1 **do**
7. *RemainingSolution(i)* = [$\boldsymbol{MW + OW_i,\ ML + OL_i}$] where *i* is the remaining indexes (*RI*) of the corrugated boxes in the *CurrentSolution(.)* and $\boldsymbol{MW}$/$\boldsymbol{ML}$ correspond to the maximum allowable width/length of the **dummy corrugated board**
8. **End for**
9. *RemainingCost* ← Calculate the fitness function value of *RemainingSolution(i)* using the reduced **MILP model**
10. **If** *RemainingCost* <= *CurrentCost* **Then**
11. **Append the *i*th index (*IB*)** of the corrugated boxes to the **set of ineffective board sizes (*IS*)**
12. **End if**
13. **End for**
14. **Find the list of** remaining corrugated box indexes (*RIC*), which are not included in the *CurrentSolution(.)*
The remaining indexes (*RIC*) = *Set of all the corrugated boxes – indexes of the corrugated boxes in the CurrentSolution(.)*
15. **If** length(*IS*) > 0 & length(*RIC*) >= length(*IS*) **Then (Case – 1)**
16. **For** k=1 to length(*IS*)
17. **Select a new random index** (*NI*) of the corrugated box from the list of the pre-specified remaining indexes (*RIC*)
18. **End for**
19. **Remove the pre-found indexes** of **ineffective corrugated board sizes** (*IS*) from the *CurrentSolution(.)*
20. *The list of remaining indexes in the CurrentSolution(.) = Indexes of the corrugated boxes in the CurrentSolution(.) – the pre-found indexes of ineffective corrugated board sizes* (*IS*)
21. **Insert the pre-selected new indexes** (*NI*) of the randomly chosen corrugated boxes into the *list of remaining indexes in the CurrentSolution(.)*
22. *Indexes of corrugated boxes in the NeighborSolution = The list of remaining indexes in the CurrentSolution(.) + new random indexes* (*NI*) *of corrugated boxes selected from the list of the remaining indexes* (*RIC*)
23. **End if**
24. **If** length(*IS*) > 0 & length(*RIC*) <= length(*IS*) **Then (Case – 2)**
25. **For** *k* =1 to *StepSize* **do**
26. **Select a new random index** (*ISS*) of the corrugated box from the list of **ineffective corrugated board sizes (*IS*)**
27. **End for**
28. **Remove the randomly chosen indexes** as *StepSize* from the **ineffective corrugated board sizes** (*IS*)
29. *The list of remaining indexes in the CurrentSolution(.) = Indexes of the corrugated boxes in the CurrentSolution(.) – the randomly chosen indexes as StepSize (ISS) from the ineffective corrugated board sizes* (*IS*)
30. **For** *k* =1 to *StepSize* **do**
31. **Select a new random index** (*NI*) of the corrugated box from the list of the pre-specified remaining indexes (*RIC*)
32. **End for**
33. **Insert the pre-selected new indexes** (*NI*) of the randomly chosen corrugated boxes into the list of remaining indexes in the *CurrentSolution(.)*
34. *Indexes of corrugated boxes in the NeighborSolution = The list of remaining indexes in the CurrentSolution(.) + new random indexes* (*NI*) *of corrugated boxes selected from the list of the remaining indexes* (*RIC*)
35. **End if**
33. **If** length(*IS*) > 0 & length(*StepSize*) > length(*IS*) **(Case – 3)**
36. **Find the set of** effective board sizes (ES) = *Indexes of the corrugated boxes in the CurrentSolution(.) – the pre-found indexes of ineffective board sizes (IS)*
37. **For** *k* =1 to (length(*StepSize*) – length(*IS*)) **do**
38. **Select a new random index** (*NIE*) of the corrugated box from the list of the effective board sizes (ES)
39. **End for**
40. **Remove the randomly chosen indexes** of effective board sizes (*NIE*) as well as the ineffective board sizes (*IS*) from the *CurrentSolution(.)*
41. *The list of remaining indexes in the CurrentSolution(.) = Indexes of the corrugated boxes in the CurrentSolution(.) – the randomly chosen indexes (NIE) from the effective corrugated board sizes* (*ES*) *– the list of indexes in ineffective corrugated board sizes* (*IS*)
42. **For** *k* =1 to *StepSize* **do**
43. **Select a new random index** (*NI*) of the corrugated box from the list of the pre-specified remaining indexes (*RIC*)
44. **End for**
45. **Insert the pre-selected new indexes** (*NI*) of the randomly chosen corrugated boxes into the list of remaining indexes in the *CurrentSolution(.)*
46. *Indexes of corrugated boxes in the NeighborSolution = The list of remaining indexes in the CurrentSolution(.) + new random indexes* (*NI*) as *of corrugated boxes* (as *StepSize*) *selected from the list of the remaining indexes* (*RIC*)
47. **End if**
48. **If** length(*IS*) <= 0 **(Case – 4)**
49. **Apply the proposed random neighborhood generation strategy** that is given in **Figure 4.6**.
50. **End if**
51. **For** *i* =1 to *NCB* **do**
52. *NeighborSolution(i)* = [$\boldsymbol{MW + OW_i,\ ML + OL_i}$] where *i* is the index of the corrugated boxes in the *NeighborSolution(.)* and $\boldsymbol{MW}$/$\boldsymbol{ML}$ correspond to the maximum allowable width/length of the **dummy corrugated board**
53. **End for**
54. **Output:** *NeighborSolution(.)*

Figure 4.8 The pseudo-code of the proposed greedy search-based neighborhood generation strategy

CHAPTER FIVE
A REAL-LIFE APPLICATION IN THE CORRUGATED CARDBOARD BOX MANUFACTURING COMPANY

In this chapter, to show the validity and applicability of the proposed matheuristic-based solution approach, a real-life application is first presented for the before-mentioned corrugated box manufacturing company in Manisa, Turkey. In this application study, the weekly towel radiator box demands of the company's most important customer with the highest sales volume are taken into consideration. This customer firm requests towel radiator boxes with different dimensions at the beginning of each week. Due to the wide variety of the produced towel radiators, there are a total of 236 different corrugated box dimensions demanded by this customer firm during the whole year. According to its weekly production plan, an average of 30 to 40 different dimensions of corrugated boxes are generally required by this customer firm for each week.

Due to this product and raw material variety (problem size), it's critical to maximize the efficiency of the matheuristic. So, a computational study is also performed using Taguchi's design of experiment method for the fine-tuning process of the algorithm parameters (i.e., LMC, MNM_{min}, and SP) and analyzing the impacts of these parameters on the results.

5.1 Data Description

An example of weekly demand data and other inputs (i.e., the actual dimensions of the corrugated boxes, the minimum widths/lengths, and unit costs of the required corrugated boards in case of zero waste manufacturing) for the towel radiator boxes with 30 different dimensions are presented in Figure 5.1. As mentioned before, the minimum widths/lengths and the unit costs of the required corrugated boards for each towel radiator box are computed by making use of Equations (3.1) – (3.3). The standard 42 different dimensions for the purchased corrugated boards that are determined heuristically by the company managers are also given in Figure 5.2. It should be emphasized here that these standard dimensions are specified by considering

236 different corrugated box dimensions demanded by this customer firm throughout the whole year. On the other hand, the company managers also desire to decrease the total number of different sizes of corrugated boards so that they can reduce the stock diversity. There is a conflict between the total number of different sizes of corrugated boards and the resulting waste cost.

Product codes	Width (WP_i)	Height (HP_i)	Length (LP_i)	Minimum width (OW_i)	Minimum length (OW_i)	Minimum unit production cost (OC_i)	Weekly demand (DE_i)
7-03-204	415	35	1220	950	1305.	7.39	105
7-03-808	465	35	920	1050	1005	6.29	20
7-03-093	465	35	1246	1050	1331	8.33	140
7-03-057	465	35	1486	1050	1571	9.84	43
7-03-094	465	35	1886	1050	1971	12.34	70
7-03-040	515	35	720	1150	805	5.52	14
7-03-096	515	35	806	1150	891	6.11	85
7-03-337	515	35	820	1150	905	6.21	85
7-03-043	515	35	1020	1150	1105	7.58	71
7-03-205	515	35	1220	1150	1305	8.95	247
7-03-095	515	35	1246	1150	1331	9.13	146
7-03-068	515	35	1486	1150	1571	10.78	140
7-03-242	515	35	1620	1150	1705	11.69	105
7-03-257	515	35	1820	1150	1905	13.07	170
7-03-092	515	35	1886	1150	1971	13.52	283
7-03-070	565	35	1886	1250	1971	14.70	60
7-03-075	615	35	806	1350	891	7.17	30
7-03-196	615	35	1020	1350	1105	8.90	70
7-03-051	615	35	1220	1350	1305	10.51	311
7-03-097	615	35	1246	1350	1331	10.72	150
7-03-463	615	35	1520	1350	1605	12.92	130
7-03-198	615	35	1620	1350	1705	13.73	70
7-03-258	615	35	1820	1350	1905	15.34	332
7-03-149	615	35	1840	1350	1925	15.50	40
7-03-099	615	35	1886	1350	1971	15.87	300
7-03-1310	620	60	1790	1410	1925	16.19	54
7-03-019	615	35	1780	1350	1865	15.02	45
7-03-1308	470	60	1790	1110	1925	12.74	25
7-03-1311	770	65	1790	1720	1935	19.85	15
7-03-1312	920	60	1790	2010	1925	23.08	15

Figure 5.1 Weekly demand and the basic data of the corrugated boxes produced by the manufacturing company

In other words, the total waste cost can be reduced when the stock diversity is increased. For this reason, company managers need to determine a balanced solution, which stabilizes the total number of different sizes of corrugated boards and the resulting waste cost. It should also be noted here that the purchasing costs of the corrugated boards will also increase when the stock selection and sizing decisions are

made inaccurately and non-optimally. The remaining data used in this case study are also determined by the company managers and/or the vendors as follows: First, the maximum width and length of any corrugated board that can be ordered are specified as 1750 and 2000 millimeters by the vendors, respectively.

Corrugated board no.	Width (WR_j)	Length (LR_j)	Corrugated board no.	Width (WR_j)	Length (LR_j)	Corrugated board no.	Width (WR_j)	Length (LR_j)
1	950	905	15	1150	1571	29	1350	1971
2	950	1105	16	1150	1705	30	1650	1971
3	950	1305	17	1150	1750	31	1060	1400
4	950	1705	18	1150	1971	32	1060	1720
5	1050	1331	19	1250	1331	33	1060	2000
6	1050	1571	20	1250	1571	34	1250	1400
7	1050	1971	21	1350	1105	35	1250	1720
8	1150	805	22	1350	1205	36	1250	2000
9	1150	905	23	1350	1305	37	1450	1400
10	1150	950	24	1350	1331	38	1450	1720
11	1150	1105	25	1350	1405	39	1450	2000
12	1150	1205	26	1350	1571	40	1750	1400
13	1150	1305	27	1350	1705	41	1750	1720
14	1150	1331	28	1350	1750	42	1750	2000

Figure 5.2 The standard corrugated board dimensions that were pre-specified by the company managers

The purchasing managers of the company and its vendors also have reached a consensus on the unit purchasing cost of the corrugated boards per square meter as 5.967 Turkish lira. For the procurement orders, the minimum total area is also determined as 500 square meters for each size of the purchased corrugated board. Finally, the constant values for the fold allowance and adhesive margin for gluing or stapling are set to 5 and 30 millimeters respectively by the process managers of the manufacturing company.

5.2 Computational results of the proposed matheuristic solution approach with Taguchi's design-of-experiment method

DOE is a statistical technique that is used to better understand the critical parameters of a process, their interactions with each other, and their impacts on the result. It is firstly introduced by R. A. Fischer in England in the 1920s to determine the optimum water, rain, sunshine, fertilizer, and soil consitions needed to produce the best crop (Roy, 2001). After World War II, it has been standardized by G. Taguchi for quality improvement, and named after him. Figure 5.3 shows the flow chart of the Taguchi's method.

Taguchi’s method is one of the most popular tools, due to its applicability. Even if it does not provide high sensitivity on parameters as full factorial design, it requires fewer experiments to identify a better combination of the parameters. After identifying the problem, the noise factors (parameters) and their levels (values) are determined to construct the orthogonal matrix. After conducting the experiments based on the orthogonal matrix, all of the results are used for data analysis.

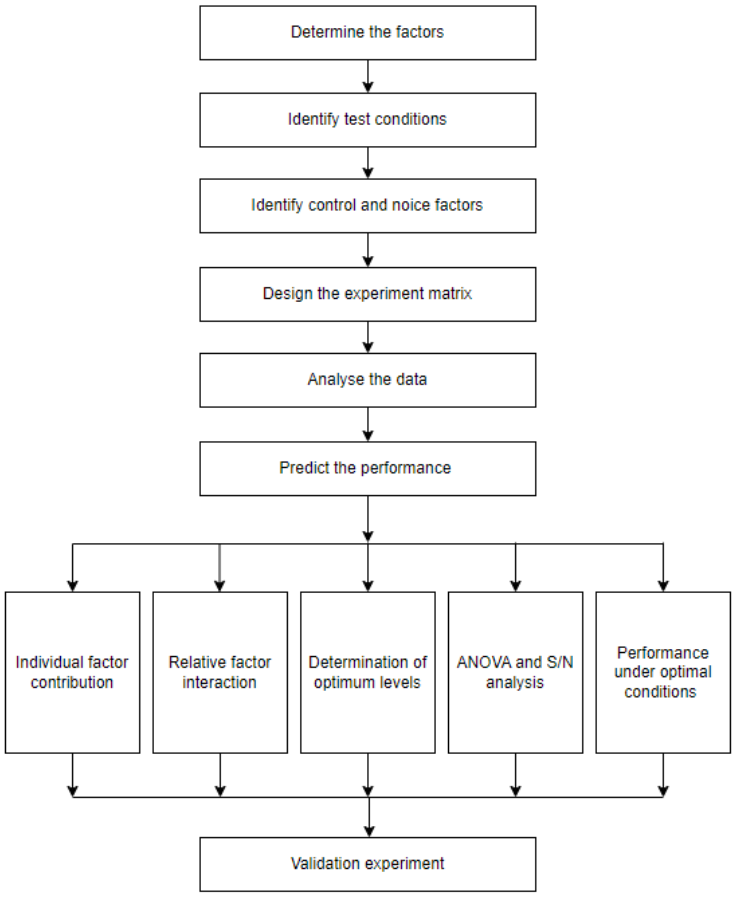


Figure 5.3 The flow chart of Taguchi's DOE (Fraley, Zalewski, Oom, & Terrien, n.d.)

To improve the cutting plan of the manufacturing company and reduce the total costs, the proposed solution approach is implemented to achieve near-optimal solutions within reasonable computing times. An experimental study is performed using Taguchi’s design of experiment method for the fine-tuning process of the algorithm parameters (i.e., *LMC*, MNM_{min}, and *SP*) and analyzing the impacts of these parameters on the computational results. On the other hand, the ideal values of the

other remaining SA parameters such as T_0, q, and el_{max} are estimated based on the following formulas in Equations (5.1) – (5.5), which were determined from empirical experimental studies by previous researchers (Baykasoğlu &Gindy, 2001; Bennage & Dhingra, 1995; Lin, Kao, & Hsu, 1993). It should be noted here that these algorithm parameters, i.e., T_0, q, and el_{max} are not included in the experimental study since their values can be easily computed by using the following equations, and also their values should be recalculated for the larger problem sizes with many corrugated box dimensions. In other words, in the scope of an experimental study, determining constant factor levels for these parameters may not be appropriate for different sizes of test problems.

$$T_0 = {f_{min} - f_{max}}\Big/{\ln(P_{acc})} \tag{5.1}$$

In order to provide a high degree of randomness in the initial stages of the stochastic search, a sufficiently high value should be specified for the initial temperature (i.e., T_0). Therefore, most of the generated neighbor solutions at the beginning of the SA algorithm are accepted since the value of acceptance probability is generally set very close to 1. In this study, its value is set to P_{acc}= 0.90. The minimum and maximum values of the total waste and purchasing costs, i.e., f_{min} and f_{max} are found as 43486.19 and 44241.63 by performing additional runs on the proposed mathematical model by taking the minimum dimension (i.e., OW_i and OL_i) values of the demanded corrugated boxes for zero waste manufacturing as the sizes of purchased corrugated boards. In other words, these minimum dimension values of the corrugated boxes are used as the sizes of purchased corrugated boards to obtain zero waste manufacturing and also total minimum cost value, i.e., f_{min}. In this case, it should be noted here that the value of the maximum number of corrugated boards of different sizes (*NCB*) is also determined the same as the number of demanded corrugated boxes, i.e., (*NCB* = 30). When the sizes of the corrugated boards are determined as the same as the minimum dimensions of the corrugated boxes with zero waste, the lower bound of this stock selection and sizing problem will be provided as f_{min}= 43486.19 by using the Gurobi MIP solver. However, it should be highlighted here that the maximum number of corrugated boards of different sizes cannot be equal to the number of corrugated

boxes of different dimensions in real-life settings. Indeed, it is generally limited due to the minimum stock diversity goal of the manufacturing companies in the corrugated box production industry. To determine the maximum objective value, i.e., f_{max}, the standard 42 dimensions for the purchased corrugated boards that are pre-specified by the company managers are utilized as input data for the proposed mathematical model. In this circumstance, the solution of the proposed model will be equal to f_{max} = 44241.63. Finally, the value of the initial temperature can be estimated as T_0 =7169.95 by using Equation (5.1). To accomplish a slow cooling, these initial and the current temperatures must be decreased gradually at each iteration by using the cooling rate (i.e., q) as in Equation (5.2). While determining the cooling rate, the final acceptance probability, i.e., P_{fin} is also needed in addition to the initial acceptance probability. In this case study, its value is set to a very smaller value closer to zero, i.e., $P_{fin} = 10^{-15}$as in the study of Baykasoğlu and Gindy (2001). Moreover, the ideal value of this cooling rate is also dependent on the maximum number of iterations (i.e., el_{max}) in the outer loop of the SA algorithm as formulated in Equation (5.3).

$$T_{el+1} = T_{el} \times q \tag{5.2}$$

$$q = \left(\frac{\ln P_{acc}}{\ln P_{fin}}\right)^{\frac{1}{(el_{max}-1)}} \tag{5.3}$$

The value of el_{max} can be decided in such a way that the final temperature, i.e., T_{fin} can be reached at the end of the last iteration as formulated in Equation (5.4). If the value of the final temperature, i.e., T_{fin} computed in Equation (5.4) is not very close or equal to its estimated value, i.e., $\hat{T}_{fin}$ in Equation (5.5) the value of el_{max} should be reselected and the value of cooling rate should also be modified until the required number of outer loop iterations is reached. In this case study, the value of el_{max} is chosen as 100 initially. Then, the value of the cooling rate is calculated as q = 0.9425 by using Equation (5.4). Afterward, the final temperature at the end of the iteration number 100 is computed as T_{fin} = 19.32 by using Equation (5.4). However, since this value is not very close to the estimated final temperature $\hat{T}_{fin} = 20.5$ in Equation (5.5) the initial el_{max} value may require a revision. For that reason, after a series of

modifications on el_{max}, its value is finally set to 300. Therefore, the cooling rate is computed as q = 0.981 by making use of Equation (5.3) and then, the final temperature is also calculated as $T_{fin} \cong 20.11$ through Equation (5.4) which is a very close value to its pre-determined estimated value, i.e., $\hat{T}_{fin} = 20.5$.

$$T_{fin} = T_0 \times q^{el_{max}} \tag{5.4}$$

$$\hat{T}_{fin} = {f_{min} - f_{max}}\Big/{\ln(P_{fin})} \tag{5.5}$$

In addition to the previously mentioned parameters of the SA algorithm, the length of the Markov chain (*LMC*), the minimum number of accepted solutions (MNM_{min}), and the step size parameter: a division factor (*SP*) also represent other parameters of the proposed matheuristic-based approach and therefore, they are specified as controllable factors in the experimental design study and have four levels. The first one, i.e., *LMC* is varied between 5 and 30 (i.e., the maximum number of corrugated box), the second one, i.e., MNM_{min} is altered between 3 and 15 (i.e., the maximum number of corrugated boards) and finally, the division factor is altered between 2 and 8. To reduce the computational effort and the total number of experimental runs of the proposed solution approach, Taguchi's L16 Orthogonal array is chosen. It should be emphasized here that Taguchi's DOE method usually ignores the interactions among these factors and focuses on the main factor effects. It is known that these parameters do not depend on each other. Subsequently, the proposed solution algorithm is implemented in Python 3.8.8 modeling and development environment for solving the integrated stock selection and sizing problem of the manufacturing company and all of the computational experiments have been conducted on a personal computer with 2.70 GHz CPU and 16 GB RAM running under a Windows 10 operating system.

For the different maximum numbers of corrugated board dimensions (*NCB* = 5, 10, and 15), the present stock selection and sizing problem of the manufacturing company has been solved three times by the proposed matheuristic-based approach with the proposed random neighbor generation strategy in each design set to generate more reliable results. It should be noted here that although the maximum number of

corrugated boards of different sizes is determined as *NCB* = 15 by the company managers, additional computational experiments are also conducted for the smaller number of corrugated board dimensions (*NCB* = 5 and 10) to analyze the effect of this parameter on the total cost value. As mentioned before, there is a trade-off between the maximum number of different corrugated board dimensions (i.e., stock diversity) and the resulting total waste cost value. As a result, a total of 48 outputs are obtained by the proposed solution approach for each *NCB* value. According to the L16 Orthogonal design matrices in Table 5.1, experimental results in terms of the best-worst found solutions, their averages, standard deviations for the total cost objective, and the average computation times are also involved for each *NCB*.

To analyze the effects of the parameters on the total waste and purchasing costs and evaluate the experimental results, ANOVA tests, analysis of means graphs, and Taguchi's S/N ratios are performed by making use of MINITAB 14 concerning Taguchi's L16 scheme. The best found and the average total cost values in Table 5.1 are utilized as response variables for analyzing the experimental designs. The main factor effects provided by the S/N ratios are first plotted in Figures 5.4 - 5.6 for each *NCB* value. Additionally, the response tables for S/N ratios are also reported in Table 5.2. It should be emphasized here that the "smaller is better" option is considered to calculate the S/N ratios since the objective function is minimizing the total waste and purchasing costs of the corrugated boards.

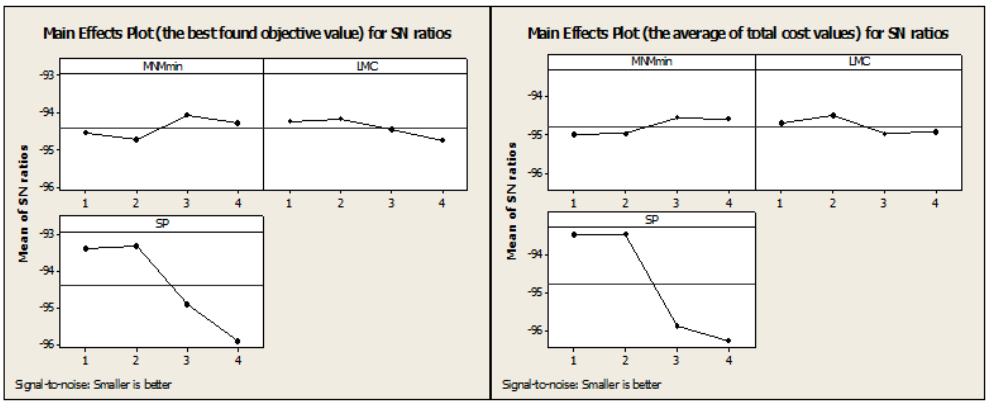


Figure 5.4 Main factor effects identified by the S/N ratios for the best found and average total cost values (NCB = 5)

Table 5.1 Orthogonal arrays for analyzing the effects of the matheuristic algorithm's parameters on the total cost value

	Exp. No.	MNM_{min}	*LMC*	*SP*	Best	Worst	Avg.	Std. dev.	Avg. CPU time
	1	3	5	2	47516.62	49425.41	48201.67	867.3787	66.45
	2	3	10	4	46129.57	47606.39	46872.01	602.9369	141.48
	3	3	15	6	55547.87	70206.82	64289.24	6308.804	48.41
	4	3	30	8	66695.03	69990.45	68755.64	1466.604	43.35
	5	5	5	4	46129.57	47641.49	47086.65	679.6178	68.95
	6	5	10	2	47374.94	47516.62	47450.77	58.27099	132.16
	7	5	15	8	64092.76	74705.85	68645.27	4462.071	66.72
***NCB* = 5**	8	5	30	6	62703.52	66260.36	64257.62	1486.325	74.94
	9	10	5	6	52057.32	66044.95	61121.62	6417.386	74.94
	10	10	10	8	58465.68	61815.19	60494.30	1456.129	146.52
	11	10	15	2	46129.57	48027.08	46762.07	894.4946	208.27
	12	10	30	4	46129.57	47460.75	47014.49	625.7418	382.12
	13	15	5	8	61307.41	63813.49	62803.04	1079.09	64.02
	14	15	10	6	53500.79	66448.90	58999.53	5463.034	142.47
	15	15	15	4	47374.94	47967.45	47598.51	262.8232	184.68
	16	15	30	2	46129.57	47606.39	46621.84	696.1805	361.54
	1	3	5	2	44241.63	44469.03	44348.52	93.33288	107.54
	2	3	10	4	43973.50	44065.33	44021.57	37.61422	174.19
	3	3	15	6	43973.50	44409.62	44118.87	205.5886	229.00
	4	3	30	8	44221.00	44374.37	44291.35	63.25154	451.35
	5	5	5	4	43973.50	44825.37	44466.88	360.6177	133.87
	6	5	10	2	44312.75	44409.62	44345.04	45.66386	221.11
	7	5	15	8	43932.59	44685.62	44306.53	307.4425	299.30
***NCB* = 10**	8	5	30	6	43941.95	44278.68	44144.04	145.512	508.79
	9	10	5	6	44374.37	44374.37	44374.37	0	124.94
	10	10	10	8	44256.62	44699.83	44519.41	190.0804	241.21
	11	10	15	2	44161.96	44241.63	44194.75	34.0162	329.42
	12	10	30	4	43973.50	44428.22	44204.41	185.7058	608.35
	13	15	5	8	44445.49	45029.18	44640.06	275.155	122.30
	14	15	10	6	43973.50	44701.35	44402.11	310.9054	236.58
	15	15	15	4	44152.53	44424.71	44285.31	111.2147	382.34
	16	15	30	2	44196.33	44278.68	44241.24	34.03377	660.30
	1	3	5	2	43596.80	43618.94	43604.18	10.43603	197.88
	2	3	10	4	43486.20	43729.54	43567.31	114.712	281.27
	3	3	15	6	43486.20	43729.54	43638.12	108.1607	369.68
	4	3	30	8	43486.20	43831.35	43672.05	142.1516	582.99
	5	5	5	4	43618.94	43729.54	43689.55	50.07908	182.74
	6	5	10	2	43596.80	43628.35	43617.84	14.87453	297.33
	7	5	15	8	43698.61	43761.09	43719.44	29.45645	368.65
***NCB* = 15**	8	5	30	6	43618.94	43698.61	43645.49	37.55604	640.55
	9	10	5	6	43628.35	43761.09	43696.02	54.22158	170.78
	10	10	10	8	43618.94	43840.76	43740.26	91.74887	332.92
	11	10	15	2	43596.80	43618.94	43611.56	10.43603	451.32
	12	10	30	4	43486.20	43596.80	43559.93	52.13797	765.10
	13	15	5	8	43698.61	43932.59	43838.31	100.7799	172.31
	14	15	10	6	43486.20	43761.09	43577.83	129.5865	318.36
	15	15	15	4	43618.94	43720.19	43655.83	45.67137	510.48
	16	15	30	2	43486.20	43628.35	43533.58	67.0125	964.48

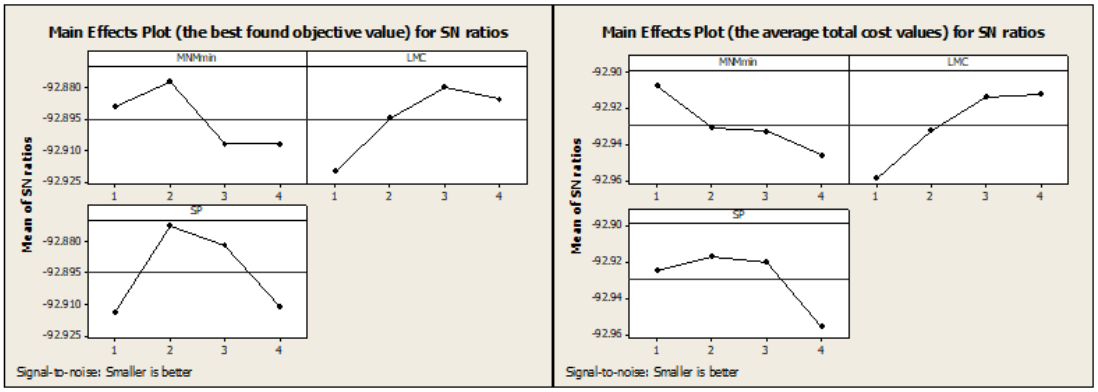


Figure 5.5 Main factor effects identified by the S/N ratios for the best found and average total cost values (NCB = 10)

According to Figures 5.4 – 5.6, all the pre-determined parameters have significant importance for the chosen response variables because they lead to substantial variability on the best found and the average total cost values. Additionally, these results are also supported by the ANOVA tests given in Table 5.3. It should be noted here that since the *p*-values of all the parameters are relatively small and $p \leq \alpha = 0.1$ (i.e., 90% confidence interval) for the average total cost values, the null hypothesis (H_0) can be rejected for *NCB* = 5 and 10. Hence, all the pre-determined algorithm parameters have a major impact on the average total cost values for the *NCB* = 5 and 10 with a sufficiently high coefficient of determination values of 99.43% and 87.85% (R-Sq.), respectively. On the other hand, except for the parameter MNM_{min} whose *p*-value was acquired as 0.461, all the others also have a substantial influence on the average total cost value in the case of *NCB* = 15. However, it should be noted here that a sufficiently high coefficient of determination value, i.e., 89.35% was also obtained when the *NCB* is set to 15.

It should be noted here that since the larger coefficient of determination values was obtained for the average total cost value as a response variable, ANOVA results for the best-found total cost values are not included in Table 5.3.

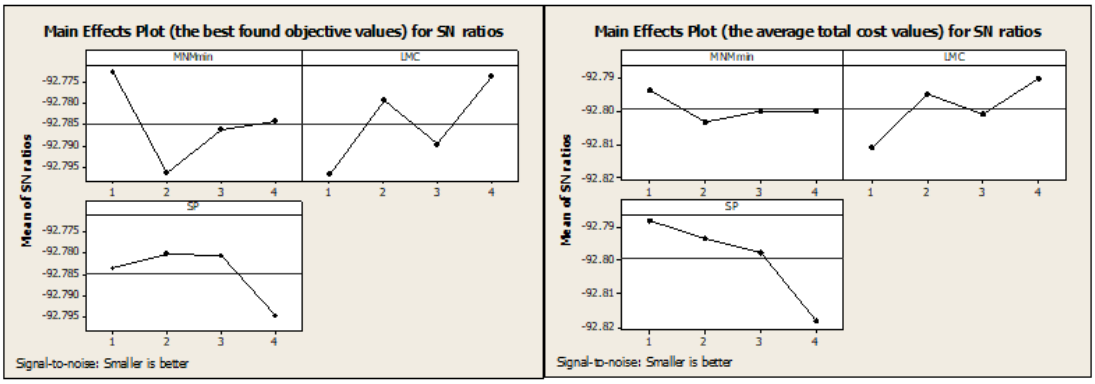


Figure 5.6 Main factor effects identified by S/N ratios for the best found and average total cost values (NCB = 15)

Table 5.2 The response tables for S/N ratios of the best and average total cost values according to different NCBs

	Response: The best found total cost values				**Response: The average of total cost values**			
NCB = 5	Level	MNMmin	LMC	SP	Level	MNMmin	LMC	SP
	1	-94.55	-94.22	-93.40	1	-95.00	-94.70	-93.49
	2	-94.72	-94.17	-93.34	2	-94.97	-94.50	-93.47
	3	-94.06	-94.46	-94.93	3	-94.55	-94.96	-95.87
	4	-94.28	-94.75	-95.93	4	-94.58	-94.93	-96.27
	Delta	0.66	0.57	2.59	Delta	0.45	0.46	2.80
	Rank	2	3	1	Rank	3	2	1
NCB = 10	Level	MNMmin	LMC	SP	Level	MNMmin	LMC	SP
	1	-92.89	-92.92	-92.91	1	-92.91	-92.96	-92.92
	2	-92.88	-92.89	-92.87	2	-92.93	-92.93	-92.92
	3	-92.91	-92.88	-92.88	3	-92.93	-92.91	-92.92
	4	-92.91	-92.89	-92.91	4	-92.95	-92.91	-92.96
	Delta	0.03	0.04	0.04	Delta	0.04	0.05	0.04
	Rank	3	2	1	Rank	2	1	3
NCB = 15	Level	MNMmin	LMC	SP	Level	MNMmin	LMC	SP
	1	-92.77	-92.80	-92.78	1	-92.79	-92.81	-92.79
	2	-92.80	-92.78	-92.78	2	-92.80	-92.79	-92.79
	3	-92.79	-92.79	-92.78	3	-92.80	-92.80	-92.80
	4	-92.78	-92.77	-92.79	4	-92.80	-92.79	-92.82
	Delta	0.02	0.02	0.01	Delta	0.01	0.02	0.03
	Rank	1	2	3	Rank	3	2	1

According to Table 5.2, it can also be concluded that these algorithm parameters may have different ranks of variability when the response variable and the *NCB* value are altered. For instance, when the *NCB* is set to 5, the *SP* factor has the largest variability (i.e., first rank) on both the best found and average total cost values. However, the ranks of *LMC* and MNM_{min} will change according to the chosen response variable. Based on the S/N ratios, the main factor levels: MNM_{min} – 3, *LMC* – 2, and *SP* – 2 are determined as the optimal factor levels that reduce both the best found and the average total cost values when the *NCB* is set to 5. Similarly, MNM_{min} – 2, *LMC* –

3, and *SP* – 2 are detected as the ideal parameter levels that decrease the best-found total cost value in the case of *NCB* = 10. On the other hand, MNM_{min} – 1, *LMC* – 4, and *SP* – 2 are determined as the ideal factor levels for the minimum average total cost value.

Table 5.3 ANOVA results for the average total cost values according to different NCB values

Analysis of Variance for the average total cost values, using Adjusted SS for Tests(NCB = 5)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
MNMmin	3	36538881	36538881	12179627	10.90	0.008
LMC	3	31032247	31032247	10344082	9.25	0.011
SP	3	1103132604	1103132604	367710868	328.95	0.000
Error	6	6706985	6706985	1117831		
Total	15	1177410717				

S = 1057.28 R-Sq = 99.43% R-Sq(adj) = 98.58%

Analysis of Variance for the average total cost values, using Adjusted SS for Tests(NCB = 10)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
MNMmin	3	80477	80477	26826	3.58	0.086
LMC	3	147554	147554	49185	6.57	0.025
SP	3	96970	96970	32323	4.32	0.061
Error	6	44944	44944	7491		
Total	15	369945				

S = 86.5488 R-Sq = 87.85% R-Sq(adj) = 69.63%

Analysis of Variance for the average total cost values, using Adjusted SS for Tests(NCB = 15)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
MNMmin	3	4765	4765	1588	0.98	0.461
LMC	3	24358	24358	8119	5.02	0.045
SP	3	52232	52232	17411	10.77	0.008
Error	6	9696	9696	1616		
Total	15	91050				

S = 40.1994 R-Sq = 89.35% R-Sq(adj) = 73.38%

Finally, when the *NCB* is set to 15, optimal parameter levels that decreased the best found total cost value comprised: MNM_{min} – 1, *LMC* – 4, and *SP* – 2 whereas one of these ideal parameter levels changed to *SP* – 1 for achieving the minimum average total cost value. It is also obviously seen in Figures 5.1 – 5.3 that the main factor effects with the calculated S/N ratios maintained the same optimal factor levels. In a nutshell, the combinations of the optimal factor levels provided by the computational experiments are given in Table 5.4.

Table 5.4 Optimal parameter levels according to different NCB values and response variables in case of using the random neighbor strategy

Controllable factors	**The bestbest-foundal cost value**						**The average total cost value**						**Most common factor levels (values)**
	Level			**Value**			**Level**			**Value**			
NCB	5	10	15	5	10	15	5	10	15	5	10	15	All of the *NCB*s
MNM_{min}	3	2	**1**	10	5	3	3	**1**	**1**	10	3	3	1(3)
LMC	2	3	**4**	10	15	30	2	**4**	**4**	10	30	30	4(30)
SP	**2**	**2**	**2**	4	4	4	**2**	**2**	1	4	4	2	2(4)

In order to select the generic best parameter values among these ideal factor levels for different *NCB* values and response variables given in Table 5.4, the most common factor levels whose frequency is the largest are used. Thus, the generic ideal parameter levels (or values) for the proposed matheuristic solution approach are determined as MNM_{min} – 1(3), *LMC* – 4(30), and *SP* – 2(4) for solving larger sized problem instances when the random neighbor generation strategy is utilized. Since a full factorial experimental design is not applied, all the factor-level combinations including these generic ideal parameter levels are not presented in the L16 orthogonal array (Table 5.1). Therefore, the proposed SA-based matheuristic algorithm is also re-run with these ideal parameter values for the confirmation runs in the subsequent section.

Lastly, it should be emphasized here again that the random neighbor generation strategy in the proposed matheuristic approach is employed while determining the optimal factor level combinations, since most of these parameters (except the step size division parameter, *SP*) have no significant importance when the proposed greedy search-based neighbor strategy is implemented. In other words, changing the values of *LMC* and MNM_{min} may not lead to considerable variability in the chosen response variables when the proposed greedy search-based neighbor strategy is applied because of its fast convergence property. Therefore, the ideal values of these parameters are set to MNM_{min} – 1(3), *LMC* – 1(5), and *SP* – 2(4) for solving all the problem instances in case of the usage of greedy search-based neighbor strategy. Note that the formerly determined most common factor levels (or generic best parameter values) in the case of random neighbor generation strategy are not changed for the MNM_{min} and *SP* parameters. Only the ideal factor level of the *LMC* parameter is decreased from 30 to

5 in the case of the usage of greedy search-based neighbor strategy since it can provide faster convergence to the global optimal solutions when compared to the random neighbor strategy.

5.3 Confirmation experiments and comparison of the optimization results

The proposed solution approach including the random neighbor strategy is re-run three times again with the formerly determined generic ideal parameter values for the NCB = 15 and the results of these additional confirmations experiments are presented in Table 5.5. In these confirmation runs, the proposed matheuristic solution approach found the optimal solution two times (or the minimum total cost value, i.e., 43486.2 Turkish lira) for NCB = 15. The convergence graphs of the confirmation run for the best-found solution are also depicted in Figure 5.7. Moreover, the standard 42 different corrugated board dimensions that are pre-specified by the company managers are also used as input data in the proposed MINLP model, and then, the result of the reduced MILP model is also reported in Table 5.5.

When compared to the existing cutting plan of the manufacturing company, the proposed matheuristic solution approach achieved a 1.7% better total cost value. Furthermore, it should be noted here that this improvement percentage may increase in case of larger demand quantities and more amount of different corrugated box dimensions. In other words, when the towel radiator boxes with more than 30 different dimensions with a larger amount of demands are considered, this improvement percentage may rise rapidly.

Apart from these standard corrugated board dimensions, three greedy heuristic search procedures are implemented to determine the corrugated board dimensions based on the dimensions of the towel radiator corrugated boxes demanded by the customer firm. The first one is to use 15 different corrugated box dimensions with the maximal demand quantities as input data in the proposed MINLP model. It should be noted here that the maximal corrugated board dimensions, i.e., MW = 1750 and ML = 2000 millimeters, are also incorporated as dummy sizes into the data set to achieve a feasible solution.

Table 5.5 Results of the confirmation runs and comparisons with the existing cutting plan of the manufacturing company

Confirmation runs of the proposed SA-based matheuristic solution approach				**MILP model**		
Best	**Worst**	**Average**	**Std. dev.**	**Optimization with standard 42 corrugated board dimensions of the company**	**Optimization with 15 different corrugated box dimensions with maximal demand**	**Optimization with 15 different corrugated box dimensions with maximal widths/lengths**
43486.2	43596.8	43523.1	52.13	44241.63	43933.53	47719.93 – 48404.59
Avg. CPU time (sec.)		734.38		0.618	0.257	0.227 – 0.235

When the resulting MILP model is solved by the Gurobi MIP solver, the optimal total cost value is provided as 43933.53 Turkish lira, which is also better than the solution with the standard 42 different corrugated board dimensions in the existing cutting plan of the manufacturing company. Fortunately, the proposed matheuristic approach has still a %1.02 better total cost value than this greedy search procedure. Furthermore, two greedy heuristic search procedures in which 15 different corrugated boxes with maximal dimensions (i.e., widths and lengths) are chosen as input data for the proposed MINLP model are also implemented. However, the resulting total cost values (i.e., 47719.93 and 48404.59 Turkish lira) produced by the reduced MILP model are comparatively worse than all the previously produced solutions (see Table 5.5). Therefore, it can be concluded that the proposed matheuristic solution approach can generate better cutting plans with lower total cost values than the existing cutting plan of the manufacturing company as well as the greedy heuristic search procedures.

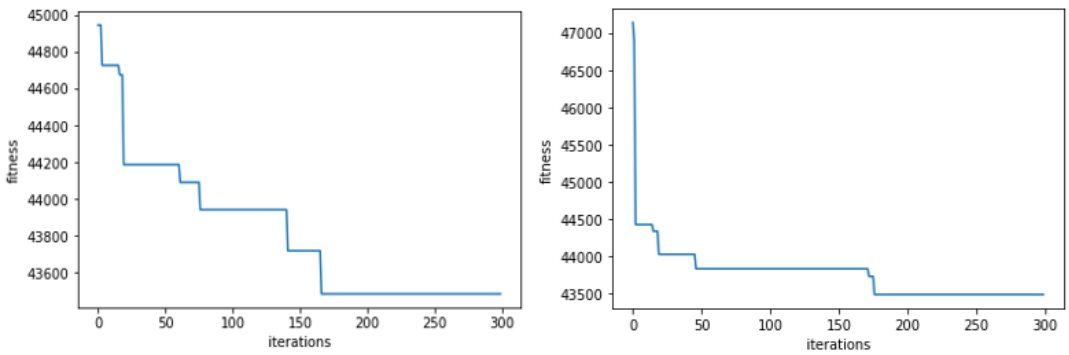


Figure 5.7 The convergence graphs of the proposed solution approach with the random neighbor generation strategy in the confirmation runs

On the other hand, although the proposed matheuristic approach outperforms these alternative solution procedures in terms of the solution quality, its computational effort (i.e., average CPU time) is considerably high since it evaluates several solution alternatives by solving the reduced MILP model many times at every iteration. Fortunately, it should be highlighted here that the investigated integrated stock selection and sizing problem is a tactical-level industrial problem and should be solved only once at the beginning of each week to generate the cutting plan and purchasing orders. In other words, since the present problem is not an operational level problem, which needs to be solved many times during each working day within a short time, the average computing time of the proposed matheuristic approach is evaluated as reasonable and acceptable by the company managers.

Finally, the details of the best-found solution are given in Table 5.6. According to the cutting plan in this table, 10 different corrugated board dimensions should be used in the production of all the towel radiator boxes with 30 different dimensions. In other words, only 10 out of a total number of 15 (i.e., *NCB*) corrugated board dimensions will be enough to satisfy the weekly demand of the customer firm with minimal waste. In addition to the optimal sizes of these corrugated boards, their purchasing order quantities are also reported in Table 5.6. Furthermore, the cutting plan that represents the stock selection and production requirements of the corrugated boards for each corrugated box is also reported. In other words, this table shows which corrugated box dimension should be obtained from the cutting of which corrugated board dimension. The optimal assignment of the corrugated box productions to the corrugated board dimensions is presented. For instance, the towel radiator corrugated boxes having the following dimensions [1250, 1971] and [1350, 1971] should be produced by using the corrugated board dimension of [1350, 1971]. Besides, the amounts of required corrugated boards for the production of each corrugated box are also included in Table 5.6. For instance, 60 and 300 units of corrugated board dimension of [1350, 1971] are required for the production of the aforementioned two corrugated box dimensions, respectively. Therefore, its purchasing order quantity is also determined as 360 units as seen in Table 5.6.

Table 5.6 The proposed cutting plan, corrugated board dimensions, their purchasing and production requirement quantities

The corrugated board dimensions		**Purchasing order quantities (units)**	**Cutting plan and production requirements [The chosen corrugated box dimensions] (Quantity of the required corrugated board)**
Width	**Length**		
1150	1305	372	[950, 1305] (105), [1050, 1005] (20), [1150, 1305] (247)
1150	1331	357	[1050, 1331] (140), [1150, 1105] (71), [1150, 1331] (146)
1150	1705	288	[1050, 1571] (43), [1150, 1571] (140), [1150, 1705] (105)
1150	1905	256	[1150, 891] (43), [1150, 905] (43), [1150, 1905] (170)
1150	1971	408	[1050, 1971] (70), [1150, 1971] (283), [1110, 1925] (25), [2010, 1925] (30)
1350	1331	531	[1350, 1105] (70), [1350, 1305] (311), [1350, 1331] (150)
1350	1705	218	[1150, 805] (7), [1350, 1605] (130), [1350, 1705] (70)
1350	1905	347	[1350, 891] (15), [1350, 1905] (332)
1350	1971	360	[1250, 1971] (60), [1350, 1971] (300)
1720	1935	154	[1350, 1925] (40), [1410, 1925] (54), [1350, 1865] (45), [1720, 1935] (15)

CHAPTER SIX
COMPUTATIONAL STUDY

Since the present integrated stock selection and sizing study is a case-oriented application and has not been studied in the literature before, the instances for benchmarking had to be generated based on the manufacturing company's previous weekly demand records of the towel radiator boxes to test the performance of the proposed matheuristic-based approach. In fact, small, medium, large, and very large-sized problem instances are generated by increasing the number of different towel radiator box dimensions (i.e., 30, 60, 95, and 130) demanded by the customer firm as well as the maximum number of different corrugated board dimensions (i.e., 5, 10, 15, 20, 30, 31, 42, 58, and 65) that will be purchased from the vendors. For example, the largest-sized problem instance has weekly demands for 130 different corrugated box dimensions and aims to determine the maximum number of 42, 58, and 65 different corrugated board dimensions for purchasing orders.

Although the customer firm generally orders between the maximum number of 30 and 40 different corrugated box dimensions, its several weekly demand data are aggregated to derive the larger-sized problem instances. All the datasets of the generated benchmark instances are given in Appendix 1. The comparative analyses are performed on these problem instances by making use of the proposed MINLP model. In other words, to give a deep insight to other researchers and practitioners in the corrugated box production industry, the proposed MINLP model is first reduced to a MILP model with the help of standard 42 different corrugated board dimensions that are determined by the company process managers and then, the resulting MILP model is solved by the Gurobi solver. Furthermore, the aforementioned three greedy heuristic search procedures are also applied in that MILP model to make further comparisons. In detail, the reduced MILP model is solved by using the pre-specified *NCB* different corrugated box dimensions with the maximal demand and maximal width/length values. Moreover, the lower bound of each problem instance is also estimated by solving the reduced MILP model with the maximal number of different corrugated board dimensions that are already equal to the total number of different corrugated box

dimensions demanded by the customer firm. It should be noted here again that the company managers desire to obtain the minimum number of required corrugated board dimensions that will provide minimal waste and purchasing costs while satisfying the weekly corrugated box demands.

For the fine-tuning process for the parameters of the proposed matheuristic algorithm, a realistic-sized benchmark instance with a total number of 30 different corrugated box dimensions is selected instead of applying Taguchi's DOE approach for each size of the benchmark instance. Fortunately, this provides less computational effort to obtain the optimization results for each design combination. Thus, the same generic ideal parameter values that are pre-specified by using the Taguchi DOE approach are utilized in the proposed matheuristic solution algorithm for solving each problem instance. It should be noted here again that these generic ideal parameter values are determined in the case of using the random neighbor generation strategy in the SA-based matheuristic algorithm. It's needed to mention here again that the length of the Markov chain (*LMC*) is only decreased from 30 to 5 when the greedy search-based neighbor generation strategy is implemented for solving each benchmark instance because of its rapid convergence property. On the other hand, the values of the remaining SA parameters (i.e., T_0, q, and el_{max}) whose values mainly depend on the range of best and worst fitness function (objective) values, are recomputed through Equations (5.1) – (5.5) for each problem instance and are given in Table 6.1.

Table 6.1 The proper values of the SA parameters for the random neighbor strategy

Problem instances	**Initial temperature (T_0)**	**Final temperature (T_{fin})**	**Cooling rate (q)**	**Max. number of iterations (el_{max})**
Small-sized ISSS-30	7169.95	20.11	0.981	300
Medium-sized ISSS-60	63108.37	179.16	0.992	750
Large-sized ISSS-95	59212.12	168.34	0.994	1000
Very large-sized ISSS-130	299104.55	852.05	0.996	1500

Unlike the other parameters of the proposed matheuristic algorithm (i.e., MNM_{min}, *LMC*, and *SP*) whose ideal values are specified by the Taguchi DOE approach, determining constant values for these parameters, i.e., T_0, q, and el_{max}, is not

appropriate and reasonable for all the benchmark instances with different sizes. It should also be noted here that only the maximum number of iterations (i.e., el_{max}) is decreased to 5, 20, 30, or 50 according to the problem size in the case of using the greedy search-based neighbor generation strategy due to its faster convergence characteristic.

By using these generic ideal parameter values as well as the recomputed values of the remaining SA parameters given in Table 6.1, optimization results of the proposed matheuristic approach with different neighbor generation strategies are provided as given in Table 6.2 and 6.3. In detail, the best/worst, average, and standard deviations of the total cost objective and the average computation times are presented for each problem instance in these tables. Moreover, the estimated lower bounds for each problem instance that are obtained from the reduced MILP model are also reported in Table 6.2 and 6.3. According to these tables, the small-sized benchmark instance can be easily solved within a reasonable computation time by using both the random and greedy search-based neighbor strategies in the proposed matheuristic approach and the best-found solution is the same with the estimated lower bound value of the problem when the maximum number of corrugated board dimensions are set to 10 and 15. However, it is clearly seen from this table that the average computation time of the proposed matheuristic algorithm will rise significantly when the number of different corrugated boxes and corrugated board dimensions is increased due to the huge expansion of the solution space. In this circumstance, the proposed matheuristic approach has to evaluate many more solution alternatives at every iteration, which cause higher computational effort and longer run times. Fortunately, when compared to the random neighbor generation strategy, the usage of greedy search-based neighbor generation strategy in the proposed algorithm can provide better quality solutions within relatively shorter computing times for the larger-sized problem instance. However, it should be noted here that the random neighbor generation strategy outperforms the greedy search-based one in terms of solution quality for some small and medium-sized problem instances in which the value of a maximum number of different corrugated board dimensions is set too low.

Table 6.2 Computational results of the proposed solution approach with a randomization-based neighborhood generation mechanism

Problem sizes			**Matheuristic solution approach with a randomization-based neighborhood generation mechanism**					
	Max. # of corrugated boxes	**Max. # of corrugated boards (*NCB*)**	**Best**	**Worst**	**Average**	**Std. dev.**	**Avg. CPU time**	**Max. # of iterations (el_{max})**
Small	30	5	**46129.6**	46880.1	46379.7	353.8	309.55	5
		10	**43486.2**	43932.6	43804.6	92.01	663.1	5
		15	**43486.2**	43596.8	43523.1	52.13	734.38	5
Medium	60	15	**223745.1**	225537.1	224889.4	811.4	2508.2	20
		20	218775.1	220287.5	219309.3	692.6	3281.9	20
		30	211215.9	214037.8	213061.9	1306	4750.8	5
Large	95	20	237006.3	242928.8	240380.6	2487	7774.8	50
		31	232870.5	233362.4	233164.3	211.9	10616.7	50
		42	228097.4	229638.2	229048.7	679.1	14465.9	5
Very Large	130	42	433704.8	435010.3	434408.4	537.8	20009.9	50
		58	423860.9	426972.7	424906.5	1461	23914.7	30
		65	420692.2	422631.2	421580.8	799.7	25245.5	10

Table 6.3 Computational results of the proposed solution approach with a greedy search-based neighborhood generation mechanism

Problem sizes			**Matheuristic solution approach with a greedy search-based neighborhood generation mechanism**					
	Max. # of corrugated boxes	**Max. # of corrugated boards (*NCB*)**	**Best**	**Worst**	**Average**	**Std. dev.**	**Avg. CPU time**	**Max. # of iterations (el_{max})**
Small	30	5	49662.7	50990.7	50354.3	543.6	**3.6**	5
		10	**43486.2**	44063.6	44017.4	32.66	**14.2**	5
		15	**43486.2**	43486.2	43486.2	0	**32.22**	5
Medium	60	15	228257.2	231933.4	230355.2	1545.1	**229.3**	20
		20	**216317.8**	224045.1	220717.8	3244.5	**408.1**	20
		30	**209946.8**	211674.9	210820.9	705.6	**223.63**	5
Large	95	20	**236695.3**	238843.5	237886.1	892.4	**1548.8**	50
		31	**225798.5**	226167.3	225951.2	157.1	**3344.2**	50
		42	**225798.5**	225798.5	225798.5	0	**796.52**	5
Very Large	130	42	**428265.6**	430604.3	429279.9	979.6	**9323.5**	50
		58	**414324.7**	415596.3	414748.6	599.4	**11767.4**	30
		65	**413153.7**	413908.6	413410.8	352.1	**5060.8**	10

For all the remaining cases where the maximum number of different dimensions for the corrugated boxes and corrugated boards are increased gradually, the proposed matheuristic approach with the greedy search-based neighbor generation strategy provides superior results in terms of both solution quality and solution time. It should also be highlighted here that the best-found solutions that are achieved by the proposed algorithm with the greedy search-based neighbor strategy are the same as the values of the estimated lower bounds for the small, medium, and large-sized problem instances. However, the proposed matheuristic approach with the random neighbor strategy has reached the lower bound value of the small-sized problem instance only. In other words, except for the small-sized problem instance, the proposed matheuristic approach with the random neighbor strategy couldn't achieve the best solutions as good as the lower bound values obtained from the reduced MILP model. Fortunately, the usage of such a greedy search-based neighbor generation strategy in the proposed matheuristic approach makes it possible to reach very close to the lower bound value even for the largest size problem instance.

Additionally, it is also clearly seen in Table 6.2 and 6.3 that the total costs can be decreased for all the benchmark instances by increasing the maximum number of different corrugated board dimensions. However, such an increase in stock diversity is not generally desired by the process managers of the manufacturing company. It should be noted that if the total number of raw materials that will be used in production were limited by a constraint, the MILP model would have included p-median problem constraint. In this case the model would become an NP-hard problem, and this would make the model unsolvable in polynomial time especially for larger sized data. Even this case shows the necessity and applicability of the developed solution approach. Table 6.4 shows the estimated lower bounds of the problems.

Comparisons of the results of the proposed solution approach with the existing cutting and procurement plans of the manufacturing company are also carried out and presented in Table 6.5 and 6.6. In addition, further comparisons with the greedy search heuristics embedded in the proposed MILP model are also involved in these tables. As shown in these tables, if the maximum number of different corrugated board

dimensions is set to a small value (i.e., 5, 15, 20, and 42 for the small, medium, large, and very large-sized problem instances, respectively) which is lower than the optimal stock variety, the total cost value will increase. On the other hand, if the stock diversity can be determined optimally (For instance, 10, 29, 31, and 58 for the small, medium, large, and very large-sized problem instances, respectively), the total cost of the manufacturing company will be improved reasonably.

Table 6.4 Estimated lower bounds of the problems

Problem sizes			**Estimated lower bounds obtained from the reduced MILP model**	
	Max. # of corrugated boxes	**Max. # of corrugated boards (*NCB*)**	**Total Cost**	**CPU time**
Small	30	5	43486.2	0.1
		10		
		15		
Medium	60	15	209946.8	1.49
		20		
		30		
Large	95	20	225798.5	3.78
		31		
		42		
Very Large	130	42	413153.03	7.33
		58		
		65		

It is also seen in Table 6.5 that significant cost savings can be provided by making use of the proposed matheuristic approach with the greedy search-based neighbor strategy for all of the benchmark instances when compared to the existing cutting and purchasing plans of the manufacturing company. According to Table 6.6, the cost-saving percentage will also rise rapidly when the order quantity and the maximum number of different corrugated box dimensions demanded by the customer firm are increased. In other words, the improvement percent accomplished by the proposed algorithm will increase for the larger-sized problem instances when compared to the existing cutting and purchasing plan of the manufacturing company. Fortunately, usage of the proposed MILP model integrated with a greedy search heuristic that employs the pre-specified number of different corrugated box dimensions with the maximal demand can also provide good quality solutions within very shorter run times

since the resulting MILP model is solved only once. However, it should be emphasized here that the best solutions found by the proposed algorithm that are also equal to the lower bound values of the test problems may not be achieved in this situation. In other words, global optimal solutions may not be found by using such a greedy search procedure integrated with the proposed MILP model.

Table 6.5 The results obtained from the MILP model with greedy search heuristics

Problem sizes			**MILP model with greedy search heuristics**					
	Max. # of corrugated boxes	**Max. # of corrugated boards (*NCB*)**	**Optimization with standard 42 corrugated boards of the company**		**Optimization with *NCB* different corrugated boxes with maximal demand**		**Optimization with *NCB* different corrugated boxes with maximal widths/lengths**	
			Total cost	**CPU time**	**Total cost**	**CPU time**	**Total cost**	**CPU time**
Small	30	5			50477.4	0.15	71525.8 57012.8	0.120.11
		10	44241.63	0.62	44531.2	0.17	55407.3 55146.6	0.180.19
		15			43933.53	0.25	47719.93 48404.59	0.220.23
Medium	60	15			**222989.8**	0.43	285997.66 280211.11	0.410.45
		20	216595.9	1.82	217504.4	0.52	253064.24 261920.21	0.550.66
		30			210757.4	0.83	237264.21 222113.19	0.860.96
Large	95	20			**235188.8**	0.89	277397.2 320656.9	1.081.01
		31	232037.1	2.05	227800.8	1.3	270209.41 273125.45	1.261.37
		42			**225798.5**	1.91	254453.26 239950.92	1.812.0
Very Large	130	42			**426691.6**	2.42	510683.8 504328.8	2.692.71
		58	444666.8	2.46	415502.1	3.53	470856.9 437989.2	3.673.60
		65			414997.4	3.95	459004.7 436082.9	3.893.95

For instance, except for the large-sized problem instance with 95 different corrugated box dimensions, global optimal solutions are not achieved for all of the remaining problem instances by using such a greedy heuristic procedure. In other words, only the global optimal solution of the large-sized problem instance (i.e., 225798.5) is found by this greedy heuristic when the maximum number of different corrugated board dimensions is set to 42. In addition to the cost-saving advantage, the proposed matheuristic approach also offers opportunities for the company managers to reduce the stock variety considerably. For the medium-size problem instance, 3.06%

better total cost value and 28.5% lower stock variety can be obtained via the proposed matheuristic approach compared to the existing cutting/purchasing plan of the company. Even for the largest-sized problem instance with 130 different corrugated box dimensions, 3.7% cost saving can be provided by the proposed matheuristic approach without increasing the existing stock variety (i.e., maximum # of 42 different corrugated board dimensions) of the manufacturing company.

Table 6.6 Comparison of the computational results with the existing cutting plan of the manufacturing company and the results of the proposed matheuristic approach

Problem sizes			**Optimization with standard 42 corrugated boards of the company**		**The best-found solutions of matheuristic approach with different neighborhood generation strategies**		**Optimal variety of corrugated board**	**Cost saving percent (%) compared to the existing cutting plan of the company**	**Stock variety improvement percent (%) compared to the existing cutting plan of the company**
	Max. # of corrugated boxes	**Max. # of corrugated boards (*NCB*)**	**Total cost**	**CPU time**	**Randomized**	**Greedy search**			
Small	30	5	44241.63	0.62	46129.6	49662.7	10	- 4.2%	88.1%
		10			**43486.2**	**43486.2**		1.7%	76.2%
		15			**43486.2**	**43486.2**		1.7%	64.3%
Medium	60	15	216595.9	1.82	223745.1	228257.2	29	- 3.3%	64.3%
		20			218775.1	**216317.8**		0.13%	52.4%
		30			211215.9	**209946.8**		3.06%	28.5%
Large	95	20	232037.1	2.05	237006.3	236695.3	31	- 2.1%	52.4%
		31			232870.5	**225798.5**		2.68%	26.2%
		42			228097.4	**225798.5**		2.68%	0%
Very Large	130	42	444666.8	2.46	433704.8	428265.6	58	3.7%	0%
		58			423860.9	**414324.7**		6.8%	- 38.1%
		65			420692.2	**413153.7**		7.1%	- 54.7%

Finally, the convergence graphs of the proposed solution approach with different neighbor generation strategies for the best-found solutions of the medium, large, and very large sized problem instances are depicted in Figures 6.1 – 6.3. According to these graphs, the number of iterations necessary to achieve the best solutions with minimal waste and purchasing costs will decrease in the case of using the greedy search-based neighbor generation strategy.

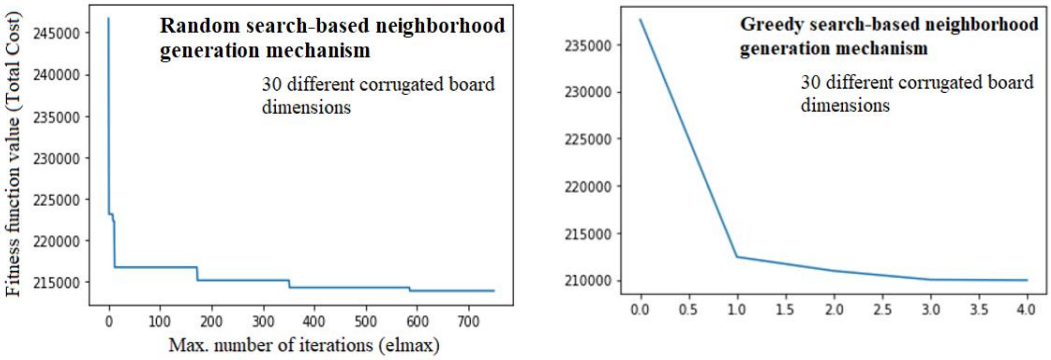


Figure 6.1 Convergence graphs of the best-found solutions for the medium-sized problem instance via the proposed matheuristic approach with different neighborhood generation strategies

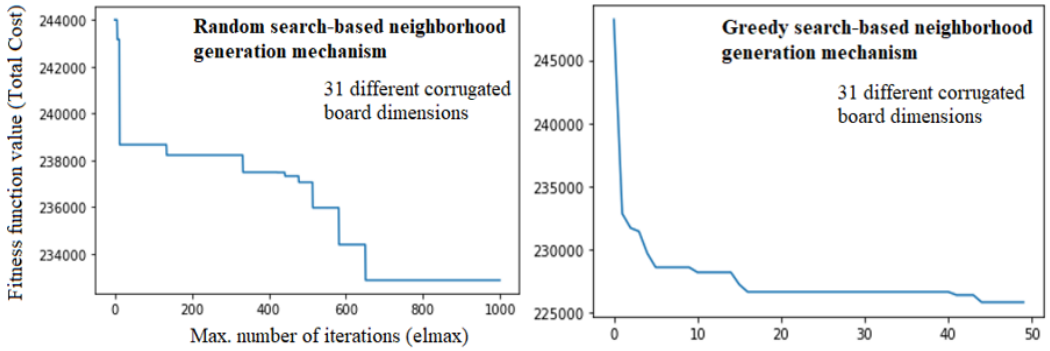


Figure 6.2 Convergence graphs of the best-found solutions for the large-sized problem instance via the proposed matheuristic approach with different neighborhood generation strategies

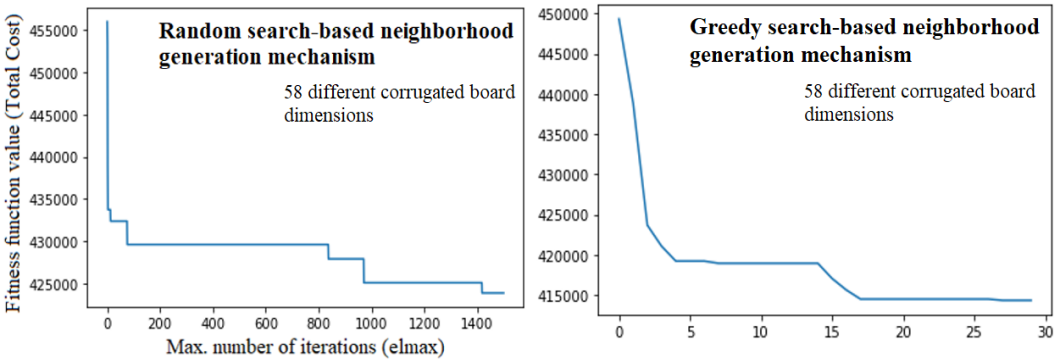


Figure 6.3 Convergence graphs of the best-found solutions for the very large-sized problem instance via the proposed matheuristic approach with different neighborhood generation strategies

For the medium, large, and very large sized problem instances, the details of these best-found solutions including the proposed cutting plans, stock selection, and the purchasing quantities of each corrugated boards are also presented in Appendix 2 - 4. It is also clearly seen from tables that the minimum required number of different

corrugated board dimensions will rise meaningfully as the number of different corrugated boxes dimensions demanded by the customer firm increases. In other words, the total required number of different corrugated board dimensions that satisfy the weekly demand of the customer firm with minimal waste and purchasing costs will increase in case of a large number and variety of demanded corrugated boxes. For instance, only 10 out of a total number of 30 (i.e., *NCB*) different corrugated box dimensions will be enough to satisfy the weekly demand of the customer firm in case of the solution of small sized problem instance whereas this value (i.e., optimal stock variety) will increase up to 58 for the largest sized problem instance with 130 different corrugated box dimensions demanded by the customer firm.

After evaluating the proposed cutting and purchasing plans by the company managers, they have announced that the produced solutions by the matheuristic solution approach offer significant improvements in terms of both waste amounts of the purchased corrugated boards and stock diversity. They have also declared that all the produced solutions by the matheuristic approach are acceptable and applicable and can also balance between the total number of different corrugated board dimensions (i.e., stock variety) and the total waste and purchasing costs. Additionally, they also declared that using such a matheuristic solution approach that produces superior cutting and purchasing plans may also contribute to the zero-waste and environmentally friendly production policies of the company.

CHAPTER SEVEN
CONCLUSION AND FUTURE RESEARCH

7.1 Summary of the Thesis and Concluding Remarks

In this thesis, a real-life integrated stock selection and sizing problem of a corrugated box manufacturing company in Manisa, Turkey is addressed. To provide optimal cutting and purchasing plans for the supplied corrugated boards that are used in the production of corrugated boxes with many different dimensions, a novel MINLP model is first developed. Due to the highly non-convex nonlinearities in the objective and several constraint functions of this mathematical model, it is not possible to transform it into an equivalent linear form by any linearization approach, and therefore, it cannot be solved by any optimization solver properly. For this reason, a matheuristic-based solution approach that integrates a MILP model into a well-known metaheuristic technique, namely the SA algorithm is also proposed. Indeed, dimensions of the purchased corrugated boards are determined by the SA algorithm instead of defining them as decision variables in the proposed MINLP model and therefore, a reduced MILP model with a fully linear objective and constraint functions that can be easily solved by any MIP solver is obtained. By making use of such a matheuristic-based approach, the present integrated stock selection and sizing problem can also be reduced to just a stock selection problem with a fully linear structure.

The proposed matheuristic approach with two different neighbor generation mechanisms (i.e., random and greedy search-based strategies) is first tested on a real-life application study for the towel radiator box products of the manufacturing company. For the fine-tuning process of the proposed solution approach's parameter values, Taguchi's DOE approach with several experimental analyses is also performed. Thereafter, a comprehensive computational study is also carried out to test the performance of the proposed matheuristic-based solution approach and make further comparisons with the existing cutting and purchasing plans of the manufacturing company. The computational results have demonstrated that superior results which can balance the total number of different corrugated board dimensions (i.e., stock variety) and the minimal waste and purchasing costs, can be effectively

provided via the proposed matheuristic approach with a greedy search-based neighbor generation strategy. Indeed, the solutions generated by the proposed matheuristic approach provide significantly lower total cost value and require less stock variety than the existing cutting and purchasing plans of the manufacturing company. It is also proven by the computational experiments that the percentage of cost saving and the stock variety reduction percent, which are provided by the proposed matheuristic solution approach will increase significantly when the demand quantities and product varieties (i.e., the number of different corrugated box dimensions demanded by the customers) are much more increased.

7.2 Future Research Directions

The uncertainty in the customer demands as well as the dimensions of the corrugated boxes cannot be handled through the proposed deterministic solution approach. Therefore, stochastic programming techniques and fuzzy logic methods can be embedded into the proposed matheuristic approach in the future to produce robust cutting and purchasing plans by considering the dynamic and uncertain nature of customer demands.

Furthermore, the development of a dynamic (or multi-period) stock selection and sizing model in which the periodic corrugated board inventories as well as the weekly demands of the corrugated boxes may change considerably from one period to another can be scheduled as future work. Moreover, the proposed matheuristic approach can also be integrated into the company's ERP system in the future.

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APPENDICES

APPENDIX 1: Datasets of the Generated Benchmark Instances

All of the datasets of the test problems for the integrated stock selection and sizing problem of the corrugated box manufacturing company can be accessed at the following link:

https://data.mendeley.com/datasets/d39dhswbmb/draft?a=ab26c6f9-2490-47ec-bbbd-e710c5bd2802

APPENDIX 2: The Details of the Proposed Cutting and Purchasing Plans for the Medium-Sized Problem Instance

Corr. board No.	The corrugated board dimensions		Purchasing quantities (units)	Cutting plan and production requirements [The chosen corrugated box dimensions] (Quantity of the required corrugated board)
	Width	Length		
1	1750	2000	143	[1720, 1935] (75), [1750, 705] (20), [850, 785] (5), [850, 1321] (15), [1450, 752] (15), [1450, 652] (10)
2	950	1305	464	[950, 1305] (389), [750, 1185] (75)
3	1050	1331	370	[1050, 1005] (20), [1050, 1331] (350)
4	1050	1971	368	[950, 905] (138), [950, 1905] (55), [1050, 1971] (175)
5	1150	805	1333	[1150, 805] (1273), [2020, 750] (60)
6	1150	891	740	[1150, 891] (740)
7	1150	905	548	[1150, 905] (508), [2150, 905] (40)
8	1150	1105	1049	[1150, 1105] (1049)
9	1150	1305	1188	[1150, 1305] (1188)
10	1150	1331	1555	[1150, 1331] (1505), [1110, 1327] (50)
11	1150	1571	555	[1050, 1571] (148), [1150, 1571] (247), [1150, 765] (160)
12	1150	1705	302	[950, 1705] (20), [1150, 1705] (282)
13	1150	1905	647	[1150, 1905] (647)
14	1150	1971	683	[1150, 1971] (648), [1150, 1971] (35)
15	1250	1971	203	[1250, 1971] (120), [1210, 965] (35), [1220, 965] (15), [1220, 1765] (30)
16	1350	1105	434	[1350, 1105] (384), [1240, 1065] (50)
17	1350	1305	723	[1350, 1305] (723)
18	1350	1331	745	[1350, 1331] (725), [1210, 1327] (20)
19	1350	1605	256	[1350, 1605] (130), [1220, 1365] (36), [1250, 1405] (30), [1350, 767] (60)
20	1350	1705	280	[1350, 1705] (280)
21	1350	1905	1987	[1350, 1905] (1987)
22	1350	1971	1665	[1350, 1925] (40), [1350, 1971] (1625)
23	1410	1925	270	[1410, 1925] (252), [1410, 1603] (18)
24	1350	1865	205	[1350, 891] (40), [1350, 905] (110), [1350, 1865] (45), [1250, 705] (10)
25	950	683	920	[950, 683] (920)
26	1150	1217	590	[1150, 1217] (590)
27	1350	1216	777	[1350, 1205] (187), [660, 1185] (10), [1350, 1216] (580)
28	1150	1749	395	[1150, 1749] (395)
29	1350	1749	300	[1350, 1749] (300)

APPENDIX 3: The Details of the Proposed Cutting and Purchasing Plans for the Large-Sized Problem Instance

Corr. Board No.	The corrugated board dimensions		Purchasing quantities (units)	Cutting plan and production requirements [The chosen corrugated box dimensions] (Quantity of the required corrugated board)
	Width	Length		
1	1750	2000	221	[1720, 1935] (75), [1420, 1365] (16), [1420, 1765] (17), [1710, 1327] (15), [1750, 705] (20), [1420, 965] (8), [1750, 505] (4), [1750, 1305] (10), [660, 1985] (8), [660, 1985] (8), [1450, 752] (15), [1450, 920] (15), [1450, 652] (10)
2	950	1305	499	[950, 1205] (15), [950, 1305] (389), [750, 1185] (75), [1850, 1305] (20)
3	950	1905	283	[950, 905] (138), [950, 1705] (20), [950, 1905] (55), [850, 785] (10), [750, 1320] (30), [830, 1900] (30)
4	1050	1331	405	[1050, 1305] (25), [1050, 1331] (350), [850, 1321] (30)
5	1050	1971	248	[1050, 705] (8),[1050, 1905] (15), [1050, 1971] (175), [2010, 1925] (30), [1050, 1505] (20)
6	1150	805	1383	[1150, 805] (1273), [1850, 705] (20), [2150, 805] (30), [2020, 750] (60)
7	1150	891	740	[1150, 891] (740)
8	1150	905	568	[1150, 905] (508), [1850, 905] (20), [2150, 905] (40)
9	1150	1105	1089	[1050, 1005] (20), [1150, 1105] (1049), [2150, 1105] (20)
10	1150	1205	873	[1150, 1205] (873)
11	1150	1305	1188	[1150, 1305] (1188)
12	1150	1331	1555	[1150, 1331] (1505), [1110, 1327] (50)
13	1150	1571	585	[1050, 1571] (148), [1150, 1571] (247), [1150, 765] (160), [1000, 1350] (30)
14	1150	1705	282	[1150, 1705] (282)
15	1150	1905	647	[1150, 1905] (647)
16	1150	1971	708	[1150, 1971] (648), [1110, 1925] (25), [1150, 1971] (35)
17	1250	1971	275	[1250, 1971] (120), [1210, 965] (35), [1210, 1925] (60), [1220, 965] (15), [1220, 1765] (30), [1250, 1805] (15)
18	1350	905	410	[1150, 869] (45), [1350, 869] (1), [1350, 891] (80), [1350, 905] (220), [2550, 805] (20), [2560, 700] (20), [1250, 705] (20)
19	1350	1105	434	[1350, 1105] (384), [1240, 1065] (50)
20	1350	1305	753	[1350, 1277] (30), [1350, 1305] (723)
21	1350	1331	745	[1350, 1331] (725), [1210, 1327] (20)
22	1350	1605	291	[1350, 1605] (130), [1210, 1365] (30), [1220, 1365] (36), [1220, 1565] (5), [1250, 1405] (30), [1350, 767] (60)
23	1350	1705	280	[1350, 1705] (280)
24	1350	1905	2032	[1350, 1905] (1987), [1350, 1865] (45)
25	1350	1971	1665	[1350, 1925] (40), [1350, 1971] (1625)
26	1410	1925	270	[1410, 1925] (252), [1410, 1603] (18)
27	950	683	920	[950, 683] (920)
28	1150	1217	590	[1150, 1217] (590)
29	1350	1216	782	[1350, 1205] (187), [1220, 1165] (5), [660, 1185] (10), [1350, 1216] (580)
30	1150	1749	395	[1150, 1749] (395)
31	1350	1749	300	[1350, 1749] (300)

APPENDIX 4: The Details of the Proposed Cutting and Purchasing Plans for the Very Large-Sized Problem Instance

Corr. Board No.	Corr. board dimensions		Purchasing qty. (units)	Cutting plan and production requirements [The chosen corrugated box dimensions] (Quantity of the required corrugated board)
	Width	Length		
1	1750	2000	145	[1720, 1935] (75), [1750, 705] (20), [1750, 505] (4), [1750, 1305] (10), [660, 1985] (8), [660, 1985] (8), [850, 785] (5), [830, 1900] (15)
2	950	905	1275	[950, 905] (275), [1850, 705] (20), [1850, 905] (20), [950, 891] (960)
3	950	1305	424	[950, 1205] (15), [950, 1305] (389), [1850, 1305] (20)
4	1050	1331	375	[1050, 1305] (25), [1050, 1331] (350)
5	1050	1971	242	[950, 1705] (20), [1050, 1905] (15), [1050, 1971] (175), [2010, 1925] (30)
6	1150	805	1363	[1150, 805] (1273), [2150, 805] (30), [2020, 750] (60)
7	1150	891	740	[1150, 891] (740)
8	1150	905	548	[1150, 905] (508), [2150, 905] (40)
9	1150	1105	1089	[1050, 1005] (20), [1150, 1105] (1049), [2150, 1105] (20)
10	1150	1205	873	[1150, 1205] (873)
11	1150	1305	1363	[1150, 1305] (1188), [1146, 1281] (175)
12	1150	1331	1555	[1150, 1331] (1505), [1110, 1327] (50)
13	1150	1571	642	[1050, 1571] (148), [1150, 1571] (247), [1150, 765] (160), [1150, 1505] (87)
14	1150	1705	282	[1150, 1705] (282)
15	1150	1905	647	[1150, 1905] (647)
16	1350	1105	435	[1350, 869] (1), [1350, 1105] (384), [1240, 1065] (50)
17	1350	1305	753	[1350, 1277] (30), [1350, 1305] (723)
18	1350	1331	725	[1350, 1331] (725)
19	1350	1605	232	[1050, 705] (8), [1350, 1605] (130), [1220, 1565] (5), [660, 1185] (10), [1250, 705] (10), [1350, 767] (60), [1150, 655] (9)
20	1350	1705	280	[1350, 1705] (280)
21	1350	1905	1987	[1350, 1905] (1987)
22	1350	1971	1665	[1350, 1925] (40), [1350, 1971] (1625)
23	1410	1925	270	[1410, 1925] (252), [1410, 1603] (18)
24	1350	1865	219	[1350, 905] (110), [1350, 1865] (45), [1350, 1855] (64)
25	1150	1971	708	[1150, 1971] (648), [1110, 1925] (25), [1150, 1971] (35)
26	1210	1925	254	[1210, 1925] (60), [1184, 939] (194)
27	1710	1327	244	[1710, 1327] (15), [750, 1185] (38), [850, 1321] (15), [750, 1320] (15), [850, 1255] (161)
28	950	683	920	[950, 683] (920)
29	1150	1217	590	[1150, 1217] (590)
30	1350	1216	767	[1350, 1205] (187), [1350, 1216] (580)
31	1150	1749	395	[1150, 1749] (395)
32	1350	1749	300	[1350, 1749] (300)
33	950	1505	746	[950, 1505] (746)
34	990	1305	610	[990, 1305] (610)
35	1150	1277	942	[1150, 1277] (942)
36	1350	805	466	[2550, 805] (20), [2560, 700] (20), [1250, 805] (366), [1350, 805] (60)
37	1650	1971	182	[1420, 1365] (16), [1420, 1765] (17), [1420, 965] (8), [1450, 752] (15), [1450, 920] (15), [1450, 652] (10), [1650, 1971] (64), [750, 985] (37)
38	950	1105	528	[950, 1105] (528)
39	950	1605	912	[950, 1605] (912)
40	950	1961	287	[950, 1905] (55), [950, 1961] (232)
41	1050	891	714	[1050, 891] (714)
42	1050	1825	863	[1050, 1825] (863)
43	1270	999	395	[1150, 869] (45), [1210, 965] (70), [1220, 965] (30), [1270, 999] (248)
44	1270	1407	336	[1250, 1405] (30), [1270, 1407] (306)
45	1270	1985	213	[1250, 1971] (120), [1220, 1765] (30), [1270, 1985] (63)
46	1146	1512	1223	[1050, 1505] (20), [1146, 1512] (1203)
47	1150	955	876	[1150, 955] (876)
48	1150	1255	770	[1150, 1255] (770)
49	1150	1605	870	[1150, 1605] (870)
50	1150	1855	655	[1150, 1855] (655)
51	1184	1605	1110	[1184, 1605] (1110)
52	1184	1739	1042	[1184, 1739] (1042)
53	1180	1205	983	[1180, 1205] (983)
54	1234	945	461	[1234, 945] (461)
55	1234	1385	1091	[1210, 1365] (30), [1220, 1365] (36), [1000, 1350] (30), [1234, 1385] (995)
56	1250	1305	572	[1220, 1165] (5), [1250, 1305] (567)
7	1250	1331	1151	[1210, 1327] (20), [1250, 1331] (1131)
58	1350	1825	762	[1350, 891] (40), [1250, 1805] (15), [1350, 1825] (707)