**DOKUZ EYLÜL UNIVERSITY**
**GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES**

# A GENETIC ALGORITHM APPROACH FOR A MULTI-PRODUCT MULTI-PERIOD CONTINUOUS REVIEW INVENTORY MODEL WITH DETERMINISTIC AND VARIABLE DEMAND

**by**
**İlkay SARAÇOĞLU**

**May, 2014**
**İZMİR**

# A GENETIC ALGORITHM APPROACH FOR A MULTI-PRODUCT MULTI-PERIOD CONTINUOUS REVIEW INVENTORY MODEL WITH DETERMINISTIC AND VARIABLE DEMAND

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**by**
**İlkay SARAÇOĞLU**

**May, 2014**
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**PH.D. THESIS EXAMINATION RESULT FORM**

We have read the thesis entitled **"A GENETIC ALGORITHM APPROACH FOR A MULTI-PRODUCT MULTI-PERIOD CONTINUOUS REVIEW INVENTORY MODEL WITH DETERMINISTIC AND VARIABLE DEMAND"** completed by **İLKAY SARAÇOĞLU** under supervision of **ASSOC. PROF. DR. ŞEYDA AYŞE TOPALOĞLU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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**DEDICATED**

**To my lovely husband Hakan for his patience, love and support.**
**To my lovely children Merve and Mert for the joyfulness they have brought.**
**To my lovely mother for her hard working to give us better life.**

*To the beloved memory of my father,*

*Ali Aksoy…*

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İlkay SARAÇOĞLU

# A GENETIC ALGORITHM APPROACH FOR A MULTI-PRODUCT MULTI-PERIOD CONTINUOUS REVIEW INVENTORY MODEL WITH DETERMINISTIC AND VARIABLE DEMAND

## ABSTRACT

This study formulates an approach for multi-product multi-period (Q, r) inventory models that calculates the optimal order quantity and optimal reorder point under the constraints of shelf life, budget, storage capacity, and "extra number of products" promotions according to the ordered quantity.

Detailed literature reviews conducted in both fields have uncovered no other study proposing such a multi-product (Q, r) policy that also has a multi-period aspect and which takes all the aforementioned constraints into consideration. A real case study of a pharmaceutical distributor in Turkey dealing with large quantities of perishable products, for whom the demand structure varies from product to product and shows deterministic and variable characteristics, is presented and an easily-applicable (Q, r) model for distributors operating in this manner proposed.

First of all, this problem is modelled as an integer linear programming (ILP) model. Next, a genetic algorithm (GA) solution approach with an embedded local search is proposed as for a solution to higher scale problems. The penalty function is employed in order to ensure the compliance of the constraints.

The results indicate that the proposed approach yields high-quality solutions within reasonable computation times.

**Keywords:** (Q, r) inventory policy, multi-product, multi-period, integer linear programming, variable demand, promotion, genetic algorithm, local search

# SABİT VE DEĞİŞKEN TALEPLİ ÇOK ÜRÜNLÜ ÇOK PERİYOTLU SÜREKLİ GÖZDEN GEÇİRMELİ ENVANTER MODELİ İÇİN GENETİK ALGORİTMA YAKLAŞIMI

## ÖZ

Bu çalışmada raf ömrü, bütçe, depo alanı kısıtları ve sipariş miktarına bağlı ekstra eklenmiş adet promosyonu altında optimum sipariş miktarı ve yeniden sipariş verme noktasını bulmak için çok ürünlü çok periyodlu (Q, r) envanter politikası geliştirilmiştir.

Bu alanda detaylı literatür taraması yapılmış ve ilgili literature göre çok ürünlü (Q, r) envanter politikası için çok periyodlu olarak ve bahsedilen kısıtları dikkate alan bir çalışmaya rastlanmamıştır. Bu tez çalışmasında, üründen ürüne değişen talep yapısı olan ve talebin deterministik ve değişken karakteristikler gösterdiği, binlerce bozulabilir ürün taşıyan Türkiye'deki bir ecza distribütörünün gerçek vaka problemi ele alınmaktadır ve bu şekilde faaliyet gösteren distribütorler için kolayca uygulanabilir bir (Q, r) modeli önerilmektedir.

Öncelikle bu problem tamsayılı doğrusal programlama (ILP) modeli ile modellenmiştir. Sonra, büyük boyutlu problemlerin çözümü için lokal arama ile desteklenmiş bir Genetik Algoritma (GA) çözüm yaklaşımı önerilmiştir. Kısıt koşulların uygunluğunu sağlamak için ceza fonksiyonu uygulanmıştır.

Sonuçlar önerilen yaklaşımın makul hesaplama süreleri içinde yüksek kalitede çözümler verdiğini göstermiştir.

**Anahtar kelimeler:** (Q, r) envanter politikası, çoklu ürün, çoklu periyot, tamsayılı doğrusal programlama, değişken talep, promosyon, genetik algoritma, lokal arama

**CONTENTS**

**Page**

THESIS EXAMINATION RESULT FORM ..... ii

DEDICATED ..... iii

ACKNOWLEDGMENTS ..... iv

ABSTRACT ..... vi

ÖZ ..... vii

LIST OF FIGURES ..... xii

LIST OF TABLES ..... xiv

**CHAPTER ONE - INTRODUCTION ..... 1**

1.1 Importance of the Problem ..... 1

1.2 Research Methodology ..... 3

1.3 Outline of the Thesis ..... 5

**CHAPTER TWO - INVENTORY MANAGEMENT SYSTEMS ..... 7**

2.1 Introduction ..... 7

2.2 Inventory Management ..... 7

2.3 Types of Inventories ..... 8

2.4 Inventory Costs ..... 9

2.5 Characteristics of Inventory Systems ..... 10

2.6 Review Strategies and Inventory Policies ..... 12

2.7 Deterministic Continuous Review (Q, r) Models ..... 14

2.8 Stochastic Continuous review (Q, r) Models ..... 18

2.8.1 Single Item Stochastic (Q, r) Models ..... 21

2.8.2 Multi - item Stochastic (Q, r) Models ..... 29

2.9 Other Continuous Review Models Applied on Perishable Inventories ..... 32

2.9.1 (Q, r, T) Inventory Policy ..... 32
2.9.2 (S-1, S) Inventory Policy ..... 33
2.9.3 (s, S) Inventory Policy ..... 33
2.10 Multi - item Multi - period Inventory Models ..... 36
2.11 Chapter Conclusions ..... 39

**CHAPTER THREE - DESCRIPTION OF GENETIC ALGORITHM AND ITS LITERATURE STUDY IN INVENTORY PROBLEMS ..... 41**

3.1 Introduction ..... 41
3.2 Genetic Algorithm ..... 44
3.3 Basic Concepts of Genetic Algorithms ..... 45
3.4 A Simple Genetic Algorithm ..... 46
3.5 Basic Steps of Genetic Algorithm ..... 47
3.6 Parameter Setting for Efficient GA ..... 50
3.7 Handling Constraints ..... 52
3.7.1 Penalty Function ..... 54
3.7.1.1 Joines and Houck's Method ..... 56
3.8 Literature Review of Genetic Algorithm Applications for Inventory Management Problems ..... 57
3.9 Genetic Algorithm Specifications ..... 61
3.10 Chapter Conclusions ..... 68

**CHAPTER FOUR - PROPOSED MATHEMATICAL MODEL AND GENETIC ALGORITHM APPROACH FOR MULTI-PRODUCT MULTI-PERIOD (Q, r) INVENTORY PROBLEM ..... 69**

4.1 Introduction ..... 69
4.2 Proposed Integer Linear Programming Model ..... 70
4.2.1 Sets, Indices, Parameters and Variables ..... 71
4.2.2 Objective Function ..... 73
4.2.3 Constraints ..... 76

4.3 Computational Study for the ILP Model ..... 81
4.4 The proposed GA with Local Search ..... 84
4.4.1 The Logic of Search in GAs ..... 84
4.4.1.1 Chromosome Representation ..... 84
4.4.1.2 Initial Population ..... 85
4.4.1.3 Calculating the Fitness Values ..... 86
4.4.1.4 Proposed Local Search ..... 90
4.4.1.5 Selection ..... 94
4.4.1.5 Genetic Operators -Crossover ..... 95
4.4.1.6 Genetic Operators -Mutation ..... 98
4.4.1.7 Termination Criteria ..... 99
4.5 Parameter Settings for Efficient GA ..... 99
4.6 Computational Study ..... 104
4.7 Chapter Conclusions ..... 108

**CHAPTER FIVE - A CASE STUDY FOR PHARMACEUTICAL DISTRIBUTOR ..... 110**

5.1 Introduction ..... 110
5.2 Pharmaceutical Market ..... 110
5.3 Pharmaceutical Supply Chain ..... 112
5.4 The Pharmaceutical Distributor Case ..... 115
5.4.1 ABC analysis for Classification Products ..... 115
5.4.2 Financial Inventory Performance Measures ..... 118
5.4.3 Demand Type ..... 118
5.4.4 Determination of Demand Distribution ..... 122
5.4.5 Calculation of the Inventory Costs ..... 125
5.4.6 Promotion on the Drugs ..... 129
5.4.7 Lifetime of the Drugs ..... 129
5.4.8 Problem Definition of the Pharmaceutical Distributor ..... 132
5.4.9 Application of Variable Demand in Genetic Algorithm ..... 132
5.4.10 Computational Results ..... 138

5.4.11 Sensitivity Analysis ..... 142
5.4.12 Different Scenarios for the Company ..... 145
5.5 Chapter Conclusions ..... 148

**CHAPTER SIX - CONCLUSION AND FUTURE RESEARCH DIRECTIONS ..... 150**

6.1 Conclusion ..... 150
6.2 Future Research Directions ..... 151

**REFERENCES ..... 154**

**APPENDICES ..... 170**

## LIST OF FIGURES

**Page**

Figure 1.1 The role of the distributors in the pharmaceutical supply chain ..... 3
Figure 2.1 Demand types according to the coefficient of variation ratio ..... 11
Figure 2.2 Inventory level for the deterministic inventory model ..... 15
Figure 2.3 Optimum order quantity ..... 16
Figure 2.4 Stochastic inventory model ..... 19
Figure 3.1 Binary chromosome representation ..... 47
Figure 3.2 The basic steps of the genetic algorithm ..... 49
Figure 3.3 One-point crossover ..... 50
Figure 3.4 Mutation on only one gene ..... 50
Figure 4.1 CPU times (hr) for two products and different number of periods ..... 83
Figure 4.2 Gene string and chromosome representation of 2xN matrix ..... 85
Figure 4.3 A sample solution for the example problem ..... 86
Figure 4.4 General structure of the GA with local search ..... 88
Figure 4.5 Calculate inventory level matrix for the deterministic demand case ..... 89
Figure 4.6 Pseudocode for calculating fitness value for deterministic demand ..... 90
Figure 4.7 The GA procedure with local search ..... 91
Figure 4.8 Pseudocode for local search ..... 91
Figure 4.9 Representation of the roulette wheel selection probabilities ..... 95
Figure 4.10 Only one row applied for crossover ..... 96
Figure 4.11 Crossover applied on the column ..... 97
Figure 4.12 Crossover applied both row and column ..... 97
Figure 4.13 Pseudocode for the crossover operator ..... 98
Figure 4.14 Main effects plot for means ..... 103
Figure 4.15 Main effects plot for S/N ratios ..... 103
Figure 4.16 Best found profit for 1000 iteration ..... 104
Figure 4.17 CPU times (sec.) for different problem types with ILP and GA ..... 105
Figure 5.1 World pharmaceutical market (2012) ..... 112
Figure 5.2 The players in pharmaceutical supply chain ..... 113
Figure 5.3 Typical ABC inventory analysis ..... 117

Figure 5.4 Histogram and CDF of an empirical distribution ..... 124
Figure 5.5 Calculate inventory matrix for variable demand ..... 134
Figure 5.6 Calculate fitness value for one demand set..... 135
Figure 5.7 Calculate fitness value for all products..... 136
Figure 5.8 Pseudocode to calculate fitness value for variable or seasonal demand. 138
Figure 5.9 Profit versus inventory holding cost % change ..... 143
Figure 5.10 Profit versus ordering cost % change ..... 144
Figure 5.11 The percentage change in profit for corresponding customer service level ..... 145
Figure 5.12 Best found profit for scenarios ($1000)..... 148
Figure 5.13 Expected shortage quantities for different scenarios ..... 148

**LIST OF TABLES**

**Page**

Table 2.1 Inventory policies for periodic review and continuous review ..... 14
Table 2.2 The published literature for deterministic single perishable items ..... 18
Table 2.3 The published literature review for single-item stochastic (Q, r) inventory models ..... 24
Table 2.4 The published literature for multi-item (Q, r) inventory policy ..... 31
Table 2.5 The published literature based on the continuous review (s, S) policy ..... 35
Table 2.6 The published literature for multi-item, multi-period inventory models ..... 38
Table 3.1 GA specifications according to parameters of GA published in inventory management problem ..... 63
Table 4.1 Product demand for each period for the example of the [3P_8T] problem ..... 81
Table 4.2 Input data for the example of the [3P_8T] problem ..... 82
Table 4.3 Extra quantity ranges for the [3P_8T] problem ..... 82
Table 4.4 Computational results for the ILP model ..... 83
Table 4.5 Selection probabilities of individuals ..... 94
Table 4.6 Experimental factors with their levels ..... 100
Table 4.7 The experimental results for [3P_8T] problem ..... 101
Table 4.8 ANOVA table of means for [3P_8T] problem type. ..... 102
Table 4.9 Response table for signal to noise ratios (larger is better) ..... 102
Table 4.10 Results obtained by proposed GA method and ILP ..... 105
Table 4.11 (Q, r) values found for small size problems by ILP and GA ..... 107
Table 4.12 (Q, r) values found for large size problems by ILP and GA ..... 108
Table 5.1 An ABC inventory classification example for class A ..... 116
Table 5.2 The items into the ABC classification ..... 117
Table 5.3 Comparison of A, B and C classes ..... 117
Table 5.4 The current value of inventory level ..... 118
Table 5.5 Products selected to represent the various groups along with their descriptions ..... 119
Table 5.6 Quarterly demands of the selected products ..... 120

Table 5.7 ANOVA table for Pro2 ..... 121
Table 5.8 Tukey's test results for Pro2 ..... 121
Table 5.9 The characteristics of the selected products ..... 122
Table 5.10 Demand data for Pro3 ..... 123
Table 5.11 Chi-square-of-fit test for Pro3 ..... 124
Table 5.12 Distribution summary for Pro3 ..... 125
Table 5.13 Drugs price strategy ..... 126
Table 5.14 The percentage of the cost elements ..... 126
Table 5.15 Ordering cost calculation ..... 127
Table 5.16 Safety factors for the service levels ..... 128
Table 5.17 Input data for the products ..... 129
Table 5.18 Extra promotional quantities offered with some products ..... 131
Table 5.20 The lower and upper bounds for (Q, r) decision variables ..... 139
Table 5.21 Results for the [9P_52T] problem type ..... 141
Table 5.22 Percentage change in holding cost versus percentage change in profit . 142
Table 5.23 Percentage change in ordering cost versus percentage change in profit 143
Table 5.24 Percentage change in shortage cost versus percentage in profit ..... 144
Table 5.25 The results of different scenarios ..... 146

# CHAPTER ONE
# INTRODUCTION

## 1.1 Importance of the Problem

Customer satisfaction has recently become one of the most important issues for companies as a consequence of globalization, increased competition, and shrinking profit margins. The most fundamental condition associated with customer satisfaction is the availability of products upon request. In order for companies to satisfy this condition, excellent inventory management is essential.

Much research has been conducted on inventory management, and results have shown that the type of inventory management needed varies according to the nature of the companies. To construct an inventory management policy, it is therefore necessary to conduct an in-depth analysis of the nature of the target company. The inventory manager must determine (1) when an order should be placed, and (2) how much should be ordered. Further, inventory, production, marketing, and finance managers should operate in concert in order to reach an agreement on how to reduce production costs and inventory investments, and increase customer responsiveness.

Good inventory management is closely linked to the inventory policy implemented, and is based upon a profound analysis of the system. Having stocks available helps to increase customer satisfaction by avoiding uncertainty; with no uncertainty existing, what the customer wants and when s/he wants it could be estimated very well in order to meet demand. However, in real life, there is usually uncertainty with regard to demand and lead time. This uncertainty can lead to desired items being out of stock, resulting in dissatisfied customers. To avoid such a scenario and to ensure that they are able to meet demand, companies frequently resort to keeping extra inventory, which is also referred to as “safety stock.”

Harris (1915) first estimated the economic order quantity to minimize the total cost of keeping stocks and placing orders. A model was developed for a single

product without any constraints on the assumption that demand is known and lead time is zero. When the constraints, such as budget, storage area, packaging dimensions, shelf life, and various discounts, were taken into consideration, what at first appeared to be a simple problem became very complicated.

This thesis analyzes a pharmaceutical distributor that buys medicines from pharmaceutical manufacturers and resells them to pharmacies. The objective of the distributor is to meet the demands of the pharmacies on time. Because a large number of distributors is available in the sector, competition is intense. Consequently, customer satisfaction is viewed with the utmost priority. However, because distributors have to operate within the confines of budget constraints, they need to make optimum use of their budgets. In addition to the budget, shelf life and storage area—for drugs that require cold storage—are additional constraints that have to be satisfied. Further, pharmaceutical manufacturers may desire to deliver an additional amount of drugs for free to induce increases in the number of medicines sold in the pharmacies, instead of offering discounts on the basis of quantity ordered. This promotion varies from product to product. A further item to bear in mind is the fact that pharmaceutical manufacturers may provide convenient payment options to the distributor who, in turn, may pass on these payment options to the pharmacies.

This dissertation constructs a multi-product multi-period (Q, r) inventory policy considering items with variable demand and determines the optimal order quantity (Q) and reorder point (r) for each item to maximize the profit for the pharmaceutical distributor in question, considering all these constraints.

In the literature survey performed until today, it is seen that (Q, r) inventory model is used as single period in multi-product problems and for nonperishable products. (Q, r) inventory model is established firstly by Hadley & Within (1963) as multi-period, but they addressed single product. The main purpose of this Ph.D. thesis is to model the problem for multi-product and multi-period perishable products and under the constraints of budget, storage space, lifetime and some discounts with deterministic and variable demand.

## 1.2 Research Methodology

The previous section states the field of study for this dissertation. It is formulated for a major distributor keeping various sorts of perishable products in stock when and how much to order under the constraints of budget, storage area, and shelf life. Along with these constraints, the promotions given away are taken into consideration. The demand for some products is deterministic, for others seasonal, and for still others variable. By taking this demand structure into account, a solution to maximize the profit and meet the constraint conditions has been sought after.

At the outset of the study, company managers are consulted and their specific problems are identified. The firm that is under scrutiny here is a big-scale firm which buys medical products from manufacturing companies, stores them in stocks, and meets the demands of the customers. The product flow between the manufacturer, distributor, and customer is shown in Figure1.1.

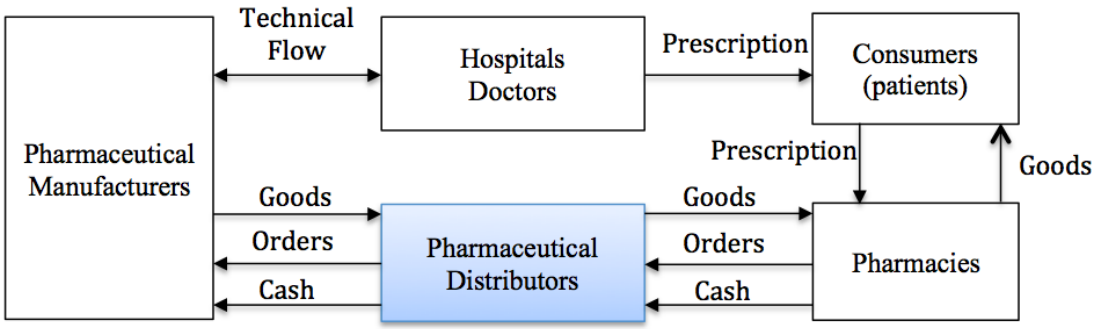



Figure 1.1 The role of the distributors in the pharmaceutical supply chain

For the solution of the problem, a current situational analysis of the company has been conducted. The necessary data about the products such as cost, demand, profitability rate, and shelf life has been gathered on a database. What the demand structure of the products looks like has been examined and a demand structure for each product has been decided upon. After a problem description has been made, a decision has been come upon with respect to the implementation of which inventory management policy would be more suitable for the company. Initially, the constraints

and the objective function are determined and the problem was formulated as a integer linear programming (ILP) model. This is a novel ILP model that is developed for the multi-item multi-period (Q, r) inventory management problem, considering various constraints such as budget, storage area, shelf life, and various discounts.

Within the ILP model, some of the decision variables are restricted as integer values solely. There is a great deal of difficulty for solving optimization models with integer variables due to the non-convex structure. The required memory and computation time increase exponentially with the number of integer variables. Mathematically, it has been observed that it is not possible to reach the optimal solution as the number of periods increases. Setting off from this point, it has been agreed upon using the Genetic Algorithm (GA) method, the most popular of the meta-heuristic methods. In order to get better results with the GA, a local search algorithm has also been added to the optimization search. To evaluate the efficiency and effectiveness of the proposed GA, the solution results of GA and ILP methods for small size problems with few products and a small planning horizon have been compared. It has been detected that it is possible to obtain results which are close to each other or just the same in reasonable computation times. An optimization of the parameters used in the GA is performed with the Taguchi analysis. In the next phase, company products have been classified by the ABC analysis and products from each group with distinctive demand structures have been chosen so as to propose a key solution for large size inventory problem.

Owing to the fact that the cost information such as inventory holding cost, ordering cost, and shortage cost should be estimated within the problem and that it is based on assumptions, a sensitivity analysis has been conducted so as to explore the impact of a change in cost on the objective function value. By coming up with different scenarios, it is aimed to be of some help to the company managers about which scenario to pick regarding the inventory holding management.

## 1.3 Outline of the Thesis

The primary focus of this thesis is to develop efficient GA solution approaches for solving what quantity order is placed and when its ordered with budget, storage area and shelf life constraints, and various promotions that offered based on the order quantity. A brief outline of this thesis is as follows.

In Chapter 2, inventory management problem is introduced by basic concepts with the importance of effective inventory management for success of companies under hard competitive conditions. This chapter includes definition of the characteristic of inventory models as well as their cost structure, variables, and policies along with the comprehensive literature review. This review study focuses on problem specifications of the published literature on inventory management problem, analyzes the problem environments and presents the solution methods suggested by other researchers to clarify the contributions of this thesis.

In Chapter 3, the solution approaches employed in this thesis, GA solution approach with local search is explained in detail and a review of recent literature on applications of GA for inventory management problems is presented. It is explained how to applied penalty function to solve constrained optimization problems with genetic algorithms.

In Chapter 4, we develop an ILP model for a multi-product, multi-period (Q, r) inventory policy. The aim of the problem is to maximize the profit under the constraints such as budget, storage area and shelf life, and various promotions. We analyze the model for problems with different number of products and periods and compared with proposed GA. The solutions show that as the number of products and periods increase, an optimum solution cannot be reached with ILP in acceptable computational time. We propose a GA approach since the case to be addressed in our study is a larger sized problem.

In Chapter 5, the modeling of this multi-product multi-period (Q, r) inventory problem is tested with some real data extracted from the pharmaceutical distributor and explained the solution process step by step. We explain how we can reach the solution with products that have different demand structures, deterministic and variable. Firstly, we explain the general information about the pharmaceutical sector and then propose the GA approach is evaluated for the large-size problem.

Finally, in Chapter 6, the conclusions of this thesis study are discussed. The future research directions are also presented.

# CHAPTER TWO
# INVENTORY MANAGEMENT SYSTEMS

## 2.1 Introduction

The principal objective of inventory management is to improve the level of customer service and to reduce the costs related to inventory. It should be noted that inventory models always attempt to attain an answer to the question of when and how much to order. Inventory is essential for businesses, yet it is crucial to decide how much inventory should be kept in stock. Thus, inventory management addresses the question of how much inventory should be kept in stock so that the desired service level could be achieved. Supply chain management, on the other hand, connects manufacturing and storage activities by way of transportation and balances demand, supply, and cash flow.

Since problems related to inventory are complex, it is indeed difficult to develop a model which comprises all the inventory related issues. As a consequence, the models are frequently developed under certain assumptions. This chapter discusses the characteristic features of inventory models as well as their cost structure, variables, and policies along with literature studies.

## 2.2 Inventory Management

Inventory management determines the necessary inventory level to counter balance the cost of capital in the case of keeping excessive inventory with the penalty cost occurring with inventory shortage.

Dealing with conflicting objectives, effective inventory control could provide a significant advantage. The key factor affecting the solution is the demand structure, whether it is deterministic or probabilistic. In real life, demand usually has a probabilistic structure. However, in some cases, the simpler deterministic approach might be adopted to come to a solution. Another important factor for inventory

models is cost. This chapter will present elucidating information on inventory related costs. A further influential factor is lead time. Lead time is the time period between the placement of an order to the supplier and its physical entering into stocks. This can be a random or fixed period of time. Thus, lead time will bring about modifications or alterations in inventory models. As well as these factors, it might be indispensable to supplement the model with certain constraint conditions. One of the most important conditions is the budget. There might also be a constraint condition with respect to storage area. Some products might deteriorate in time. Hence, shelf life is a significant element, as well. Furthermore, the suppliers could offer various promotions as long as they receive the order for certain quantities. All these factors play a significant role in developing inventory models and determining the kind of policy to be followed. This chapter provides a detailed literature study in the field concerning a variety of models generated by alterations in these factors.

## 2.3 Types of Inventories

Materials or products to be kept in stocks by business organizations are classified in the literature as follows (Gaither, 1996; Arnold, Chapman & Clive, 2008):

*Raw materials* are items which constitute the purchased material inputs for the production process and which are provided by the supplier. The purchased materials consist of component parts and subassemblies.

*Subassemblies* are items which have been fabricated during the production process of the purchased raw materials and which are yet unfinished. They might be composed of a number of raw materials.

*Products* or *finished goods* are final products whose production process has come to an end and which are ready for sale and storage.

Classification of individual parts shows a variety depending on which stage they take place in the production process. For instance, whereas the microchip is a final product for the supplier, it is a component for a TV factory.

## 2.4 Inventory Costs

In the framework of inventory management, cost minimization and/or profit maximization is one of the major objectives. Even though different problems have different characteristics, the major elements of inventory costs encompass inventory holding cost, ordering cost, and shortage cost (Tersine, 1994; Arnold, Chapman & Clive, 2008):

***Inventory Holding Cost:*** Inventory holding cost might also be referred to as carrying cost or inventory cost. It entails the sum of all the expenses incurred by physical stock-keeping of the business organization. The larger the quantity of inventory is held, the more the inventory holding cost is incurred. Inventory holding cost incorporates storage cost, capital cost, obsolescence, damage, shrinkage, deterioration and spoilage cost, and insurance and tax costs. It is not easy to estimate the ratio of the inventory holding cost to the total value of inventory. Although various different ratios are to be found in the literature, in practice, the annual holding cost is assumed to be, in accordance with the company structure, within the interval of 15-30% of the inventory investment value.

***Ordering / Setup Cost:*** Ordering cost entails the expenses incurred while preparing the order to be placed to the supplier. The cost of placing an order does not show an alteration according to order quantity. It is comprised of components such as order preparation, revision, entry review processes of the received products, invoice receipt and payment stages, and expediting. If the item is manufactured by the company itself, the expenses required for setting up the machines used for production and for work order preparation are termed as setup cost. The annual ordering cost depends on the number of orders placed in a year. The number of orders could yet be

reduced; however, since the order quantity will increase, inventory level and, hence, the cost for holding inventory will increase, as well.

***Stockout or Shortage cost:*** Stockout cost is also referred to as shortage cost or penalty cost. It is the cost which comes into question when there is no inventory available to meet the customer's demand. In case that demand happens to be more than expected, the part that cannot be met is either backordered for later delivery or the sale is lost. If the order is backed, the stockout cost is built up on keeping the order in store and on delay cost and this cost is called the backordering cost. The customer's refusal to wait any longer when the demanded product is not in inventory leads to lost sales. In other terms, it might be regarded as the loss of revenue which could have been otherwise obtained from the sale. In both cases, loss of goodwill is also to be considered. In practice, it is rather difficult to foresee goodwill loss. The shortage cost could be reduced by holding extra inventory in order to satisfy a potential increase in demand. This time, however, doing so will raise the inventory holding cost. Demand could be satisfied by holding safety stock just in case demand occurring during lead time happens to be more than anticipated.

## 2.5 Characteristics of Inventory Systems

The most significant factor which determines the characteristics of inventory management systems is demand. Demand can be divided into two types: dependent demand and independent demand. The independent demand is the demand for a final product. It does not in any way rely on the demand of other items kept in inventory. The independent demand can be estimated via actual customer orders or demand forecasting. The dependent demand, on the other hand, relies on the demand for other items. Material requirements planning (MRP) should be used for these items to count their usages according to the bills of materials. Demand structure also plays a crucial role in constructing an inventory model. The fact that demand is deterministic or probabilistic might be the cause for the model to be complex.

Demand could be subject to fluctuation as regards time. That is, the market interest for a product might show variations over time. For example, the Christmas tree is a product which has demand once a year, merely in the preceding months of the New Year. Similarly, flu vaccine is supposed to be applied merely once a year, necessarily in October or December. Likewise, the demand for seasonal products exists in certain times of the year. Furthermore, due to weather conditions, dramatic increases or decreases in the demand for a specific product can be observed in the same months every year.

How do we decide which model is suitable for our demand structure when we deal with the problem? Initially, we need to calculate the mean value and the standard deviation of demand for a given number of periods. Next comes the calculation of the coefficient of variation (V) according to the formula below. We can come to a conclusion about demand structure according to whether this value is over or under 20 per cent.

$$V = \frac{Standard\ deviation}{Mean}\ x\ 100 \tag{2.1}$$

Taha (2007) distinguishes four separate types of demand structure as can be noted in Figure 2.1:

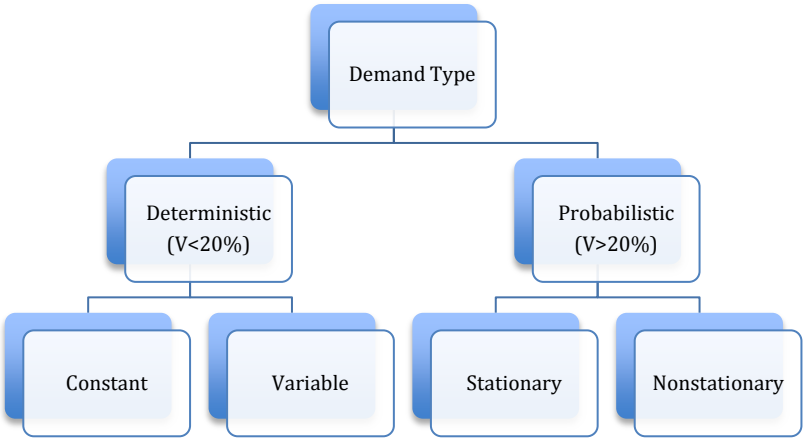



Figure 2.1 Demand types according to the coefficient of variation ratio

Demand data of past years are surveyed for a given number of periods, for example monthly or quarterly. We use the monthly periods to show the calculation procedure. The calculated V values are compared with each other and based on the differences observed from month to month, it is determined which of the above four distinct demand types is at hand here, in line with the instructions below:

1. If the V value is less than 20% and the average monthly demand for all months of data is constant, this demand type can be acknowledged as deterministic and constant.
2. If the V value is less than 20% and the average monthly demand for all months of data displays an alteration from month to month, it is regarded as deterministic but variable demand.
3. If the V value is more than 20% and close to one another on a monthly basis, the demand at hand is probabilistic and stationary.
4. If the V value is more than 20% but varies from month to month, the demand in question is considered probabilistic and nonstationary.

Another characteristic feature to be used in determining inventory models is *lead time*. Particularly in cases where demand is uncertain, the length of lead time is of vital importance. As lead time is stretching, the probability of shortage is thereby increasing. So as not to become stockless, the organization can resort to keeping safety stock. Since safety stock quantity will scale up in line with the long lead time, it will also result in an increase in inventory holding cost. As a consequence, companies attempt to keep their lead time short so that they can reduce the inventory holding cost. Diverse researches have been conducted in the literature dependent upon the condition of lead time: whether it is constant, random, or zero.

## 2.6 Review Strategies and Inventory Policies

There are hundreds of articles written about inventory management. In this thesis, a multi-item perishable inventory management system is addressed. Many products may lose their market value in the process of time due to becoming out of date,

which is named as perishability. Some products lose their value faster than others. In general, products that have a chemical structure are called perishable products. Food, blood, drugs, vitamins and photographic films are examples of fixed-life perishable inventories. Also, some products are outdated due to high technology and changing market conditions. These include technological products such as computers, computer components, phones, digital cameras, etc.

Perishability is divided into two categories in terms of fixed or random lifetime. Fixed lifetime is used for items that have a certain physical life such as photographic film, drugs, vitamins and ready packed food, and milk.

Random lifetime is used for items whose spoilage time cannot be estimated such as fresh vegetables, fruits and meat. The first study about perishable items is the fixed lifetime and exponential decay inventory study performed by Van Zyl (1964).

The issuing policy takes an important place in perishable inventory problems and the First in First out (FIFO) policy is most widely used for fixed lifetime inventory problems. FIFO is most widely used in fixed lifetime inventory problems. However, customers prefer buying the latest arriving products in supermarkets. Therefore, the LIFO policy is observed to be more widely used in such supermarkets. Subsequently, the FIFO policy is also applied for our case of the pharmaceutical distributor that is subject to this study since they have the control of product distribution.

Our control policy is one in which the inventory level is continuously reviewed, (Q, r). In this policy, an order quantity Q is placed when the inventory level is at or below reorder level r. In this continuous review policy, the inventory level is continuously monitored and determined, and replenishment decisions can be taken at any time. In periodic reviews, the inventory is controlled at certain periods. In other words, the inventory level is only known at discrete points in time. In the conditions prevalent today, the periodic review system is considered 50 years behind since companies have shifted to automation. It is also required that this time be added to the lead time in the periodic review system. Furthermore, formulating the inventory

problem with periodic review policy under stochastic demand is very complicated; therefore, the continuous review policy is used more extensively nowadays (Nahmias, 2011). We can collect the policies used in the literature study performed with these assumptions under six titles. After shortly reviewing these policies, we essentially aim to focus on the (Q, r) inventory policy which this thesis is based on. Let us assume $I(t)$ as the inventory level that is observed at time $t$. In Table 2.1, we can briefly summarize the continuous review and periodic review policies most widely used in literature as follows.

Table 2.1 Inventory policies for periodic review and continuous review

| Policy | Monitoring time (*) | Reorder point | Order Quantity |
|---|---|---|---|
| EOQ | t | 0 | Q |
| (Q, r) | 0 | r | Q |
| (Q, r, T) | 0 | r or T | Q |
| (t, s, S) | t | s | Q = S- s |
| (S-1, S) | 0 | S-1 | 1 or S – I(t) |
| (s, S) | 0 | s | Q = S-s |

(*): Policies with monitoring time t period show the periodic review policy whereas policies with 0 period show the models that are continuously reviewed. T shows the remaining shelf life.

## 2.7 Deterministic Continuous Review (Q, r) Models

Manufacturers, wholesalers and retailers generally make new purchases in accordance with the deterministic continuous review inventory policy as the inventory level decreases The simplest model for this policy is the economic order quantity (EOQ) model. It is required to start from the EOQ model which is the first theoretical step in inventory management and explains its fundamentals. First of all, let us review the situation of single item and deterministic demand for nonperishable products. Deterministic models are the ones in which all the parameters and variables are known and definitely computable. Demand rate and lead time are constant and known. Deterministic models are a perfect approach used to start inventory management. EOQ formula is first developed by F. W. Harris in 1915. Later, this formula is extended by Wilson (1934) and has gained an extensive usage in industry. Although it is developed by Harris, this formula is usually known as

Wilson EOQ formula. EOQ and its variations still have an extensive usage in independent demand inventory management.

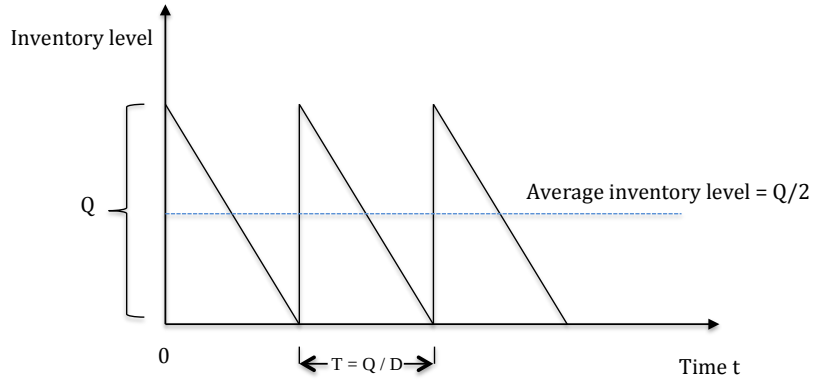



Figure 2.2 Inventory level for the deterministic inventory model

The aim of EOQ model is to minimize the total annual inventory cost. Classical deterministic inventory model is shown in Figure 2.2 (Gaither, 1996). It starts with the assumption that the amount of stock is the order quantity (*Q*) at time zero. Order is placed when the inventory level is positive or zero depending on the lead time. The lead time is zero in the simplest form of EOQ scenario. The new order is added to the stock when the inventory level becomes zero. This situation is not realistic. The situation that is generally encountered is fixed lead time.

Annual purchasing cost is calculated through multiplying one unit purchasing cost (*P*) by annual demand (*D*) amount. Annual ordering cost is the number of orders given in a year (*D/Q*) multiplied by the cost of giving single order (*K*). Annual holding cost is the cost of holding one unit (H) multiplied by the average inventory (*Q/2*). The total inventory cost of any purchased item TC (*Q*) is equal to the total of these three costs.

Total cost = purchasing cost + ordering cost + holding cost

$$TC\ (Q) = PD + \frac{KD}{Q} + \frac{HQ}{2} \tag{2.2}$$

In order to calculate the EOQ amount required to obtain minimum cost, the first order derivative of the total inventory cost formula is taken, the result is equated to zero and then the equation is solved for *Q* as follows.

$$Q^* = \sqrt{\frac{2KD}{H}} \tag{2.3}$$

where $H = i*P$, *i=annual interest rate*

Figure 2.3 shows the change of costs in terms of order quantity (Gaither, 1996). As the order quantity increases, holding cost will increase in direct proportion. Since the number of orderings will decrease as the order quantity increases, ordering cost will decrease too. Purchase cost will remain at the same level since there is no discount. Therefore, when we find the intersection point of the holding cost and ordering cost, we get the point of minimum total inventory cost. The order quantity corresponding to this point is the optimum order ($Q^*$) quantity. When EOQ quantity is known, the number of orderings throughout the year, $N = \frac{D}{Q^*}$and the time between orders, $T = \frac{1}{N} = \frac{Q^*}{D}$can be determined.

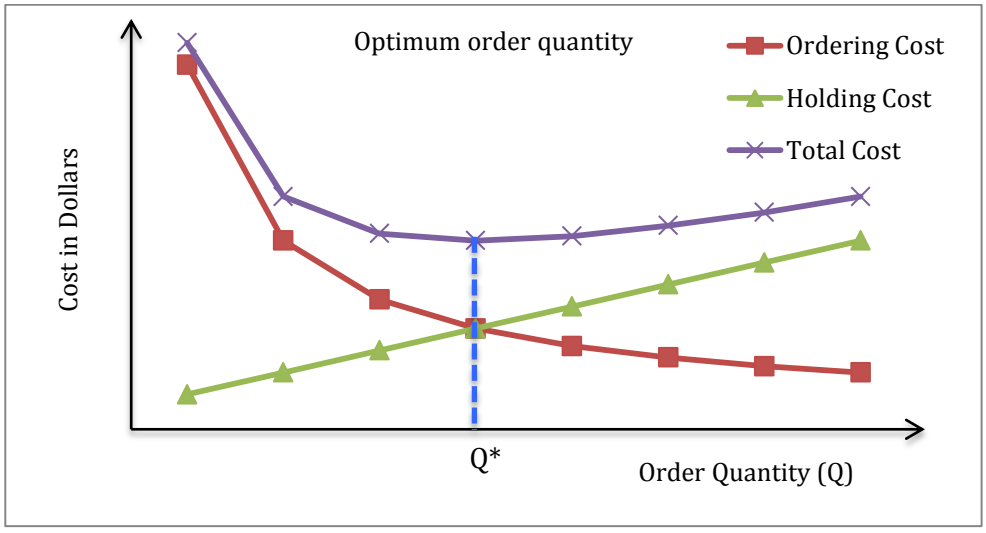



Figure 2.3 Optimum order quantity

If the lead time ($L$) is positive, reorder point, $r$, should be determined when the inventory reaches zero point so that the new order can be added to the stock. Reorder point occurs when the inventory level decreases to lead time demand point ($\lambda L$). In the case of fixed lead time, reorder point is calculated as follows in deterministic inventory models:

Reorder point ($r$) = demand rate ($\lambda$) x Lead time ($L$)

In deterministic perishable inventory models, optimal policy means making an order which will prevent the product from spoiling. Let us assume that the simple EOQ model is used for a product with fixed lifetime, $m$. It is easily seen that the optimal order quantity that will guarantee prevention of any product from spoiling is $\propto = \min(Q^*, \lambda m)$ . The fixed lifetime and exponential decay inventory study performed by Zyl (1964) is among the first perishable items study conducted. No constraints are taken into account in this model. Another study with the same assumptions but with random lifetime is firstly performed by Ghare & Schrader (1963). Examples to literature studies for deterministic demand with random and fixed lifetime are given in Table 2.2.

Nahmias (1982), Rafaat (1991) and Goyal & Giri (2001) reviewed the current literature on ordering policies for perishable items.

Table 2.2 The published literature for deterministic single perishable items

| author/year | Method | Policy | Leadtime | Constraints |
|---|---|---|---|---|
| **Fixed lifetime** | | | | |
| Zyl, 1964 | A | EOQ | 0 | |
| Aggarwal & Jaggi, 1995 | A | EOQ | 0 | |
| Wee, 1995 | A | lot-size | 0 | |
| Hariga, 1996 | L | EOQ | 0 | service level |
| Chu et al., 1998 | A | EOQ | 0 | |
| Giri & Chaudhuri, 1998 | A | EOQ | 0 | |
| Chung et al., 2000 | A | EOQ | 0 | |
| Hsu, 2000 | DP | lot-size | 0 | |
| Liao et al., 2000 | A | EOQ | 0 | permissible delay |
| Sarker et al., 2000 | A | EOQ | 0 | permissible delay /inflation |
| Cooper & Tweedie, 2002 | M/Sm | critical number policy | 0 | |
| Yang & Wee, 2002 | A/H | lot-size | F | |
| Ouyang et al., 2003 | A/H | optimal ordering | 0 | Inflation |
| Rau et al., 2003 | A | EOQ | 0 | |
| Liao, 2007 | A | EOQ | 0 | permissible delay |
| Min & Zhou, 2009 | A/H | optimal ordering | 0 | |
| Ouyang et al., 2009 | A | EOQ | 0 | permissible delay |
| Chang et al., 2010 | A | optimal ordering | 0 | quantity discount, permissible delay |
| Lia & Huang, 2010 | A | optimal ordering | F | storage, permissible delay |
| Shen et al., 2011 | A | optimal ordering | 0 | minimum volume inventory |
| Ghiami et al., 2013 | A/GA | optimal ordering | 0 | Storage |
| **Random Lifetime** | | | | |
| Ghare & Schrader, 1963 | A | EOQ | 0 | |
| Liu & Shi, 1999 | A | (s, S) | 0 | |
| Chung &Liao, 2004 | A | EOQ | F | permissible delay |
| Dye & Ouyang, 2005 | A | EOQ | 0 | |
| Chung & Huang, 2007 | A | EOQ | F | storage, permissible delay |

F: Fixed; A: Analytical; DP: Dynamic Programming; H: Heuristics; M: Markov; Sm: Simulation; GA: Genetic Algorithm

## 2.8 Stochastic Continuous Review (Q, r) Models

Inventory management models are considered in two types in the case of uncertainty: 1) periodic review policy and 2) continuous review policy. Periodic review policy can be for a single planning period or multiple planning periods. The single period model is usually used for high trend items, food that needs to be consumed very fast and newspaper. Such problems are known as newsboy model in literature. Continuous control (Q, r) policy is the most widely used inventory model in cases that demand is uncertain throughout the replenishment lead time. In this policy, an order of quantity $Q$ is placed when the inventory level drops to or below reorder level r. Unlike the EOQ model, $Q$ and $r$ are considered as independent

decision variables since demand is random. In inventory problems, optimization is typically to find the rules that will lead to minimum total inventory cost. Nevertheless, when demand is random, cost will be random too. Therefore, it is aimed to minimize the expected cost in all stochastic optimization techniques applied in inventory problems. Lead time demand and lead time standard deviation should be estimated in order to determine the reorder point level. Let us assume the lead time demand has f($x$) probability density function (p.d.f) and F($x$) cumulative distribution function (c.d.f) with continuous random variable D. In this case, the lead time demand will be $\mu = E(D)$and standard deviation will be $\sigma = \sqrt{var(D)}$. We will form the expression that will minimize the expected average annual cost with (Q, r) decision variables as in EOQ model. Different than the EOQ formula, holding cost should be considered as the following. The expected inventory level will vary between safety stock, *SS*, and *Q* + *SS*, as Figure 2.4 demonstrates (Taha, 2007).

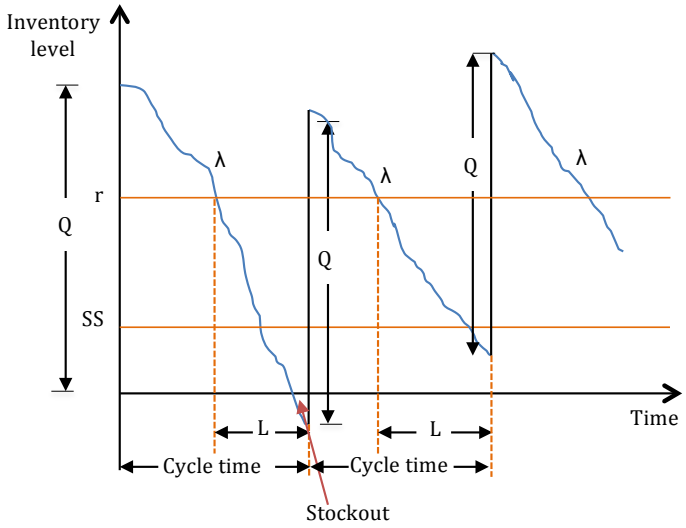



Figure 2.4 Stochastic inventory model

*SS* level is calculated as $SS = r - \lambda L$. Thus, expected average inventory level is found as $SS + Q/2 = r - \lambda L + Q/2$. When demand is uncertain, shortage can occur. For this reason, penalty cost, $p$ should be added to the expected cost expression. When the lead time demand is higher than *r* level, then this situation will cause shortages. The expected number of shortages is expressed as follows (Nahmias, 1993):

$$E\ (\max(D-r,0)) = \int_{r}^{\infty}(x-r)f(x)dx \tag{2.4}$$

This equation is defined as *n(r)* that shows the number of shortages that can occur per cycle. The expected number of shortages incurred in a cycle is written as $\frac{n(r)}{T} = \lambda n(r)/Q$. Under these circumstances, the average expected annual cost of holding, ordering and shortages will be:

$$G(Q,r) = h(Q/2 + r - \lambda L) + K\lambda/Q + p\lambda n(r)/Q \tag{2.5}$$

The aim is to determine *Q* and *r* by minimizing *G (Q, r)*. Optimum solution is achieved by iteratively solving these two equations.

$$Q^{*} = \sqrt{\frac{2\lambda[K + pn(r)]}{h}} \tag{2.6}$$

$$1 - F(r) = Qh/p\lambda \tag{2.7}$$

The solution is found by these steps as follows:

1. *Q* value is calculated as EOQ.
2. *r* value is found by Equation (2.7).
3. This *r* value is used for the calculating *n*(*r*) value.
4. *Q*1 value is calculated with *n*(*r*) value in the Equation (2.6).
5. *r*1 value is found by the *Q*1 value in the Equation (2.7).
6. The steps are repeated until the *Q* and *r* values are found successfully.

Safety stock level is calculated if average demand amount is subtracted from the found value, *r*. Equation (2.6) and (2.7) are found on the assumption that the all excess demand are backordered. Recently, the competitive conditions bring out the situation of unsatisfied excess demand. This situation is named lost sales in literature. The lost sales version of the Equation (2.7) is:

$$1 - F(r) = Qh/(Qh + p\lambda) \tag{2.8}$$

The effect of the Equation (2.7) is caused by increasing the value *r* in order to meet stockouts conditions.

Continuous review policy (Q, r) has been used for nonperishable items in a number of studies. The first such study was conducted by Berk and Gurler (2008). The (Q, r) optimal policy that is used for nonperishable items is not used extensively with perishable items. In the nonperishable continuous review system, an order is made as soon as the inventory level decreases to reorder point level, *r*. However, if spoilage occurs before level *r*, the inventory level will immediately decrease to zero. Therefore, a slight change invariably has to be made to the policy. From this point of view, studies about (Q, r) policy for nonperishable items are primarily discussed since more policies related to them exist.

### *2.8.1 Single Item Stochastic (Q, r) Models*

The inventory models (Q, r) which are reviewed continuously, is one of the most common policies in cases which the demand is uncertain and has lead time for supply. When lead time is zero, optimum order amount can be calculated with basic EOQ formula and reorder point will be zero. The (Q, r) policies applied articles are summarized in Table 2.3 in regard to item type, demand structure, used methods and constraint provisions. As it is seen in the table, the single item is worked with (Q, r) policies by Kim & Park (1985). They considered a heuristic treatment of a lot-size reorder point policy with stochastic demand in which a fraction, $\beta$, of the demand during the stockout period was backordered and the remaining fraction, $1 - \beta$, was lost. Lead time demand was supposed that is a continuous density function, *f (x),* with a mean of $\mu$. If the lead time demand is *x*, then the expected shortage is given by,

$$\int_r^{\infty} (x - r) f(x) dx \equiv \eta(r) \tag{2.9}$$

$\beta\eta(r)$: expected number of backorders per cycle

$(1-\beta)\eta(r)$: expected demand lost per cycle

Yano (1985) presented an algorithm to determine the optimal order quantity and reorder point systems with service level constraints and complete backordering while minimizing the total cost. A new heuristic algorithm was presented that is more efficient computationally than the optimal procedure and provides excellent solutions.

Federgruen & Zheng (1992) derived a simple and efficient algorithm for computing optimal (Q, r) inventory control policies with constant lead time and a single item.

Zheng (1992) conducted a sensitivity analysis on the stochastic (Q, r) system. He determined the reorder point for any chosen order quantity. Cost function included holding cost, backorder cost and ordering cost. His model is a single item continuous review inventory system with fixed lead time. All stockouts are backordered. Hadley & Within (1963) considered these three costs separately. However, Zheng (1992) has shown the holding and backorder penalty cost as a single term in inventory cost. Thus the model became:

$$G(y) = E\,[h(y-D)^{+} + p(D-y)^{+}] \tag{2.10}$$

where

$G(y)$ : expected inventory cost

$D$: demand during the lead time

$y$ : inventory level at time t

Then the cost function takes the following form: $C(Q,r) = \frac{\lambda K + \int_{r}^{r+Q} G(y)dy}{Q}$ in Equation (2.10), the inventory level is uniformly distributed between ($r$, $r+ Q$] and independent of the lead time demand $D$.

When the demand system is discrete and follows a stationary point process, the cost function is formulated as in the following:

$$\tilde{C}(Q,r) = \frac{\lambda K + \sum_{y=r+1}^{r+Q} G(y)}{Q} \tag{2.11}$$

He assumed that average inventory *I* (Q, r) and average backorders *B* (Q, r) for a given (Q, r) policy are as follows:

$$I(Q,r) = \frac{1}{Q}\int_{r}^{r+Q} E(y-D)^{+}dy \tag{2.12}$$

$$B(Q,r) = \frac{1}{Q}\int_{r}^{r+Q} E(D-y)^{+}dy \tag{2.13}$$

Thus, holding and backorder cost are considered as follows in a single inventory term.

$$I(Q,r) = \frac{Q}{2} + r - \lambda L + B(Q,r) \tag{2.14}$$

Table 2.3 The published literature review for single-item stochastic (Q, r) inventory models

| Author, Year | No constraint | Budget | Service level | Storage | Inventory investment | Quantity discount | Batch size | Order frequency | Manpower time | Backorder | One replenishment | Lead Time | Lifetime | Demand | Method |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **single item + multi period + nonperishable** | | | | | | | | | | | | | | | |
| Hadley & Within, 1963 | * | | | | | | | | | | | 0 | | St | DP |
| **Single item + single period + nonperishable** | | | | | | | | | | | | | | | |
| Kim & Park, 1985 | * | | | | | | | | | | | R | | St | A |
| Yano, 1985 | | | | | * | | | | * | | | F | | St | L/H |
| Zheng , 1992 | * | | | | | | | | | | | F | | St | A |
| Federgruen & Zheng, 1992 | * | | | | | | | | | | | F | | St | H |
| Axsäter, 1993 | * | | | | | | | | | | | F | | St | A |
| Kim & Benton, 1995 | * | | | | | | | | | | | F | | St | A/I |
| Rabinowitz et al., 1995 | | | | | | | | | | * | | F | | St | L |
| Johansen & Thorstenson, 1996 | | | | | | * | | | | | | F | | St | I |
| Hopp et al., 1997 | | | * | | | | | * | | | | F | | St | L/H |
| Gallego, 1998 | | | | | | | * | | | | | F | | St | A |
| Matheus & Gelders, 2000 | | | * | | | | | | | | | F | | St | M |
| Johansen & Hill, 2000 | | | | | | | | | | | * | F | | St | M/I |
| Lau et al., 2002 | | | * | | | | | | | | | F | | St | A/I |
| Lau & Lau, 2002 | | | * | | | | | | | | | F | | St | A |
| Axsäter, 2006 | | | * | | | | | | | | | F | | St | St/H |
| Axsäter, 2007 | * | | | | | | | | | | | F | | St | St/H |
| Chen &Chang, 2007 | * | | | | | | | | | | | R | | Se | MIP |
| Zhao et al., 2007 | | | | * | | | | | | | | F | | St | St/H |
| Darwish, 2008 | | | | | | * | | | | | | F | | St | I |
| Lau & Lau, 2008 | * | | | | | | | | | | | F | | St | A |
| Wang & Hu, 2008 | | * | | | | | | | | | | F | | St | H |
| Mattson, 2010 | * | | | | | | | | | | | F | | Se | Sm |
| Hajiaghaei et al., 2011 | * | | | | | | | | | | | F | | St | A |
| Katehakis & Smit, 2012 | | | | | | * | | | | | | F | | St | M/I |
| Ang et al., 2013 | * | | | | | | | | | | | F | | St | St |
| **Single item + single period + perishable item** | | | | | | | | | | | | | | | |
| Berk & Gürler, 2008 | * | | | | | | | | | | | F | F | St | M |

St: Stochastic; Se: Seasonal; D: Deterministic; F: Fixed; A: Analytical; DP: Dynamic Programming; MIP: Mixed Integer Programming; H: Heuristics; I: Iterative Procedure; M: Markov; Sm: Simulation; L: Lagrangian; R: Random; GA: Genetic Algorithm

Consequently, the average cost is reformulated as:

$$C(Q,r) = \frac{\lambda K}{Q} + h\left({}^{Q}/_{2} + r - \lambda L\right) + (h+p)B(Q,r) \tag{2.15}$$

Kim & Benton (1995) considered a continuous review inventory control (Q, r) system. The objective of this study is to minimize the total annual cost as follows:

$$TC = \frac{D}{Q}K + \left(\frac{Q}{2} + SS\right)h + \left(\frac{D}{Q}S\right)p \tag{2.16}$$

where

$S$ = expected number of stockouts per order

They presented an iterative algorithm that determines lot size and safety stock simultaneously.

Rabinowitz, Mehrez, Chu, & Patuwo (1995) considered the partial backorder policy within the traditional stochastic (Q, r) inventory system. The aim of the study was to minimize the expected annual cost and compute the optimal parameters of the reorder point, with minimum number of backorder and order quantity. The problem was solved by using the Kuhn-Tucker conditions.

Johansen & Thorstenson (1996) proposed an (Q, r) inventory control policy to minimize the expected total discounted infinite horizon cost for all inventory levels. They developed a policy-iteration algorithm to compute the optimal solutions.

Hopp, Spearman, & Zhang (1997) formulated a constrained optimization model to minimize the inventory investment with the customer service and order frequency constraints. The aim of their study was to provide an easily implementable inventory policy for a manufacturer of mail processing equipment. They solved the (Q, r)

inventory policy problem and formulated it for a single item and then for multiple items. Their model was formulated as follows:

Minimize inventory investment
Subject to: Order frequency $\leq F$
Service level $\geq S$

where $F$ and $S$ are target order frequency and service level specified by the user. They offered three heuristic methods according to Type I and Type II and a hybrid service level by Lagrangian conditions to solve the problem. Type I service is the probability of not stocking out in any order cycle and Type II service is the probability of being able to meet a demand when it occurs.

Gallego (1998) developed lower and upper bounds on the optimal cost and the optimal batch size of operating under a (Q, r) policy.

Johansen & Hill (2000) considered periodic review (Q, r) inventory model with lost sales during a stockout. They assumed that there will never be more than one order outstanding at any time, which means that $Q > r$.

Mathens & Gelders (2000) proposed methods for calculating the reorder point $r$ of a (Q, r) inventory policy with a target service level. In their study, the service level $\beta$ is defined as the percentage of the total number of units provided directly from stock on hand.

$$\beta = 1 - \frac{expected\ demand\ that\ goes\ short\ in\ one\ cycle}{expected\ total\ demand\ in\ one\ cycle} \tag{2.17}$$

Lau, Lau, & Robinson (2002) developed a continuous review (Q, r) policy with stochastic demand and complete backordering. They took the service level constraint into consideration in order to minimize the total cost.

Lau & Lau (2002) extended the average inventory level of Hadley & Within's expression (*1/2Q + safety stock*) by adding stockout probability in a continuous review (Q, r) policy with backordering. They developed the average inventory level according to the service level. If the service level is less than 0.6 then the average inventory level, $I$ is:

$$I = \frac{1}{2}Q + SS + \frac{1}{2}SS^2/Q \tag{2.18}$$

Axsäter (2006) provided a standard single-echelon inventory system with a continuous review (Q, r) policy under the normally distributed demand and complete backlogging. The aim of this study is to determine *Q* and *r* in order to minimize the sum of the holding and ordering costs under the fill rate constraint.

Chen & Chang (2007) proposed an approximate model for seasonal demand inventory problem with variable lead time. The objective function is to minimize the total relevant cost in order to find the best order quantities and order points for seasonal demand inventory problem. Shortages and backlogging were not allowed.

Axsäter (2007) considered a single warehouse with Poisson demand under the continuous review (Q, r) policy for fixed lead time and emergency lead time. Such orders incurred additional cost in the ordering cost, but have much shorter lead time. He suggested a heuristic method to solve the emergency order problem and then evaluated it in a simulation study. The performance of his study was compared to Johansen & Thorstenson (1998).

Wang & Hu (2008) developed a (Q, r) model to find the optimal lot size and reorder point for a single-item inventory with interactions between necessary and optional components with backorders. The proposed model was formulated as a nonlinear optimization problem and suggested a heuristic approximating method for the solution.

Lau & Lau (2008) presented the current standard "textbook formulation" for stockout cost and then the improved formulation was introduced, followed by explanations of computational procedures and numerical illustrations.

Darwish (2008) considered a continuous review inventory model to find the lot-size and reorder point with all-units and incremental quantity discounts and all-weight and incremental freight discounts. He revealed that lot-size and safety stock are affected by transportation and purchasing issue. He expected total cost per unit time to be the sum-total of purchasing cost, ordering cost, in-transit holding cost, shortage cost, destination holding cost. Iterative procedure was presented and it was further assumed that the lead time demand is continuous random variable.

Mattson (2010) studied an instance of seasonal variations of the demand in the traditional reorder point model. First, they calculated the demand during the lead time according to seasonal index as follows: $DDLT = \sum_{i=d}^{d-1+lt} e_i.\overline{D}$ where $\overline{D}$ is the average demand per day, $s_i$ is the seasonal index for day *i* and $lt$ is the lead time for replenishment inventory. Simulation study is used to solve the problem by Microsoft Excel and macros written in Visual Basic.

Hajiaghaei-Keshteli, Sajadifar, & Haji (2011) studied a three-echelon inventory system with two warehouses and one retailer that applied continuous review (Q, r) policy for a single item. They assumed that all unmet demand was backlogged and there is no capacity constraint. They showed how to derive the total cost expression of the inventory system.

Katehakis & Smit (2012) developed a continuous review (Q, r) stochastic inventory model with Poisson arrivals and a fixed lead time under the multi-breakpoint discount pricing.

Ang, Song, Wang, & Zhang (2013) considered single-item (Q, r) and (s, T) inventory systems to find the optimal order quantity and reorder points. Here, the inventory position was reviewed every *T* periods and if inventory level was below s,

then a sufficient amount of order was placed to bring it back to s. The demand process was a Poisson process with fixed lead time. The cost function was inclusive of the fixed ordering cost, a per-period unit holding cost, and a per-period unit backorder cost. They compared the (Q, r) policy via the EOQ formula. This study gives us an idea of how the EOQ model compares to the stochastic model. This study was extended on the basis of Gallego‘ s research (1998).

### *2.8.2 Multi Item Stochastic (Q, r) Models*

In the previous section, studies involving single item inventory model were reviewed. In actual inventory systems, however, multi-item storage is an issue commonly encountered. For instance, the wholesalers, big distribution chains, or department stores manage an inventory system for a wide range of products. Furthermore, these products may possess different characteristics. They may vary in terms of their price, storage space, shelf life, lead time, size, and/or weight. Nevertheless, those items with distinct characteristics eventually have to utilize the same resources such as budget or storage space. Thus, the inventory management gets more complicated under these circumstances. Studies which applied the (Q, r) policy under multi-item stochastic demand are outlined in Table 2.4.

Garman (1976) analyzed the multi-product stochastic inventory problem under total inventory value constraints. Not only did he set the levels of reorder point and order quantity in his study, but he also determined the optimum interest rates to ensure the inventory value.

Gardner (1983) presented approximate decision rules and an efficient solution procedure for several objective functions regarding customer service under the reordering workload and inventory investment constraints for multi-item inventory models. These objective functions were listed as: the dollar value of sales that must be backordered, the number of times when a shortage condition occurs, and the number of customer requisitions that must be backordered. This article gives some basic information about how to approach a multi-item inventory problem. Since all

model functions used were convex, there was no direct solution concerning any decision variables in the problem. Next, a successive approximation algorithm was proposed in order to find the solution.

As Hopp et al. (1997) worked on both single item and multi-item (Q, r) policies: their study was discussed in section 2.8.1 on single item inventories.

Jeddi, Shultes & Haji (2004) examined a multi-item stochastic inventory system with backorders, shortages and budget constraints and one in which payment is due upon order arrival. The objective function was to minimize the expected total annual cost which was defined by Hadley & Within (1963). They adapted the Hadley and Whitin procedure to the constrained problem. Lagrange multiplier method was used to solve the constrained optimization problem.

Betts & Johnston (2005) built up a multi-item (Q, r) model, as a general form of the risk-adjusted profit model. They employed an approximate solution approach to solve the model under the deterministic demand and then generalized the case where demand is stochastic. The aim of this study was to determine an optimal inventory replenishment policy, by reducing risks and increasing profit which are apparently competing factors.

Zhao, Fan, Liu, & Xie (2007) tackled an optimization problem with a storage-space constraint for computing the single-item (Q, r) policy, and an efficient algorithm with polynomial time computational complexity was proposed to obtain the optimal solutions. Then, they formulated this problem for enhanced multi-item (Q, r) policies.

Wang & Hu (2010) studied a continuous review (Q, r) model to find the optimal lot size and reorder point for a multi-item inventory model with interactions among items under a budgetary constraint. The problem was formulated as a nonlinear one and a simple heuristic approximating procedure was constructed to solve it.

Table 2.4 The published literature for multi-item (Q, r) inventory policy

| Author, Year | Budget | Service level | Storage | Inventory investment | Order frequency | Backorder | Resource constraint | Extra quantity | Lead Time | Lifetime | Demand | Method |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **multi item** | | | | | | | | | | | | |
| **single period** | | | | | | | | | | | | |
| **nonperishable item** | | | | | | | | | | | | |
| Garman, 1976 | | | | * | | | | | F | | St | L/H |
| Gardner, 1983 | | * | | | | | | | F | | D | L/H |
| Hopp et al., 1997 | | * | | | * | | | | F | | St | L/H |
| Jeddi et al., 2004 | * | | | | | | | | F | | St | L/H |
| Betts & Johnston, 2005 | | | | * | | | | | F | | D / St | A |
| Zhao et al., 2007 | | | * | | | | | | F | | St | St/H |
| Wang & Hu, 2008 | | * | | | | | | | F | | St | H |
| Wang & Hu, 2010 | * | | | | | | | | F | | St | H |
| Pasandidehet al., 2011 | * | | | | * | * | | | F | | D | GA |
| Kundu & Chakrabarti, 2012 | * | | | | | | | | R | | St | L |
| Zhao et al., 2012 | | | | | | | * | | F | | St | I/LS |
| **multi period** | | | | | | | | | | | | |
| **perishable item** | | | | | | | | | | | | |
| presented model in this thesis | * | | * | | | | | * | F | F | St,D,Se | GA/LS |

St: Stochastic; Se: Seasonal; D: Deterministic; F: Fixed; A: Analytical; H: Heuristics; I: Iterative Procedure; L: Lagrangian; GA: Genetic Algorithm; LS: Local Search; R: Random

Kundu & Chakrabarti (2012) analyzed uncertain lead time and demand condition for multi-item production. They created a mathematical model which can determine the safety factor and order quantity. They proposed a continuous review model for a multi-product system with both backorders and lost sales under a budget constraint.

Zhao, Qiu, Xie, & He (2012) investigated single-item and multi-item (Q, r) inventory systems with a stochastic demand and limited resources, constant lead time and backorders. They developed a solution approach for finding an optimal or near-optimal solution. According to this approach, ordering cost, holding cost and penalty cost were considered in the cost function. They used lower and upper bounds on the optimal solution to solve the multi-item inventory problem. An iterative procedure and a local search scheme were employed as a solution approach.

It has been observed in the literature review conducted that the (Q, r) inventory policy has not yet been carried out as multi-item multi-period. In this dissertation, a multi-item multi-period (Q, r) inventory policy is proposed together in a model for perishable products under the constraints of budget, storage area, shelf life and various promotions in an attempt to maximize profit.

## 2.9 Other Continuous Review Models Applied on Perishable Inventories

Other continuous review models for perishable products studied in the literature are summarized in this section. The policies in question, in a way parallel to (Q, r) policy, aim to estimate the quantity to be ordered once the inventory level $I(t)$ drops to the desired level. Different computation methods of the inventory level are employed for calculating the order quantity.

### *2.9.1 (Q, r, T) Inventory Policy*

This policy denotes ordering as much as Q the moment inventory level drops to r or when the age threshold $T$ for the product is reached. This policy was put into use by Tekin, Gürler, & Berk (2001). They studied a continuous review perishable

inventory system under the Poisson demand and all unmet demand was lost and lead time was positive and derived the (Q, r, T) policy for lost sales perishable inventory model under the age-based policy. They described the policy as: when/if a replenishment order size *Q* drops to *r* level, or when T units of time have elapsed since the last instance at which the inventory level hit *Q*, whichever occurs first.

### *2.9.2 (S-1, S) Inventory Policy*

This policy refers to ordering as much as $S - I(t)$ whenever a demand occurs or a unit deteriorates. It is mainly utilized in slow moving, quite expensive or spare part inventory control. It is time to place an order for the unit as soon as the inventory level drops - due to demand or deterioration.

Schmidt & Nahmias (1985) suggested (S-1, S) policies for a single item whose lifetime is fixed and known. Excess demand is assumed to be lost rather than backordered. (S-1, S) policy is related in terms that an order is placed for exactly one unit at each occurrence of a demand and at each occurrence of outdating of a unit.

Kalpakam & Shanthi (2001) applied the (S-1, S) policy in order to solve perishable inventory problems with Poisson demand under the exponential lifetime and fixed lead time constraints.

Ollson & Tydesjö (2010) studied (S-1, S) inventory policy with a single product and a single location with Poisson demand and service constraint. They assumed that the product has a fixed lifetime and aging is to begin when the order is placed.

### *2.9.3 (s, S) Inventory Policy*

In a way similar to (Q, r) policy, (s, S) inventory policy is a system that procures order when the inventory level drops to the level of reorder point. Yet, unlike (Q, r) policy, order amount (quantity) is hereby calculated by the equation $Q = S - s$. *S*, is

the order-up-to level. This has been one of the most commonly applied policies in the literature.

To begin with, Sivazlian (1974) analyzed continuous review (s, S) policy where demand is defined as a discrete valued process. He presumed that the (s, S) policy is allowed for complete backlog of unsatisfied demand. He considered that the procurement cost consists of a fixed cost, a variable cost and holding cost.

Next, Şahin (1979) analyzed the continuous review inventory system with constant lead time and complete backlog of unsatisfied demand under (s, S) policy.

Weiss (1980), on the other hand, considered continuous review perishable inventory models with lost-sales and backlogging. Expected average cost is tried to be minimized under the (s, S) inventory policy with zero lead time and fixed lifetime assumptions and the demand shows a stationary Poisson process.

Liu & Shi (1999) analyzed a continuous review (s, S) model for inventory systems in which items have finite random lifetimes.

Liu & Yang (1999) further examined the (s, S) model for exponential lifetime. Markov chain process was used to solve the problem under the fixed lead time that includes the waiting and processing time.

Lian & Liu (2001) later considered a continuous review model with batch demands, allowing backorders. Firstly, they constructed a Markov renewal model for cases with zero lead times and derived a long-run average cost function with a probabilistic approach and then analyzed the properties of the optimal replenishment policy numerically. They defined the expected per cycle costs as the holding cost and the replacement costs.

Gürler & Özkaya (2003) developed a model by using general distribution for demand instead of the Poisson based study of Lian & Liu (2001).

Gürler & Özkaya (2008) also examined on a continuous review perishable inventory system under (s, S) policy with renewal batch demands and negligible replenishment lead times.

Lian, Liu, & Zhao (2009) considered a continuous review (s, S) model for items with an exponential random lifetime whereby the lead time is zero.

Finally, Sivakumar (2009) studied a continuous review (s, S) policy for a single item with random lifetime and random lead time.

The existing works under continuous review (s, S) policy can be categorized as models with lifetime, lead time and constraints that are considered in the model depicted in Table 2.5. As shown in table, all studies on (s, S) policy have no constraints in the model.

Table 2.5 The published literature based on the continuous review (s, S) policy

| Author, Year | No constraint | Lead Time | Lifetime | Demand | Method |
|---|---|---|---|---|---|
| **single item** | | | | | |
| **single period** | | | | | |
| **nonperishable item** | | | | | |
| Sivazlian, 1974 | * | 0 | | St | St |
| Şahin, 1979 | * | F | | St | St |
| **multi item** | | | | | |
| **single period** | | | | | |
| **nonperishable item** | | | | | |
| Kumar & Chandra, 2002 | * | F | | St | H |
| **perishable item** | | | | | |
| Weiss, 1980 | * | 0 | F | St | A |
| Liu & Shi, 1999 | * | 0 | R | D | A |
| Liu & Yang, 1999 | * | R | R | St | M |
| Lian & Liu, 2001 | * | 0 | F | St | M/H |
| Gürler & Özkaya, 2003 | * | 0 | F | St | M |
| Gürler & Özkaya, 2008 | * | F | R | St | H |
| Lian et al., 2009 | * | 0 | R | St | M |
| Sivakumar, 2009 | * | R | R | St | M |

St: Stochastic; Se: Seasonal; D: Deterministic; F: Fixed; A: Analytical; H: Heuristics; R: Random; M: Markov; H: Heuristic

## 2.10 Multi - item Multi - period Inventory Models

When multi-period studies are reviewed, they are observed to be mostly used for lot-sizing problems in production. Firstly, Hadley & Within (1963) addressed single-item multi-period inventory problem with (Q, r) policy. The problem is solved with dynamic programming leading to extensive backordering. They tried to minimize the cost without any constraints.

Veinott (1965) considered a dynamic nonstationary multi-product inventory model in which the system was reviewed in equal lengths of time periods. Unsatisfied demand was backlogged and deterioration of stock did not occur in storage.

Lapierre & Ruiz (2007) analyzed the logistic system of a hospital to minimize the sum of inventory and labor costs. Multi-product, multi-period logistics system was modeled as a mixed-integer program under the limitation of storage and manpower capacities.They developed a tabu search (TS) metaheuristic to solve the problem due to the large scale of the problem.

Liu (2008) analyzed an economic lot sizing (ELS) problem for single item and multiple periods with both upper and lower inventory bounds. Inventory lower bounds are set as safety stocks to protect against shortage. Demand was deterministic and known for each period. They introduced the mathematical formulations of the bounded inventory ELS problem to minimize the sum of holding and ordering costs and then solved them via dynamic programming.

The studies about multi-item and multi-period models are summarized in Table 2.6. The only multi-period study for perishable items is performed by Zhang and Ekşioğlu (2009). They presented a mixed-integer programming (MIP) with multi-mode replenishment options and perishable inventory that can be stored for a limited number of periods and considered a single item.

Kiesmüller, Kok, & Dabia (2011) developed a periodic review (R, S, $Q_{min}$) policy for single item single stage inventory system with stochastic demand. Inventory level was reviewed every *R* period and if the inventory level was below the level *S*, an order quantity that is equal to or exceeds $Q_{min}$ was ordered. Markov chain method was used to determine the optimal *S* level that minimizes the cost function.

Table 2.6 The published literature for multi-item, multi-period inventory models

| Author/year | lifetime | demand | excess demand | method | policy | Lead time | budget | storage | quantity discount | labor time | resource constraint | lifetime | extra quantity |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **multi-item + multi-period nonperishable** | | | | | | | | | | | | | |
| Veinott, 1965 | | St | backlogging | St/Sb | optimal ordering | 0 | | | | | | | |
| Gen & Syarif, 2005 | | D | no shortages | HGA | optimal ordering | 0 | | | | | * | | |
| Lapierre & Ruiz, 2007 | | D | no shortages | TS | optimal ordering | 0 | | * | | * | | | |
| Kannan et al., 2010 | | D | no shortages | GA | supply chain | F | | * | | | * | | |
| Pasandideh et al., 2013 | | D | no shortages | GA | optimal ordering | 0 | | * | * | | | | |
| **Single-item + multi-period perishable** | | | | | | | | | | | | | |
| Zhang & Ekşioğlu, 2009 | F | D | no shortages | H | Lot-size | F | | * | | | | | |
| **multi-item + multi-period perishable** | | | | | | | | | | | | | |
| developed model in this thesis | F | St, Se, D | shortages | GA / LS | (Q, r) | F | * | * | | | | * | * |

St: Stochastic; Se: Seasonal; D: Deterministic; F: Fixed; H: Heuristics; GA: Genetic Algorithm; LS: Local Search; R: Random; HGA: Hybrid Genetic Algorithm

Minner & Silver (2005) studied nonperishable multi-item (Q, r) inventory policy with stochastic demand under budget and storage area constraints. Hadley & Within (1963) considered (Q, r) inventory policy with single-item multi-period. Kiesmüller et al. (2011) studied (R, S, $Q_{min}$) policy for single non-perishable item with multi-period. In the literature survey performed until today, it is seen that (Q, r) inventory model is used as single period in multi-product problems and for nonperishable products. (s, S) policy means that an order is triggered as soon as the inventory position declines to or below the reorder point s and difficult to determine the S values. The order size is chosen so that the inventory position increases to S. Therefore, we decide to apply (Q, r) inventory policy to solve the problem. Thus, we analyzed multi-item multi-period (Q, r) inventory policy under budget, storage area, shelf life constraints and various promotions for perishable products and all of constraints are considered in one model.

## 2.11 Chapter Conclusions

Inventory management problems have a significant place in literature since they play an important role in supply chain management. In this chapter, the basic terms of inventory management problems and different inventory management models for both single item and multiple items are explained. A literature review of these models such as EOQ model for perishable items, single-item stochastic demand (Q, r) inventory policy, multi-item single period nonperishable (Q, r) inventory policy, continuous review (s, S) policy and multi-item multi-period nonperishable (Q, r) policy) is done and summarized in tables along with the assumptions considered in the models.

It is especially focused on the literature about the (Q, r) inventory management policy which is subject to this thesis.

In the literature survey performed until today, it is seen that (Q, r) inventory model is used as single period in multi-product problems and for nonperishable products. Multi-product multi-period (Q, r) inventory policy is modeled with budget,

storage area, shelf life and various promotions by developing a mixed integer linear mathematical programming model and proposed solution approach in the next chapter.

# CHAPTER THREE
# DESCRIPTION OF GENETIC ALGORITHM AND ITS LITERATURE STUDY IN INVENTORY PROBLEMS

## 3.1 Introduction

The methods used for solving inventory management problems may vary in terms of whether the demand is deterministic or stochastic, or whether there is a single product or there are multi-products. In deterministic systems and single product problems, mostly mathematical methods have been used to obtain solutions. As in most of the inventory management problems, unfortunately, the model is naturally complex for the multi-product case and it is intractable to control optimal policies analytically due to the size of the solution domain. Therefore, it is observed that iterative procedures such as heuristics and metaheuristics are used in order to reach a solution in multi-product inventory systems.

Since our problem is a multi-product inventory problem, it is mathematically modeled and solved using a metaheuristic approach, which is genetic algorithm (GA). It is intractable to solve most combinatorial optimization problems (COPs). These problems are defined as NP-hard problems in literature due to the solution run-time grow exponentially. The methods for solving COPs can be divided into two main categories as exact and heuristic methods. Exact algorithms guarantee to obtain the optimum solution. However, the solution run-time grows dramatically with the problem size. For bigger problems, usually heuristic algorithms are recommended. Although they do not guarantee optimal solutions, a well-designed heuristic method enables to obtain close to optimal solutions in reasonable computation times (Raidl & Punchinger, 2008).

Regarding exact solution approaches, branch-and-bound, dynamic programming, constraint programming and integer (linear) programming techniques can be counted among the important methods. Operation research (OR) researchers have been striving to develop new methods for years. In recent years, they have developed

powerful metaheuristics. Metaheuristics are quite useful methods in practice and the key feature of these methods is their ability to escape from local optimum. Simulated annealing, tabu search, genetic algorithm and ant colony optimization can be counted among the important metaheuristic methods.

Tabu Search (TS) algorithm, created by Glover (1986) for solving optimization problems, is an iterative local research algorithm. The basic approach in TS is to forbid or penalize previously visited solutions in order to prevent circular movement toward final solution. Thus, TS algorithm guides local-heuristic research to seek solutions ahead of local optimal solutions by reviewing new solutions.

TS algorithm's basic principle to overcome local optimality depends on the selection of the movement with the highest evaluation value in each iteration by evaluation function to form the next solution. For this, a tabu list is generated and the original goal of this list is to avoid cycling in order to escape from a local optimal point.

Tabu list has a chronological structure and uses a flexible memory structure. Although TS algorithm has been described as marking unfavorable points, it is also used for marking favorable points. It is seen that TS is used especially for solving production scheduling, routing and travelling salesman problems. Hung & Chien (2000), Gopalakrishnan, Ding, Bourjolly, & Mohan (2001), Karimi & Ghomi (2002), Berretta, França, & Armentano (2005) can be given as the examples of the studies performed on these problems. TS algorithm is used for solving inventory problems by Lapierre & Ruiz (2007). They used TS algorithm for solving multi-product, multi-period logistics system problem modeled with MIP under storage space and labor capacity constraints.

Simulated Annealing (SA) algorithm is a probabilistic search algorithm which is successfully applied in many combinatorial optimization problems. It works with the principle of slowly cooling solids starting from a certain initial temperature. The algorithm starts searching from a randomly determined initial solution. In calculation

steps, neighbour solutions of the current solution are obtained by the help of suitable neighbourhood mechanisms. Goal value of the generated neighbour solution is calculated and if it is better than the current solution, it is accepted as the new solution. If it is worse, then it is accepted or rejected according to the determined criteria. At the end of the calculation steps, temperature is lowered with respect to a certain function and this process continues until previously determined iteration number or previously determined temperature value is reached. SA algorithm has parameters such as, initial temperature, number of solutions to be generated at each temperature value, temperature lowering function and stop condition.

SA algorithm is proposed by Kirkpatrick, Gerlatt, & Vecchi (1983). SA is generally used in solving many combinatorial optimization problems such as travelling salesman problem, job shop scheduling, vehicle routing problem and capacitated lot-size problem. Taleizadeh, Aryanezhad, & Niaki (2008) and Hsiau & Lin (2009) solved deterministic demand, multi-product inventory problem by using SA algorithm.

Ant Colony Optimization (ACO) is a population-based approach that can be successfully applied to NP-hard optimization problems. It is firstly applied to Travelling Salesman Problem (TSP) (Dorigo, Maniezzo, & Colorni, 1996). Starting from a point, visiting each point once and returning to the starting point through the shortest path is the definition of TSP and it is seen that ACO is quite effective in the solution of TSP. ACO is developed with inspiration from the behaviors of real ant colonies. In cases where there are alternative paths, ants primarily spread to these alternative paths in equal probability and after a while they focus on the shortest path. It has been observed that all ants use the shortest path in the process of time. For this, they use trail pheromones remained from previous transitions. Basic rule is that the probability of preferring the path with intense trail pheromone quantity is higher. Ants' eyesight is not developed very much and they make their path choices according to the trail pheromone.

The trail pheromone quantity accumulated in the shortest path is higher than the long paths. Since transition from the shortest path will be faster, the number of ants transiting in unit time will be higher with respect to the long path. Therefore, pheromone quantity accumulated on the path between the two nodes is inversely proportional with the length of the path. One of the most basic principles of this technique is the pheromone chemical showing the quality of the solution in the problems and used as a communication tool. Pheromone is a chemical secreted from ants' bodies and used as a communication and navigation tool by real ants. Trail pheromones are updated by ants and represent information. Intense trail pheromones on a path show the quality of the path and increase its preference probability. In ACO, artificial ants search the shortest path on the model created by considering the real distances. Trail pheromones on paths are artificially updated according to the transition frequency of the ants. Until today, many ACO algorithms have been developed. The first of them is developed by Dorigo et al. (1996) with the aim of applying to TSP.

Genetic Algorithm (GA) is a stochastic search method. GA, simulating the development process of the biological systems based on “the survival of the fittest” principle of Darwin, is primarily proposed by Holland (1975) (Goldberg, 1989). It is the first used and most popular algorithm among other metaheuristics. It is also one of the most widely used algorithms in inventory problems with respect to other heuristics.

This section includes the basic definitions of GA, how the algorithm works and information on the studies about inventory management problems.

### 3.2 Genetic Algorithm

GA, which is developed by Holland in 1975, has become a useful tool for difficult problems. Since GA is a population-based, stochastic and heuristic method, it may not find the optimum result for a given problem, but it provides near optimal solutions to problems that cannot be solved with known methods or to problems

having a run-time that grows exponentially with the problem size. In the beginning, GA was applied to continuous non-linear optimization problems and then is successfully applied to combinatorial optimization problems such as travelling salesman, vehicle routing, facility layout and allocation, job shop scheduling, supply chain and transportation, inventory management problems (Gen & Cheng, 1997).

GA differs from normal optimization and search procedures due to its following features (Gen & Cheng, 1997; Goldberg, 1989):

1. GA works with a coding of the parameter sets, not with the parameters themselves.
2. GA searches from a population of solutions, not from a single solution.
3. GA uses objective function, not derivatives or other ancillary information.
4. GA uses probabilistic translation rules, not deterministic rules.

### 3.3 Basic Concepts of Genetic Algorithms

The modern field of genetics provides a further explanation of evolution and the natural selection involved in the survival of the fittest. GAs start with an initial set of random solutions that is called *population* and then choose *parents* to reproduce. Each individual in the population is called a *chromosome.* In natural systems, one or more chromosomes combine to form the total genetic prescription for the construction and the operation of some organism. The set of all chromosomes is called a *genotype*, and the resulting organism is called a *phenotype.* Each chromosome comprises a number of individual structures called *genes*, which may take on some number of values called *alleles*. Each gene is located at a particular *locus* (position) on the chromosome structure and represents a particular characteristic.

### 3.4 A Simple Genetic Algorithm

This section will examine how a GA works and describe the operations that take the initial population and generate successive populations improving the solutions. A simple GA that renders good results in many practical problems is composed of three operators:

1. Selection
2. Crossover
3. Mutation

The efficiency of GAs depends on many parameters, such as the initial population, the representation of individuals, the selection strategy, and the recombination of crossover and mutation operators.

*Selection*: This operator is utilised to guide the search process towards more promising regions in a search space. Selection is an artificial version of natural selection, a Darwinian survival of the fittest among string creatures. It refers to choosing parents for recombination; a new population is formed by replacing parents with their offspring. Several selection methods, such as roulette wheel, tournament, ranking and elitism might be used as described in Gen & Cheng (1997) and Goldberg (1989). Roulette wheel selection is one of the most commonly used techniques in such a proportionate selection mechanism.

*Crossover:* Crossover operator takes two parents and generates two offspring that. This operator randomly chooses a locus to mate pairs of chromosomes to create offspring. There are many types of crossover operators such as one-point, two-points, multiple-points and uniform crossover. Crossover operates on two parent chromosomes that are designated as *parent1* and *parent2* at a time and generates two offspring that are called *child1* and *child2* by recombining the features of both the parent chromosomes.

*Mutation:* This operator arbitrarily selects genes to counteract premature convergence and to maintain enough diversity in the population. Mutation is applied to the genes of the chromosomes with a low probability (mutation probability or mutation rate). Normally, offspring are mutated after being created by reproduction. Mutation options specify how the genetic algorithm makes small random changes in the individuals in the population to create mutation children. Thus, mutation provides genetic diversity and enables the genetic algorithm to search from a broader space. Mutation is employed to maintain the diversity of the entire population by changing individuals bit by bit with a small probability $P_m \in [0, 1]$. Mutation prevents the algorithm to escape from the trap of local minimum.

Crossover and mutation operators ensure that almost all children are different from their parents.

### 3.5 Basic Steps of Genetic Algorithm

In this section, we will explain how GAs work. Before executing the steps of a GA, it is necessary to create a chromosome structure according to the problem type. The chromosomes in a GA population are usually, but not necessarily, a binary bit string that is illustrated in Figure 3.1. Binary representations are not always the most suitable ones for the problem; therefore, in such a case, real values or integer representations are used for denoting the structure of the chromosome.

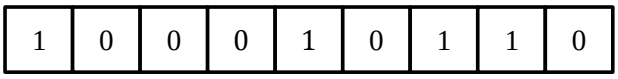



Figure 3.1 Binary chromosome representation

The basic steps involved in the GA can be summarized as follows:

1. Randomly generate initial population of *n* chromosomes.
2. Determine the fitness values *f*(*x*), which is the objective function, and their corresponding probabilities for all the chromosomes in the population in

order to determine their selection for the next generation. Solutions with higher fitness have more chance to be picked out than the *unfit* ones and lean to survive into the next generation.

3. Repeat the following steps until *n* offspring have been created:
    a. Select a pair of parent chromosomes from the current population.
    b. Crossover is applied on the pair of selected parent chromosomes with probability $P_c$ (called "crossover rate" or "crossover probability") at a randomly chosen point.
    c. Mutate the two offspring at each gene with probability $P_m$ (called "mutation rate" or "mutation probability") and place the resulting chromosomes in the new population.
4. Replace the current population with the new population.
5. Define the stopping criteria. Various stopping conditions are proposed for the *termination* or *stopping criteria*. The most widely used ones are to stop the algorithm after a specific number of generations, or after a given time period. Another way is to stop the search when the fitness values for several consecutive generations do not improve.
6. If stopping criteria is not true, then go to step 2; otherwise, stop the algorithm.

The completion of the steps described above is called iteration. GAs steps are shown in Figure 3.2. After a certain number of iterations in GA, sometimes no important change occurs at most of the solutions in the population and the solution will be far from the optimal solution. In such cases, mutation operator enables important changes on the solution set. By increasing the number of generations, it is tried to reach an optimum solution or get close to it. After a certain number of generations or if there is no positive change in the solution set, the procedure is stopped. Operation of one-point crossover and mutation operators is shown in Figure 3.3 and Figure 3.4.

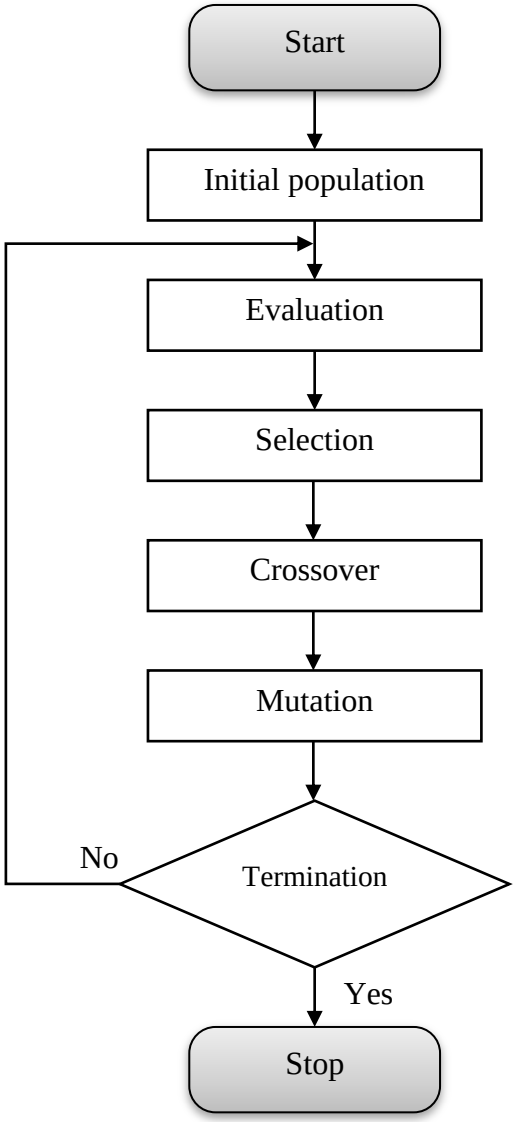



Figure 3.2 The basic steps of the genetic algorithm

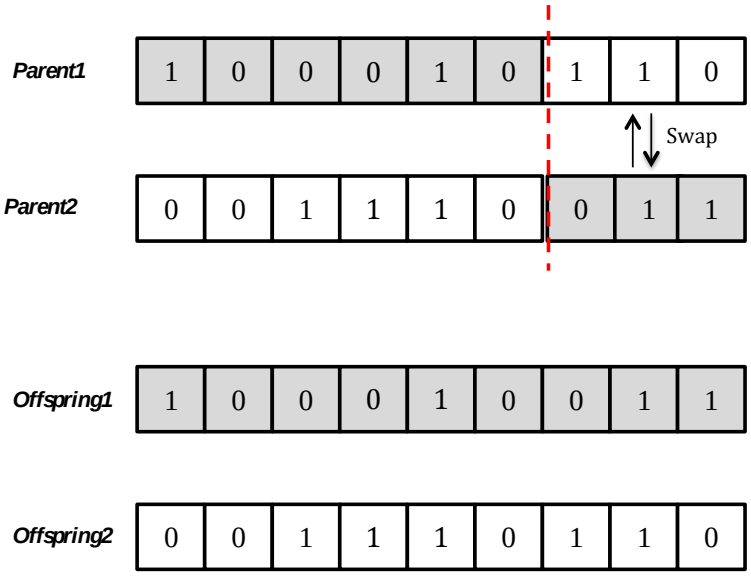



Figure 3.3 One-point crossover

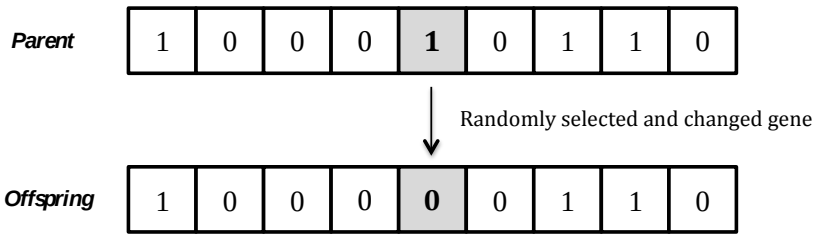



Figure 3.4 Mutation on only one gene

## 3.6 Parameter Setting for Efficient GA

As mentioned in the previous section, crossover has various parameters including mutation rate, population size and number of generations. The values of these parameters affect the GA performance, in other words, whether GA can find an optimal or near optimal solution. Therefore, the most important decision in many GA applications is to define effective GA control parameters. The values of these parameters greatly determine whether the algorithm will find an optimal or near-optimal solutions and whether it will find such a solution in a short time. In

conclusion, defining effective GA parameters has a vital role in the convergence rate of GA application and in obtaining a good quality solution.

Eiben & Smith (2007) divided the parameter selection into two groups as "parameter tuning" and "parameter control". Parameter tuning is a commonly used approach for determining good quality parameter values before starting the genetic research. It starts with the parameter values determined at the beginning and these values remain constant throughout the research. In parameter control, which is another commonly used approach, research starts with some initial parameter values and these values change throughout the research.

In parameter tuning approach, the algorithm is run with different parameter values and the values that give the best result for the addressed problem are selected. However, as the possible number of parameters and their levels increase, the number of trials also increases and thus, it becomes a very time-consuming test.

Pongcharoena, Hicksa, Braidena, & Stewardson, (2002) firstly studied on which parameters have effect on algorithm by applying factor analysis for 3 factors and 3 levels on job shop scheduling problem and then calculated the prediction function with regression analysis. Gupta, Bhunia, & Goyal (2009), Talaizadeh, Niaki, Aryanezhad, & Tafti (2010), Pasandideh, Niaki, & Tokhmenhchi (2011), Yang, Wee, Pai, & Tseng (2011) and Zhang, Song, Zhang, Wu, & Yin (2012) determined the parameter set giving the best result by using different parameter values in the studies on inventory problems conducted with GA applications. Gen & Syarif (2005) applied parameter optimization with fuzzy logic controller which is an automatic-tuning method. Pasandideh, Niaki, & Mousavi (2013) divided the problems into three categories as small-size, medium-size and large-size problems, and then applied Taguchi analysis (Taguchi, 1986) method with 3 factors and 3 levels. They determined different parameter sets for each type of problems. Taguchi Method is an experimental design technique trying to minimize the effects of intractable factors by using orthogonal sequences. This method allows obtaining a better result with much fewer trials in quality improvement as well as being effective in the quality

improvement process. Also in this thesis, Taguchi analysis is used as parameter optimization method. Detailed information about the application of this analysis can be found in section 4.5.

Adaptive genetic operators are a new trend in which the operator rates are changed throughout the research to determine the parameter set in GA. It is seen that this analysis is especially used in lot-sizing problems in literature. Özdamar & Birbil (1998), Özdamar & Barbarasoğlu (1999), and Özdamar, Birbil, & Portmann (2002) can be given as example to the studies using adaptive genetic operator for determining crossover and mutation rates with this method.

### 3.7 Handling Constraints

One of the main problems about applying GA is how to handle constraints when there are infeasible offsprings in the solution of constrained optimization problems. Constrained optimization problems are related to optimizing an objective function with equality and/or inequality constrains. The following information can be given about constrained optimization problems:

$$max \quad f(x) \tag{3.1}$$

$$s.t. \quad g_i(x) \leq 0, \qquad i = 1, \dots, m_1 \tag{3.2}$$

$$h_i(x) = 0 \qquad i = m_1 + 1, \dots, m\ (= m_1 + m_2) \tag{3.3}$$

$$x \in X \tag{3.4}$$

where $f, g_1, g_2, \dots, g_{m_1}, h_{m_1+1}, h_{m_2+1}, \dots, h_m$ are real valued functions defined on $E^n$, X is a subset of $E^n$, where $E^n$ is a search space including unconstrained solutions and $x$ is an n-dimensional real vector. The set of X might typically include lower and upper bounds on the decision variables that is called *domain constraint*. The Equation 3.2 is called an *inequality constraint* and the Equation 3.3 is called an *equality constraint*. A vector $x \in X$ satisfying all the inequality and equality constraints is called a *feasible solution* for the problem. The collection of all such solutions forms the *feasible region*. The function $f$ is called the objective function

and the above problem must be solved for the values of the variables x that minimize the function *f*. In contrast to linear programming problems, traditional solution methods are very complex and quite inefficient for solution of nonlinear programming problems. In recent years, it is focused on using GA in the solution of nonlinear programming problems. Various techniques are proposed for handling constraints with GA. Michalewicz (1996) has published a very successful research about this problem. Infeasible solutions are obtained when constrains are violated. Gen & Cheng (1997, p. 49-59) have divided the current techniques used to handle the constraints into four groups:

1. Rejecting strategy
2. Repairing strategy
3. Modifying genetic operators strategy
4. Penalyzing strategy

These strategies are briefly summarized below:

All infeasible chromosomes generated during the *Rejecting Strategy* evaluation process are rejected. This strategy is popularly used in most GAs. This strategy is suitable when the search space is convex however, it can cause serious issues in constrained optimization problems when the initial population solely consists of infeasible chromosomes.

In *Repairing Strategy*, infeasible chromosomes are turned into a feasible chromosome by using some repair procedures. It can be easily generated in most of the combinatorial optimization problems. The weakness of this strategy is that it depends on the problem. For this reason, a specific repair algorithm should be designed for each problem.

*Modifying Genetic Operator Strategy* requires specialized genetic operators to maintain the feasibility of chromosomes. Michalewicz & Schoenauer (1996) has

pointed out that such systems are usually more reliable than genetic algorithms based on penalty approach.

*Penalty Strategy* is the most commonly used technique for handling infeasible solutions in solving constrained optimization problems with GA. This technique penalizes infeasible solution with a penalty term that is added to the objective function for any violation in these technical constraints and thus the constrained problem is turned into an unconstrained problem. A more detailed explanation is given below about how this process is performed.

### *3.7.1 Penalty Function*

In GA, the penalty technique maintains a certain number of infeasible solutions in each generation in order to obtain a forced optimal solution from both feasible and infeasible solutions. Since infeasible solutions can provide more useful information than some feasible solutions, they cannot be simply rejected in each generation. The key concern is how to determine the penalty term which will decide to maintain some of the infeasible solutions and reject others.

Solution space generally consists of two parts: feasible area and infeasible area. Detailed information about how the penalty function will be used in GA for solution of constrained optimization problems can be found in the book of Gen & Cheng (1997). Penalty technique turns constrained problems into unconstrained problems by penalizing infeasible solutions. The penalty term can be added to the evaluation function in two ways. The first way is the addition form of the penalty term, added to the objective function, which can be explained as follows:

$$eval(x) = f(x) + p(x) \tag{3.5}$$

Here, *x* shows a chromosome, *f(x)* shows the objective function of the problem and *p(x)* is the penalty term. The penalty term is usually shown as follows in the maximization problems:

$p(x) = 0$ If $x$ is feasible (3.6)

$p(x) < 0$ otherwise (3.7)

The second way is explained as the multiplication form of the penalty term. Detailed information about these methods can be obtained from Gen & Cheng (1997, p. 53).

Various penalty techniques are proposed to handle constraints with GA for solution of constrained optimization problems. In literature, the penalty approach is generally divided into three categories as constant, dynamic and adaptive. Constant penalty approach does not depend on the number of generations and a constant penalty is applied for infeasible solutions. This method is proposed by Homaifar, Lai, & Qi (1994). Adaptive penalty function is specific to the problem and is totally adaptive since it includes the specific population and specific chromosome information. This approach is firstly developed by Bean & Hadj-Alouane (1992). The proposed penalty function is formed in a way that it considers the number of infeasible chromosomes in the current population, number of total population and distance from the feasible solution. In dynamic penalty function, the penalty value varies according to the number of generations. The main idea in this approach is to let smaller penalty at the beginning and then increase the severity in next generations. When increasing the severity of penalty in next generations, the objective is to help searching the section near optimal or suboptimal section feasible solutions. Dynamic penalty approach includes two dynamic parameters:

1. The number of feasible/infeasible solutions in the population pool
2. Number of generations

More extensive information can be obtained about dynamic penalty functions from the studies of Michalewicz & Schoenauer (1996), Gen & Cheng (1997) and Dadios & Ashraf (2006). In this thesis, the dynamic penalty function proposed by Joines & Houck (1994) is used.

*3.7.1.1 Joines and Houck's Method*

In real-valued GAs used for solution of constrained optimization problems, dynamic penalty function is a function of the number of generations and the penalty value increases as the number of generations increases. Thus, GA behaves more and more selective to find the feasible solution as the penalty increases. Joines & Houck proposed the following function to evaluate the individuals in each generation t.

$$eval(x) = f(x) + p(t,x) \tag{3.8}$$

$$p(t,x) = p_t^{\propto} \sum_{i=1}^{m} d_i^{\beta}(x) \tag{3.9}$$

eval(x): evaluated fitness function value

p(t, x): the penalty function value

where $t$ is the generation number of GA, and $\alpha$ and $\beta$ are parameters used to adjust the scale of penalty value. Penalty term of $d_i(x)$ is given as follows:

$$d_i(x) = \begin{cases} 0 & if\ x\ is\ feasible \\ |g_i(x)| & otherwise\ for\ 1 \le i \le m_1 \\ |h_i(x)| & otherwise\ for\ m_1 + 1 \le i \le m \end{cases} \tag{3.10}$$

$p_t = C \times t$, where $p_t$ is variable penalty factor and $C$ is a constant. The penalty on infeasible chromosomes is increased along with the evolutionary process due to the variable penalty term of $p_t$. It varies with the iteration of the genetic algorithm. The experimental results of Joines & Houck (1994) indicate that the solution quality is very sensitive to the values of three parameters: $\alpha$, $\beta$ and $C$. The recommended values for the parameters of $\alpha$, $\beta$ and $C$ are: 1, 1 or 2, and 0.5, respectively.

## 3.8 Literature Review of Genetic Algorithm Applications for Inventory Management Problems

Usually, conventional mathematical methods and markov processes are used to solve the inventory problem. However, as the size of the problem increases, heuristic methods are applied. In the last 30 years, GA, one of the important metaheuristic methods, has gained popularity in finding near optimal solutions in solving large-scale optimization problems. This section gives a review of studies where GA is applied to inventory management problems.

Man, Tang, & Kwong (1996) introduced the application of GAs for industrial engineers and provided a basic framework for GAs. GA was used in scheduling, production and lot-sizing problems in the 1990s. Aytuğ, Khouja, & Vergara (2003) provided a review of how GA has been used to solve production and operations management (POM) problems.

Sarker & Newton (2002) determined an optimal batch size for a final product and an optimal purchasing policy of associated raw materials under the limited storage space and transportation fleet capacity restrictions. This problem was formulated as a constrained non-linear integer programming (NLIP) model and GA was applied to solve the problem. Three different penalty functions were analyzed in order to find out which one of them yielded good results for the problem.

Kleniau & Thonemann (2004) presented an alternative approach for solving inventory-control problems based on Genetic Programming. They showed that GA can reach a solution, considering decision variables as reorder point and order quantity. In addition, they asserted that it is important to determine suitable GA parameters in inventory problems. In their study, single-echelon with deterministic demand, single-echelon with stochastic demand and two-echelon with stochastic demand problems were addressed.

Mahapatra, Bhunia, & Maiti (2005) developed a deterministic multi-objective inventory model of wholesaler-retailers problem with a single product. They assumed that the storage capacity is to be finite. A real-coded GA (RCGA) was proposed to solve the problem. They compared the GA solution with generalised reduced gradient method.

Pal, Das, Panda, & Bhunia (2005) applied a RCGA for mixed-integer non-linear programming (MINLP). They assumed a two-warehouse inventory system for deteriorating items under the storage space constraint, and determined an optimal replenishment number and lot size. The objective of this study was to determine the optimal values of total number of replenishment cycles and the fraction of the scheduling period by maximizing the profit function.

Gen & Syarif (2005) proposed a hybridized tree-based genetic algorithm to solve multi-period production/distribution and inventory problem. They adopted the fuzzy logic controller to improve the performance of the algorithm. The authors took into account multiple products with independent demand, multiple shared resources and multi-periods.

Maiti, Bhunia, & Maiti (2006) proposed a RCGA to solve the proposed MINLP model under the storage space constraints. They considered two owned storage facilities and a rented warehouse. In this model, optimal shipments and lot size were determined to maximize the profit function.

Chan, Cheng, & Huang (2006) formulated a mixed-policy algorithm to determine the ordering quantities for agents in a supply chain to minimize the total cost. They proposed a GA to solve the problem and conducted computer simulations for different experimental setups.

Gupta, Bhunia, & Goyal (2009) developed a RCGA to solve mixed-integer constrained optimization problem. They proposed an (Q, r) inventory policy to show the effect of advance payment by the retailer to the wholesaler. The aim of this paper

was to determine the reorder level and the number of replenishments for the time horizon by maximizing profit function.

Taleizadeh et al. (2010) analyzed a MINLP model for multi-product inventory problem, and genetic algorithm and simulated annealing approach was set forth to solve it. They considered the warehouse space and budget constraints and incremental discount policy used. The aim of this paper was to maximize profit. They also analyzed the uniform and exponential rate for replenishment. The chromosome structure of the problem reveals the inventory levels of the products and is given in the real-coded mode.

Kannan, Sasikumar, & Devika (2010) proposed a GA to solve the mixed-integer linear programming model for a multi-echelon, multi-period, multi-product closed-loop supply chain network model. This model was applied on a lead acid battery production. They made a comparison of the small-size problems and large-size real-world problems by solving problems via an optimization software GAMS and GA. Storage space and processing time restrictions were considered in the model while minimizing the total cost. Two products and two periods were taken into consideration in this model.

Pasandideh, Niaki, & Nia (2011) developed a multi-product EOQ model for a two-level supply chain system consisting of one supplier and one retailer. The capacity of the supplier warehouse is limited and there is an upper bound on the number of orders. The objective of this study was to determine the order quantities and the maximum backorder level of the products to minimize the total inventory cost. The problem was formulated as a NLIP model and a GA was proposed to solve it. The chromosome here represented the order quantities and the maximum backorder levels of the products.

Yang et al. (2011) analyzed the supplier selection model for a single buyer with stochastic demand and multiple products under the constraints of service level and budget. The goal of this problem was to maximize expected profit. The decision

variables of the problem are order quantity and the decision of which product is supplied from which supplier. They formulated a NLIP model and then a GA to solve it.

Hongjie, Ruxian, Zhigao, & Ruijiang (2011) developed two inventory control models for deteriorating items according to time-based (S, T) and quantity-based (Q, J) integrated delivery strategies for suppliers under vendor management information (VMI) model. In the (S, T) policy, in which S means the order-up-to level to meet the total demand within the delivery cycle T. In (Q, J) delivery policy, fixed Q quantity is ordered for J times in random delivery. A bi-level programming model was established and a GA was developed to solve the problem. Bi-level programming is the study of planning and management problems of the two-level system by using upper level programming and lower level programming respectively.

Zhang et al. (2012) presented an (r, nQ) policy to analyze the two-stage, multi-item inventory problem with stochastic demand. A hybrid genetic algorithm (HGA) was proposed to solve the non-linear programming model. The aim of this paper was to determine reorder point and order quantity at each stage by minimizing expected total cost. They used Monte Carlo method in order to simulate the stochastic demand. The HGA was improved based on golden section method, and evaluating the cost function by Monte Carlo method.

Pasandideh, Niaki, & Mousavi (2013) developed a multi-period multi-item inventory control problem with random demand and limited storage capacity. The aim was to determine the optimal order quantities of the products in different periods and to minimize the total inventory cost. They considered the all-unit and incremental quantity discount by means of a combination of the problem. In this study, memetic algorithm and GA were put into use to solve it.

### 3.9 Genetic Algorithm Specifications

In this section, the past research about GA related solution methods involved in inventory problems is reviewed according to the following 10 criteria: proposed GA method, genetic demonstration of chromosome structure, formation of initial population, size of population, genetic operators that are used in formation of new individuals (crossover and mutation), selection method for mating of individuals, penalty function for feasibility and termination criteria. Table 3.1 shows published literature on inventory problems according to these ten criteria in chronological order.

Most of the GAs developed for the solution of inventory problems have common characteristics. As seen in Table 3.1, most of the researchers used standard GA procedure. In the reviewed articles, it is seen that different heuristic methods are used along with standard GA. Zhang et al. (2012) used Golden Search method along with GA. Ghiami, Williams, & Wu (2013) applied *Neighbouring Search* method with GA. Gen & Syarif (2005) used spanning tree based GA method. In recent years, the trend is to create hybrid GA by using different heuristic and metaheuristic methods along with GA in order to increase the efficiency of GA. Such studies are called hybrid genetic algorithm (HGA) in literature.

Formation of chromosome structure is one of the most important steps of GA procedure. Different chromosome structures are encountered in inventory management problems depending on decision variables. In general, the matrix structure is formed based on decision variables such as order quantity, inventory level, reorder point and transportation period. The number of rows and columns in the matrix increases depending on the number of products, number of periods or presence of different storages. It is remarkable that mostly binary coding chromosome structure is used in lot-size scheduling problems in literature whereas real numbers are used in inventory problems. In most cases, using binary coding is not adopted to solve optimization problems with a large number of variables (Gupta, Bhunia, & Goyal, 2007). Sarker & Newton (2002) solved lot-size scheduling

problem with GA and formed the chromosome structure with binary coding. Gen & Syarif (2005) initially solved the problem with GA and formed the chromosome structure with Prüfer number. Chan et al. (2006) preferred binary chromosome structure since they aimed to make selections such as order-up-to level S from various inventory policies (s, S), (Q, r), (EOQ). In the remaining 14 studies, GA is applied with real numbers.

Table 3.1 GA specifications according to parameters of GA published in inventory management problem

| Literature Reviewed | Method | Chromosome representation | Initial Population | Population size | Crossover type& rate | Mutation type& rate | Selection type | Termination Criteria | Penalty | Parameter Optimization |
|---|---|---|---|---|---|---|---|---|---|---|
| Sarker & Newton, 2002 | Standard | Binary ; lot size scheduling | Randomly | Pop_size= 50 | Two-point & Pc=100% | Bit flip & Pm = 1/string length of the chromos ome | Ranking | Max_gen= 200 | Joines and Houck Dynamic Penalties & Static penalties & Adaptive penalties | - |
| Gen & Syarif, 2005 | Standard &Spaning tree based GA | Prüfer numbers; plant and customer number | Randomly | Pop_size= 10, 20,25, 50,75 | One-point & Pc= 40% | Inversion mutation & Pm=20% | Roulette wheel | Max_gen= 1000,1500,3 000,4000,50 00 | - | Fuzzy logic controller |
| Mahapatra et al., 2005 | Standard | Real number; order quantity and transport time | Randomly | Pop_size= 400 | Pc= 80% | Pm= 10% | Roulette wheel | Max_gen= 6000 | sum of squares of relative deviations of the objectives | - |
| Pal et al., 2005 | Standard | Real number; number of replenishment s and lot size for two warehouse | Randomly | Pop_size= 200 | proper fraction & Pc= 80% | Uniform & non-uniform & Pm= 10% | Ranking | Max_gen= 1000 | - | - |
| Chan et al., 2006 | Standard | binary; policy agent | Randomly | Pop_size= 100 | one point | Bit flip | Roulette wheel | Converge | - | - |

Table 3.1 GA specifications according to parameters of GA published in inventory management problem (cont.)

| Literature Reviewed | Method | Chromosome representation | Initial Population | Population size | Crossover type& rate | Mutation type& rate | Selection type | Termination Criteria | Penalty | Parameter Optimization |
|---|---|---|---|---|---|---|---|---|---|---|
| Maiti et al., 2006 | Standard | Real number; initial inventory level and number of shipment | Randomly | Pop_size= 100 | Arithmetic & Pc=80% | Non-uniform & Pm=10% | Roulette wheel | Max_gen= 15 | - | - |
| Gupta et al., 2007 | Standard | Real number; initial inventory, reorder point and frequency of advertisiment | Randomly | Pop_size= 100 | Arithmetic & Pc=90% | Non-uniform & Pm=10% | Ranking | Max_gen=5 00 | - | - |
| Maiti & Maiti, 2008 | Standard | Real number; optimum inventory level for ith item; 1x2 matrix | Randomly | Pop_size= 100 | Arithmetic & Pc=30% | Replace by randomly within boundary | Roulette wheel | Max_gen=5 000 | Feasible solution 1, infeasible solution 0 | - |
| Gupta et al., 2009 | Standard | Real number; order quantity or shortage quantity and order number;1x2 matrix | Randomly | Pop_size= 100 | Arithmetic & Pc=90% | Non-uniform & Pm=10% | Ranking | Max_gen=5 00 | - | Graphically the effects of changes of GA parameters |
| Radhakrishnan et al., 2009 | Standard | Real number; stock level for different agent; | Randomly | Is not given | Two-point & Pc=NA | 4 mutation points randomly selected, changed | Ranking | Max_gen= 100 | - | - |

Table 3.1 GA specifications according to parameters of GA published in inventory management problem (cont.)

| Literature Reviewed | Method | Chromosome representation | Initial Population | Population size | Crossover type&rate | Mutation type&rate | Selection type | Termination Criteria | Penalty | Parameter Optimization |
|---|---|---|---|---|---|---|---|---|---|---|
| Kannan et al., 2010 | Standard | Is not given | Randomly | Pop_size= 10, 25, 40 | Pc = 70% | Pm= 5% | Is not given | Max_gen= 50, 100, 150, 200 | - | - |
| Hongjie et al., 2011 | Standard | Real number; delivery cycle | Randomly | Pop_size= 1000 | Arithmetic & Pc=85% | Pm= 5% | Roulette wheel | Max_gen=400 | Penalty function but which method is used is not given | - |
| Pasandideh et al., 2011 | Standard | Real number; order quantity and backorder; 2x10 matrix | Randomly | Pop_size= 80, 100, 120, 140, 160, 180 | One-point & Pc= 65, 68, 70, 75, 80% | Two randomly selected genes interchanged eachother; Pm= 1, 2, 2.5, 3, 4, 5 % | Roulette wheel | Max_gen= 400 | Larger coefficient (penalty) is given to more infeasible solution: static penalty function | Different parameters values are used |
| Pasandideh and Niaki, & Tokhmenchi, 2011 | Standard | Real number; batch size and reorder points for two warehouse;4x 6 matrix | Randomly | Pop_size= 50, 100, 150, 200 | Binary chormosom e created; Pc= 10, 50, 100% | Randomly changed & Pm= 1, 40, 95% | Roulette wheel | Converge | Sum of squared violation of each constraint | Fine-tuning procudure (different parameters values are used) |
| Yang et al., 2011 | Standard | Order quantity (real number) and supplier selection (0 or 1); 6x5 matrix | Randomly | Pop_size= 30, 70 | Pc= 40, 50, 60% | Pm= 5,10, 15% | Roulette wheel | Max_gen= 30000, 100000 | - | Different parameters values are used |

Table 3.1 GA specifications according to parameters of GA published in inventory management problem (cont.)

| Literature Reviewed | Method | Chromosome representation | Initial Population | Population size | Crossover type&rate | Mutation type&rate | Selection type | Termination Criteria | Penalty | Parameter Optimization |
|---|---|---|---|---|---|---|---|---|---|---|
| Zhang et al., 2012 | HGA; Standard & Golden search method | Real number;reorder point and order quantity | Randomly | Pop_size= 60 | Pc= from 10 to 90% | Pm= from 10% to 90% | Is not given | Max_gen=95, 110 | - | Different parameters values are used |
| Ghiami et al., 2013 | Standard &Neighbouring Search | Real number; time inventory level reaches zero; 1x3 matrix | Randomly | Pop_size= 200 | One-point & Pc= 75% | Change within the bounds; Pm= 15% | Roulette wheel | Max_gen=200 | - | - |
| Pasandideh et al., 2013 | Standard | Order quantity and period; 4x4 matrix | Randomly | Pop_size=40, 50 | Uniform; Pc= 65, 70, 75% | Pm= 10, 15, 20, 25 % | Roulette wheel | Max_gen= 500, 1000, 1500 | Sum of squared violation of each constraint | Taguchi analyzes |
| This Thesis, 2013 | Standard & Local search | Real number; order quantity and reorder point; 2*9 matrix | Randomly | Pop_size=20, 50, 100 | One-point & Pc= 25, 33, 50% | Change within the bounds; 5, 15, 40% | Roulette wheel | Is not given | Joines & Houck Dynamic Penalties | Taguchi analyzes |

As mentioned before, a GA starts with a set of individuals which is called as initial population. In the GA methods that are employed in inventory problems, it is seen that the initial population is generated randomly provided that the values of the decision variables are between the lower and upper limits. After the initial population is generated, fitness value is calculated for each individual, selection operator is used for selection of individuals based on fitness values to generate offspring. Either roulette wheel selection or ranking method is used for parent selection in most of the GA related literature for inventory problems (see Table 3.1). As for the constraint handling techniques, Talaizadeh et al. (2010) used rejecting strategy which is one of the methods used in literature to obtain feasibility of constraints in inventory problems. Joines & Houck's methods, static penalties and adaptive penalties methods, are separately applied by Sarker & Newton (2002) and their results are compared. As an alternative method, total error sum of squares for each constraint is calculated as penalty value and then added to the fitness value. This method is used by Pasandideh, Niaki, & Tokhmenchi (2011) and Pasandideh, Niaki, & Mousavi (2013). There is no clear information about how constrains are handled in the articles other than these studies.

Researchers generally selected the most suitable parameters by trial and error method for selection of GA parameters that have the biggest impact on the results of GA. The following studies included constant number of population, crossover rate and mutation rate among these parameters: Ghiami et al. (2013), Gupta et al. Hongjie et al. (2011), Mahaptra et al. (2005), Maiti et al. (2006), Maiti & Maiti (2008), Pal et al. (2005), Sarker & Newton (2002). Gupta et al. (2009), Pasandideh, Niaki, & Nia (2011), Yang et al. (2011) and Zhang et al. (2012) are examples to the studies that determine the parameter values giving the best solution by applying different parameter values. Gen & Syarif (2005) made parameter optimization with fuzzy logic controller method. Pasandideh et al. (2013) used Taguchi analysis method which is one of the parameter optimization methods. Taguchi method is applied by researches for solution of different types of GA problems.

### 3.10 Chapter Conclusions

In this section, the logic behind the GA solution algorithm that is proposed in this thesis is simply explained. Other heuristic methods (i.e. Simulated Annealing and Tabu Search) are briefly mentioned. It is explained how the penalty function used for GA is applied in mathematical models with constraints. Also, a literature survey is presented about the inventory management problems in which GA is used.

In the next chapter, multi-item multi-period (Q, r) inventory policy is formulated by an integer linear programming model (ILP) and then genetic algorithm (GA) is applied on the model. The performance of the proposed GA is also compared with the performance of ILP model. The results are compared for small size problems to illustrate the complexity of solving the problem with the ILP model.

# CHAPTER FOUR
# PROPOSED MATHEMATICAL MODEL AND GENETIC ALGORITHM APPROACH FOR MULTI-PRODUCT MULTI-PERIOD (Q, r) INVENTORY PROBLEM

## 4.1 Introduction

In chapter 2, we have presented the summary of the studies on (Q, r) inventory problems. Also in chapter 2, we have discussed the effects of many criteria such as number of products, number of periods, the policy applied, lead time, the structure of demand, product type and constraints on establishing the model in inventory problems. (Q, r) inventory model is established firstly by Hadley & Within (1963) as multi-period, but they addressed a single product. In the literature survey performed until today, it is seen that (Q, r) inventory models are used in single period multi-product problems and for nonperishable products. The main purpose of this thesis is to develop a mathematical model for the multi-product, multi-period (Q, r) inventory problem and to develop an effective GA approach for its solution.

Let us assume a big distributor who makes a general distribution between the manufacturer and the customers for perishable products. This distributor wants to decide when to make an order and how much order to give in order to meet the customer needs in a way that maximizing the profit under a certain budget constraint. Some products need cold storage and therefore there is storage area constraint for these products. Meanwhile, manufacturing companies may offer extra quantities as promotion according to the given order quantity due to competition. The products have certain lifetimes. For this reason, FIFO rule is applied, that is, the product already on hand is sold prior to the sale of more recently delivered product. Since the demand varies for each product and the demand for some products is uncertain or seasonal, it is aimed to apply the (Q, r) model which is a continuous review system. Accordingly, a specified order quantity ($Q$) is given whenever the stock level is at or below $r$ level for each product in this system. The opened order arrives at the storage

with a delay of certain lead time. If there is a promotion for an order at the quantity of *Q*, the quantity that will be added to our storage equals to "*Q + extra quantity"*.

This section is organized as follows. Assumptions of the model are presented in Section 4.2. In Section 4.3, sets, indices, parameters and decision variables used in the model are shown. Objective function and constraints forming the model are explained in Section 4.4. In Section 4.5, a numerical example about the solution of the model is given. Calculation results are presented in Section 4.6. In Section 4.7, problem solution with the proposed GA approach is explained. Finally, discussion and comments are presented in 4.8.

## 4.2 Proposed Integer Linear Programming Model

The problem at hand is to formulate a model that maximizes the total profit subject to constraints on budget, storage area, shelf life, and various promotions. The following notations are used to derive the ILP formulation of the proposed multi-product multi-period (Q, r) continuous review inventory model.

- There are multiple products.
- There is one warehouse.
- Demand is deterministic, seasonal or variable according to products.
- Lead time is fixed and constant.
- There is storage limitation for some products that especially require cold chain.
- There is finite budget limitation enforced by the bank.
- Backorders are not allowed.
- Shortages are allowed.
- There are some extra quantities supplied from the manufactures as "promotions" according to a given order quantity.
- Planning horizon is finite and planning is periodical.
- There are payment terms to the manufacturers and to the customers and these are different for each product.

- Lifetime is fixed and constant, and changes according to products.

### *4.2.1 Sets, Indices, Parameters and Variables*

The sets, indices, parameters, and decision variables used in the model are given as follows:

*Indices and Sets:*

| | | |
|---|---|---|
| *n* | index for the product where, | $i = 1, \dots, N$ |
| *t* | index for the planning period where, | $t = 1, \dots, T$ |
| *k* | index for the break point for extra quantity where, | $k = 1, \dots, K$ |
| *PC* | set of products that need cold storage environment | |

*Parameters:*

| | |
|---|---|
| $Q_{i,t,E}$ | extra order quantity for product *i* in period *t* |
| $x_{i,t}$ | expected demand for product *i* in period *t* |
| $md_i$ | expected mean demand during the lead time for product i |
| $l_i$ | lead time for product *i* |
| $sp_i$ | sales price for product *i* |
| $pp_i$ | unit purchasing price for product *i* |
| $cs_i$ | ordering cost for product *i* per order |
| $sc_i$ | unit shortage cost for product *i* |
| $h_i$ | unit holding cost for product *i* per period |
| $I_e$ | interest rate per period |
| $M_i$ | the warehouse's payment term offered by manufacturer in unit time for product *i* |
| $N_i$ | the customer's payment term offered by distributor in unit time for product *i* |
| $w_i$ | storage space required for product *i* |
| $I1_{i,0}$ | initial inventory level for product *i* |

$shf_i$ lifetime period for product *i*

$upper_r_i$ reorder point upper bound for product *i*

$upper_Q_i$ order quantity upper bound for product *i*

$U_k$ upper break point for quantity range *k*

$L_k$ lower break point for quantity range *k*

$EX_{i,k}$ extra good for quantity range *k* of product *i*

$B$ total available budget

$storage$ total storage capacity for all products

$BigM$ big number

*Decision Variables:*

$I_{i,t}$ ending inventory level for product *i* in period *t*

$I1_{i,t}$ positive inventory level for product *i* in period *t*

$I2_{i,t}$ inventory shortage for product *i* in period *t*

$Q_{i,t}$ order quantity for product *i* in period *t*

$QS_i$ order quantity for product *i*

$r_i$ reorder point for product *i*

$Y_{i,t}$ binary variable: equal to one if an order is placed in period *t*; otherwise, zero

$A_{i,t}$ binary variable: equal to one if the beginning inventory level of product *i* is less than the reorder point in period *t*, and zero if the beginning inventory level is greater than the reorder point, indicating whether an order should be placed.

$B_{i,t}$ binary variable: equal to one if the ending inventory level of product *i* in period *t* is positive, and zero if the inventory level is negative.

$D_{i,t,k}$ binary variable: equal to one if a purchase is made in period *t* for product *i*, and zero if no purchase is made in period *t* for product *i*, with quantity break *k*.

### *4.2.2 Objective Function*

Our aim is to maximize profit; therefore, the objective function is the total of the sales revenue and money earned as a result of the payment terms provided by the manufacturers, minus the total purchasing, holding, ordering, and shortage costs, and money lost because of the payment terms provided to customers.

Consequently, the objective function of the multi-product, multi-period (*Q*, *r*) policy model is given in the equation below:

*Maximize Total Profit = Expected sales revenue + Earnings resulting from payment term provided by manufacturer – Total cost – Loss resulting from payment term provided to customer* (4.1)

*Total Cost = Purchasing cost + Holding cost + Ordering cost + Shortage cost* (4.2)

$$Max\ Z = (Q_i, r_i) \tag{4.3}$$

The components in the objective function can be calculated as following relations:

***Expected sales revenue:***

$$\text{ETSR} = \sum_{t=1}^{T}\sum_{i=1}^{N} x_{i,t}.sp_i - \sum_{t=1}^{T}\sum_{i=1}^{N} I2_{i,t}.sp_i \tag{4.4}$$

The total sales revenue obtained during the planned *T* periods is the product of the demand quantity and unit sales price. If the incoming demand is not completely satisfied, sales loss will occur as a result of the unmet demand. Expected sales revenue is calculated by subtracting this sales loss from the sales revenue.

***Earnings resulting from the payment term provided by manufacturer:***

$$\text{TPFS} = \sum_{i=1}^{N}\sum_{t=1}^{T} Q_{i,t} \,.\, pp_i . I_e . M_i \tag{4.5}$$

Manufacturing companies may provide distributors with convenient payment terms for purchased products. The earnings that a distributor can obtain as a result of this non-cash payment for the purchased quantity is equal to the product of the order quantity, purchasing cost, and periodic interest and payment term. The earnings that the distributor will obtain during the planning period as a result of the payment terms is calculated as in Equation (4.5).

***Purchasing cost:***

$$\text{TPC} = \sum_{t=1}^{T}\sum_{i=1}^{N} Q_{i,t} \,.\, pp_i \tag{4.6}$$

Each period, consideration is given to whether an order should be placed or not and, if an order is placed, the total purchasing cost that will accrue during the planning period is equal to the product of the unit sales price and order quantity.

***Loss resulting from the payment term provided to customer:***

$$\text{TLPFC} = \sum_{i=1}^{N}\sum_{t=1}^{T} x_{i,t} . sp_i . I_e . N_i \tag{4.7}$$

A distributor may also provide customers with convenient payment terms similar to those provided by the suppliers of the distributor. In this case, the distributor's loss is the product of the sales amount, payment term, and interest.

***Holding cost:***

$$\text{THC} = \sum_{i=1}^{N}\sum_{t=1}^{T} h_i . I1_{i,t} \tag{4.8}$$

The total holding cost (THC) during the planning period is the product of the available stock quantity at the end of each period and the periodic holding cost.

**Ordering cost:**

$$Q_{i,t} \leq BigM . Y_{i,t} \qquad i = 1, \dots, N, t = 1, \dots, T \tag{4.9}$$

$$\text{TOC} = \sum_{i=1}^{N}\sum_{t=1}^{T} Y_{i,t} . cs_i \tag{4.10}$$

If $Q_{i,t} > 0$ , then an order has been placed. Constraint (4.9) is used to determine whether an order has been placed or not. The total ordering cost (TOC) is the product of the number of orders given during the total planning period and the ordering cost.

***Expected total shortage cost:***

$$\text{ETSC} = \sum_{i=1}^{N}\sum_{t=1}^{T} sc_i . I2_{i,t} \tag{4.11}$$

If the demand is higher than the level of the reorder point, then there can be a shortage, in which case, the expected total shortage cost (ETSC) is the product of the expected shortage quantity and the shortage cost.

### *4.2.3 Constraints*

The constraints of the model can be grouped into six sets: budget, storage area, lifetime, inventory balance equations, and extra quantity. These constraints are explained in detail in the ensuing subsections.

*Inventory balance equations:*

If

$$I_{i,t-1} \leq r_i \qquad i = 1, \dots, N, t = 1, \dots, T \qquad (4.12)$$

Then

$$Q_{i,t} + Q_{i,t,E} - I1_{i,t} + I2_{i,t} + I1_{i,t-1} - x_{i,t} = 0$$

$$i = 1, \dots, N, t = 1, \dots, T \qquad (4.13)$$

Else

$$I1_{i,t-1} - I1_{i,t} + I2_{i,t} - x_{i,t} = 0$$

$$i = 1, \dots, N, t = 1, \dots, T \qquad (4.14)$$

and

$$Q_{i,t} = 0 \qquad i = 1, \dots, N, t = 1, \dots, T \qquad (4.15)$$

$$I1_{i,t-1} - r_i + 1 \leq BigM.(1 - A_{i,t})$$

$$i = 1, \dots, N, t = 1, \dots, T \qquad (4.16)$$

$$r_i - I1_{i,t-1} \leq BigM.A_{i,t} \qquad i = 1, \dots, N, t = 1, \dots, T \qquad (4.17)$$

$$Q_{i,t} + Q_{i,t,E} - I1_{i,t} + I2_{i,t} + I1_{i,t-1} - x_{i,t} \geq -BigM.(1 - A_{i,t})$$

$$i = 1, \dots, N, t = 1, \dots, T \qquad (4.18)$$

$$Q_{i,t} + Q_{i,t,E} - I1_{i,t} + I2_{i,t} + I1_{i,t-1} - x_{i,t} \leq BigM.(1 - A_{i,t})$$

$$i = 1, \dots, N, t = 1, \dots, T \qquad (4.19)$$

$$I1_{i,t-1} - I1_{i,t} + I2_{i,t} - x_{i,t} \geq -BigM.(1 - A_{i,t})$$

$$i = 1, \dots, N, t = 1, \dots, T \qquad (4.20)$$

$$I1_{i,t-1} - I1_{i,t} + I2_{i,t} - x_{i,t} \leq BigM.(1 - A_{i,t})$$

$$i = 1, \dots, N, t = 1, \dots, T \qquad (4.21)$$

$$Q_{i,t} \leq BigM.A_{i,t} \qquad i = 1, \dots, N, t = 1, \dots, T \qquad (4.22)$$

According to the relations given in Constraints (4.12) to (4.15), if the stock level at the beginning of a period is lower than the reorder point implied by Constraint (4.12), an order quantity $Q_{i,t}$ is placed and the inventory balance equation is enforced by Constraint (4.13). According to Constraint (4.13), for product *i*, inventory level $I_{i,t}$ at the end of period *t* is found by adding order quantity $Q_{i,t}$ and extra quantity $Q_{i,t,E}$ to the inventory level $I_{i,t-1}$ at the beginning of the period, and subtracting expected demand $x_{i,t}$. Conversely, if the stock level at the beginning of a period is greater than the reorder point, no order is placed and the inventory balance equation is provided by Constraint (4.14). In order to model these relations linearly, Constraints (4.16) to (4.22) were developed for the proposed ILP model.

*Order quantity constraints:*

$$Q_{i,t} - QS_i \leq BigM.(1 - A_{i,t}) \qquad i = 1, \dots, N, t = 1, \dots, T \qquad (4.23)$$

$$Q_{i,t} - QS_i \geq -BigM.(1 - A_{i,t}) \qquad i = 1, \dots, N, t = 1, \dots, T \qquad (4.24)$$

Constraints (4.23) and (4.24) assert that the order quantities should be equal for all the periods if an order decision has been placed, which is indicated by $QS_i$ for each product.

*Inventory level constraints:*

$$I2_{i,t} \leq BigM.B_{i,t} \qquad i = 1,\dots,N, t = 1,\dots,T \qquad (4.25)$$

$$I1_{i,t} \leq \mathrm{B}igM.(1 - B_{i,t}) \qquad i = 1,\dots,N\ t = 1,\dots,T \qquad (4.26)$$

$$I_{i,t} = I1_{i,t} - I2_{i,t}, \qquad i = 1,\dots,N\ t = 1,\dots,T \qquad (4.27)$$

Constraints (4.25) and (4.26) define positive and negative inventory as $I1_{i,t} = \max\{I_{i,t}, 0\}$ and $I2_{i,t} = \max\{-I_{i,t}, 0\}$, respectively. In period *t*, the inventory holding cost is $(h_i.I1_{i,t})$, and the inventory shortage cost is $(sc_i.I2_{i,t})$. Constraint (4.27) finds the on hand inventory level or shortage quantity.

*Storage space constraints:*

$$\sum_{i \in PC} w_i\left(I1_{i,t-1} + Q_{i,t} + Q_{i,t,E}\right) \leq storage \qquad t = 1,\dots,T \qquad (4.28)$$

The sum of the inventory level at the beginning of a period plus the given order quantity with the relevant extra quantity supplied should be below the total storage capacity at each period. This constraint ensures that the maximum inventory level is less than or equal to the storage area capacity in each period. Because some products in the storage need a cold environment, storage constraint has to be taken into account for such products.

*Budget constraints:*

$$\sum_{i=1}^{N}\sum_{t=1}^{T} Q_{i,t}.pp_i \leq B \tag{4.29}$$

The product of the order quantity planned for the entire planning period and the purchasing price should be below the budget. The distributor should complete the purchasing process without exceeding this credit limit.

*Lifetime constraints:*

$$\frac{I1_{i,t-1} + Q_{i,t} + Q_{i,t,E}}{md_i} \leq shf_i \qquad i = 1,\dots,N, t = 1,\dots,T \tag{4.30}$$

Products have specific lifetimes; the products have to be destroyed if these lifetimes expire. Therefore, the time obtained by dividing the total inventory at the beginning of the period and the incoming orders by the average demand should be shorter than the lifetime.

*Extra quantity constraints:*

$$Q_{i,t,E} = \begin{cases} EX_{i,1}, & L_1 \leq Q_{i,t} < U_1 \\ EX_{i,2}, & L_2 \leq Q_{i,t} < U_2 \\ & \vdots \\ EX_{i,k}, & L_k \leq Q_{i,t} < U_k \end{cases} \tag{4.31}$$

In an effort to stay ahead of the competition, manufacturing companies may send various extra quantities, $EX_{i,k}$, of a product according to the quantity purchased by the distributor. Constraint (4.31) shows the extra quantities offered by a manufacturing company in return for the quantity ordered. If the distributor places an order for $i$ product between $L_1$ and $U_1$ in $t$ period, s/he will obtain an extra quantity

of $EX_{i,1}$. These products arrive in storage at no cost and are sold at the normal sales price to the customer.

$$Q_{i,t} - L_k \geq BigM.D_{i,t,k} \qquad i = 1,\dots,N\ t = 1,\dots,T\ k = 1,\dots,K \quad (4.32)$$

$$Q_{i,t} - U_k + 1 \leq BigM.(1 - D_{i,t,k}) \qquad i = 1,\dots,N, t = 1,\dots,T, k = 1,\dots,K \quad (4.33)$$

$$\sum_{k=1}^{K} D_{i,t,k} = 1 \qquad i = 1,\dots,N\ t = 1,\dots,T \quad (4.34)$$

$$Q_{i,t,E} = \sum_{i=1}^{K} EX_{i,k}.D_{i,t,k} \qquad i = 1,\dots,N\ t = 1,\dots,T \quad (4.35)$$

$$Q_i \leq upper_Q_i \qquad i = 1,\dots,N \quad (4.36)$$

$$r_i \leq upper_r_i \qquad i = 1,\dots,N \quad (4.37)$$

$$A_{i,t}, B_{i,t}, Y_{i,t}, D_{i,t,k} \in \{0,1\} \qquad i = 1,\dots,N\ t = 1,\dots,T\ k = 1,\dots,K \quad (4.38)$$

$$r_i, QS_i, Q_{i,t}, I_{i,t}, I1_{i,t}, I2_{i,t} \geq 0, integer \qquad i = 1,\dots,N\ t = 1,\dots,T \quad (4.39)$$

Constraints (4.32), (4.33) and (4.34) determine the quantity range within which the given ordered quantity falls into. Constraint (4.35) determines the extra quantity in regard to the quantity purchased. Constraints (4.36) and (4.37) give ordered quantities and reorder points should be less than or equal to the upper bound values specified for each product. Constraints (4.38) and (4.39) ensure that the value and integer restrictons for all the variables are used in the model.

## 4.3 Computational Study for the ILP Model

In this section, to evaluate the performance of the proposed ILP model, a computational study is conducted by running the model for randomly generated problem instances. The generated problems are different in terms of the number of products and number of periods. For instance, the [3P_8T] problem shows a 3-product 8-period problem case. It is assumed that the product demand is independent and known for each product, and different for each period. For this problem, the demand at each period is given in Table 4.1, parameter values are given in Table 4.2 and extra quantity ranges are given in Table 4.3.

Table 4.1 Product demand for each period for the example of the [3P_8T] problem

| Period | Product 1 | Product 2 | Product 3 |
|---|---|---|---|
| 1 | 101 | 0 | 0 |
| 2 | 56 | 0 | 43 |
| 3 | 121 | 0 | 1 |
| 4 | 70 | 10 | 5 |
| 5 | 46 | 77 | 22 |
| 6 | 58 | 24 | 178 |
| 7 | 95 | 8 | 57 |
| 8 | 43 | 55 | 44 |

The proposed ILP model has been coded in LINGO 9.0 software. We have attempted to solve the test problems on Intel® Core ™ Duo with CPU P8700@ 2.53 GHz and 4.00 GB RAM personal computer and the run time has been limited up to 5 hours for each problem. The computational results are given in Table 4.4.

As seen in Table 4.4, as the number of products and periods increase, it becomes unfeasible to find the optimum solution by the proposed ILP model. Furthermore, the computational times of finding the optimal solutions depend on the number of products and the number of periods. Figure 4.1 shows different number of periods for two products and the computational CPU times for the solution of proposed ILP model.

In ILP model, some decision variables are constrained with only integer values. However, integer variables make it more difficult to solve optimization problem due

to non-convex solution space structure. Required memory and computational time grow exponentially depending on the number of integer variables. In conclusion, it is clearly seen in Table 4.4 and Figure 4.1 that as the number of products and periods increase, optimum solution cannot be obtained in acceptable computational time. From this point of view, in this thesis, genetic algorithm is proposed to solve the problem for obtaining optimum or near optimum solutions in reasonable computation time.

Table 4.2 Input data for the example of the [3P_8T] problem

| i | Product 1 | Product 2 | Product 3 |
|---|---|---|---|
| $l_i$ | 1 | 1 | 1 |
| $md_i$ | 74 | 22 | 44 |
| $sp_i$ | 64 | 19 | 59.26 |
| $pp_i$ | 59.7 | 17.26 | 55.39 |
| $cs_i$ | 10 | 10 | 10 |
| $sc_i$ | 4.18 | 1.38 | 3.87 |
| $h_i$ | 0.21 | 0.06 | 0.20 |
| $I_e$ | 15 | 15 | 15 |
| $M_i$ | 6 | 6 | 6 |
| $N_i$ | 4 | 4 | 4 |
| $w_i$ | 1 | 1 | 1 |
| $I0_i$ | 0 | 0 | 0 |
| $shf_i$ | 12 | 12 | 12 |
| $upper_r_i$ | 295 | 87 | 175 |
| $upper_Q_i$ | 126 | 127 | 100 |

Resource available: Storage: 500 units ; Budget: $300000 ; BigM: 500000

Table 4.3 Extra quantity ranges for the [3P_8T] problem

| Quantity break | Purchased | Extra Quantity Units | | |
|---|---|---|---|---|
| (k) | Quantity | Product 1 | Product 2 | Product 3 |
| 1 | 0-9 | 0 | 0 | 0 |
| 2 | 10-19 | 2 | 20 | 1 |
| 3 | 20-49 | 5 | 40 | 3 |
| 4 | 50-99 | 42 | 70 | 7 |
| 5 | 100-499 | 80 | 100 | 30 |
| 6 | 500 or more | 250 | 150 | 50 |

Table 4.4 Computational results for the ILP model

| Problem type | No. of products | No. of periods | No. of variables | No. of constraints | CPU time (hr) | Results |
|---|---|---|---|---|---|---|
| 1P_4T | 1 | 4 | 70 | 127 | 0.001 | Global optimal |
| 1P_8T | 1 | 8 | 130 | 243 | 0.002 | Global optimal |
| 1P_12T | 1 | 12 | 190 | 359 | 0.022 | Global optimal |
| 1P_24T | 1 | 24 | 370 | 707 | 5 | Feasible |
| 2P_4T | 2 | 4 | 131 | 241 | 0.002 | Global optimal |
| 2P_8T | 2 | 8 | 251 | 465 | 0.019 | Global optimal |
| 2P_12T | 2 | 12 | 371 | 697 | 0.259 | Global optimal |
| 2P_13T | 2 | 13 | 401 | 754 | 1.271 | Global optimal |
| 2P_14T | 2 | 14 | 431 | 811 | 5 | Feasible |
| 2P_15T | 2 | 15 | 461 | 868 | 5 | Feasible |
| 2P_24T | 2 | 24 | 731 | 1381 | 5 | Unknown |
| 3P_4T | 3 | 4 | 196 | 355 | 0.007 | Global optimal |
| 3P_8T | 3 | 8 | 382 | 689 | 4.794 | Global optimal |
| 3P_10T | 3 | 10 | 466 | 865 | 5 | Feasible |
| 3P_21T | 3 | 21 | 961 | 1800 | 5 | Unknown |
| 4P_4T | 4 | 4 | 259 | 469 | 0.015 | Global optimal |
| 4P_8T | 4 | 8 | 499 | 921 | 5 | Feasible |
| 4P_15T | 4 | 15 | 919 | 1712 | 5 | Unknown |
| 5P_4T | 5 | 4 | 322 | 583 | 0.045 | Global optimal |
| 5P_8T | 5 | 8 | 622 | 1147 | 5 | Feasible |
| 5P_12T | 5 | 12 | 922 | 1711 | 5 | Unknown |
| 6P_4T | 6 | 4 | 385 | 697 | 0.166 | Global optimal |
| 6P_8T | 6 | 8 | 745 | 1373 | 5 | Feasible |
| 6P_11T | 6 | 11 | 1015 | 1880 | 5 | Unknown |
| 7P_4T | 7 | 4 | 448 | 811 | 0.781 | Global optimal |
| 7P_8T | 7 | 8 | 868 | 1599 | 5 | Feasible |
| 7P_10T | 7 | 10 | 1078 | 1993 | 5 | Unknown |
| 8P_4T | 8 | 4 | 511 | 925 | 5 | Feasible |
| 8P_9T | 8 | 9 | 1111 | 2050 | 5 | Unknown |

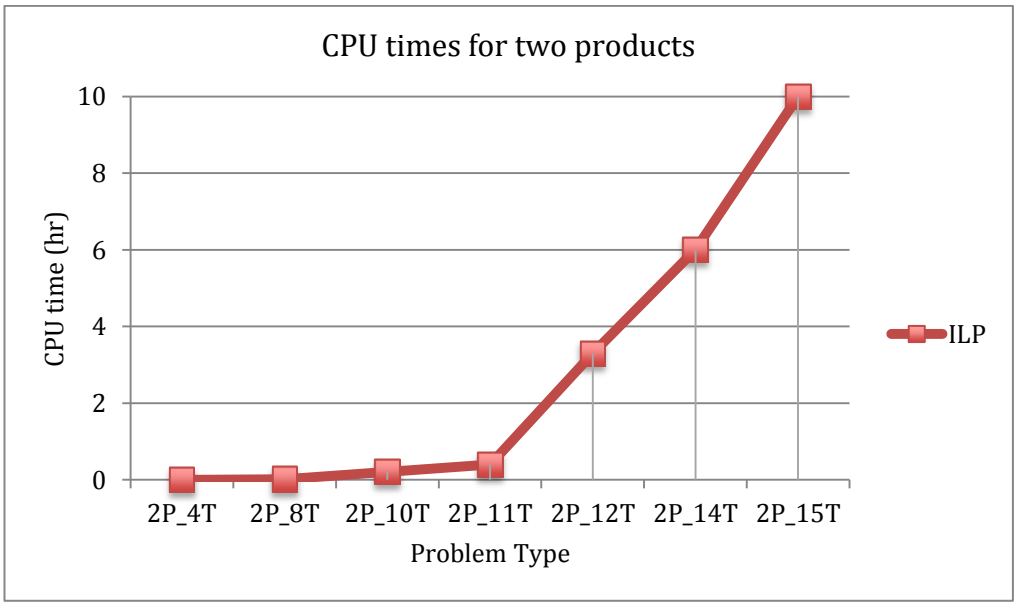



Figure 4.1 CPU times (hr) for two products and different number of periods

## 4.4 The Proposed GA with Local Search

In this section, GA approach with an embedded local search is proposed for solving a multi-product, multi-period (Q, r) inventory policy. The basic elements of GA approach and proposed methodology are presented in the following section.

### *4.4.1 The Logic of Search in GAs*

The purpose of the proposed approach is to search near optimum solutions in acceptable computational time for a multi-product, multi-period inventory problem. Primarily, ILP model is proposed in Section 4.2 for finding at what quantity orders should be placed and when these orders should be given in multi-product, multi-period inventory problem that has a known demand. In the model, an inventory policy (Q, r) is considered which aims to give an order of *Q* when the inventory level decreases to r level or below. The details of the objective function and information about general GA features and parameters are given in Chapter 3. In this section it is shown step by step how to solve the numerical example given in previous section by using GA approach.

#### *4.4.1.1 Chromosome Representation*

The first and most important step of GA approach is deciding on the chromosome structure. GA approach should begin with the numerical demonstration of decision variables and data involved in the chromosome structure. In this study, the decision variables of the proposed model are taken, which are respectively reorder point, *r*, and order quantity, *Q*. Matrix notation is used for the demonstration of a chromosome. The first and second rows of the matrix include the variables of order quantity ($Q_i$) and reorder point ($r_i$), respectively ($i = 1, \dots, N$; the number of products). The columns of the matrix show the products. Therefore, a chromosome consists of a 2x$N$ matrix including two rows and as much columns as the number of products. The demonstration of chromosome structure is given in Figure 4.2.

It is remarkable that mostly binary coding chromosome structure is used in lot-size scheduling problems in literature whereas real numbers are used in inventory problems. In most cases, it is not adopted to solve large decision variable optimization problems with binary coding (Gupta et al., 2007). Also in our problem, the chromosome structure is formed by using real values.

*4.4.1.2 Initial Population*

GA starts research with an initial population. It is well known that the performance of metaheuristics is affected by the selection of the initial solution(s). If the initial solution is well enough, the possibility of reaching better solutions will increase and convergence to the near optimum or optimum result will be faster. In order not to depend only on the initial population, a local search considering neighbour values is proposed with GA in this study.

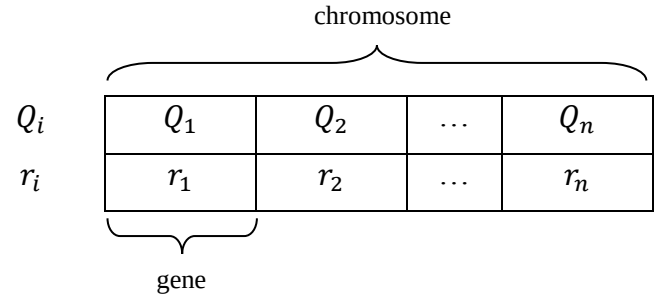



Figure 4.2 Gene string and chromosome representation of 2xN matrix

In this problem, initial population is determined randomly so that the decision variables $Q$ and $r$ are between the lower and upper limits. For each product, economic order quantity is calculated as $EOQ_i = \sqrt{2cs_i.X_i/h_i}$ where $X_i = \sum_{t=1}^{T} x_{i,t}$ and reorder point is calculated as $r_i = md_i * l_i$. EOQ and $r$ are calculated as $EOQ$ = {32, 32, 25} and $r$ = {74, 22, 44} for the three products in the [3P_8T] example problem. Lower limit is assumed zero, whereas upper limit is determined as four times the calculated values of $EOQ_i$ and $r_i$. If we assume that our first chromosome generated for the [3P_8T] problem considered in our numerical example consists of the levels of the calculated economic order quantity and reorder point, it is formed as in Figure 4.3:

| | | | |
|---|---|---|---|
| $Q_i$ | 32 | 32 | 25 |
| $r_i$ | 74 | 22 | 44 |

Figure 4.3 A sample solution for the example problem

Other chromosomes in the population are selected randomly between $Lower_Q_i = \{0, 0, 0\}$ and $Upper_Q_i = \{128, 128, 100\}$ range for the first row. They are selected randomly between $Lower_r_i = \{0, 0, 0\}$ and $Upper_r_i = \{296, 88, 176\}$ range for the second row. Therefore, all the chromosomes of the initial population are generated randomly as the number of population (pop_size).

*4.4.1.3 Calculating the Fitness Values*

Each chromosome in the initial population is evaluated by the fitness values of the individuals in order to apply natural selection. While the individuals with the worse fitness values die, the individuals with the better fitness values survive. The fitness values of the solutions can be considered as the objective function values of the solutions. Since the application of the selection method depends on the fitness value, it is important to calculate the fitness values properly. Penalty function is used because the problem addressed in this study has constraints. Since the aim is to maximize the profit, the fitness value is calculated by subtracting the penalty function from the objective function as follows.

Fitness function = objective function - penalty function

Therefore, an individual having the maximum profit in a population has the maximum fitness value. The penalty function developed by Joines & Houck (1994) and shown in Section 3.7.1.1 is used. In this penalty function, it is important to choose the parameter values correctly. They recommended the values 1, 1 or 2 and 0.5 for the parameters $\alpha$, $\beta$ and $C$ respectively. $\beta$ value is taken as 5, $\alpha$ value is taken as 1 and $C$ value is taken as 0.5 since the high-quality solutions could not be obtained with their recommended parameter values in our problem. Therefore, the penalty function in Equation 3.9 can be written as follows:

$$p(t,x) = 0.5.t \sum_{i=1}^{m} d_i^5(x) \qquad \text{where, } t = \text{iteration number; } i = 1,\ldots,m \text{ constraint number} \qquad (4.40)$$

$d_i$(x) is calculated for each iteration as in Equation 3.10. The aim is to increase the penalty value as the number of iterations increases and not to choose infeasible solutions. The flowchart of GA procedure applied in this study is shown in Figure 4.4. A four column table, which includes beginning inventory level, order quantity, demand and the stock level at the end of the period, is generated for calculating the fitness value. The calculation procedure of the inventory level matrix and fitness value for the products having deterministic demand structure is given in Figure 4.5 and Figure 4.6, respectively. The process of A and B is shown in Figure 5.9.

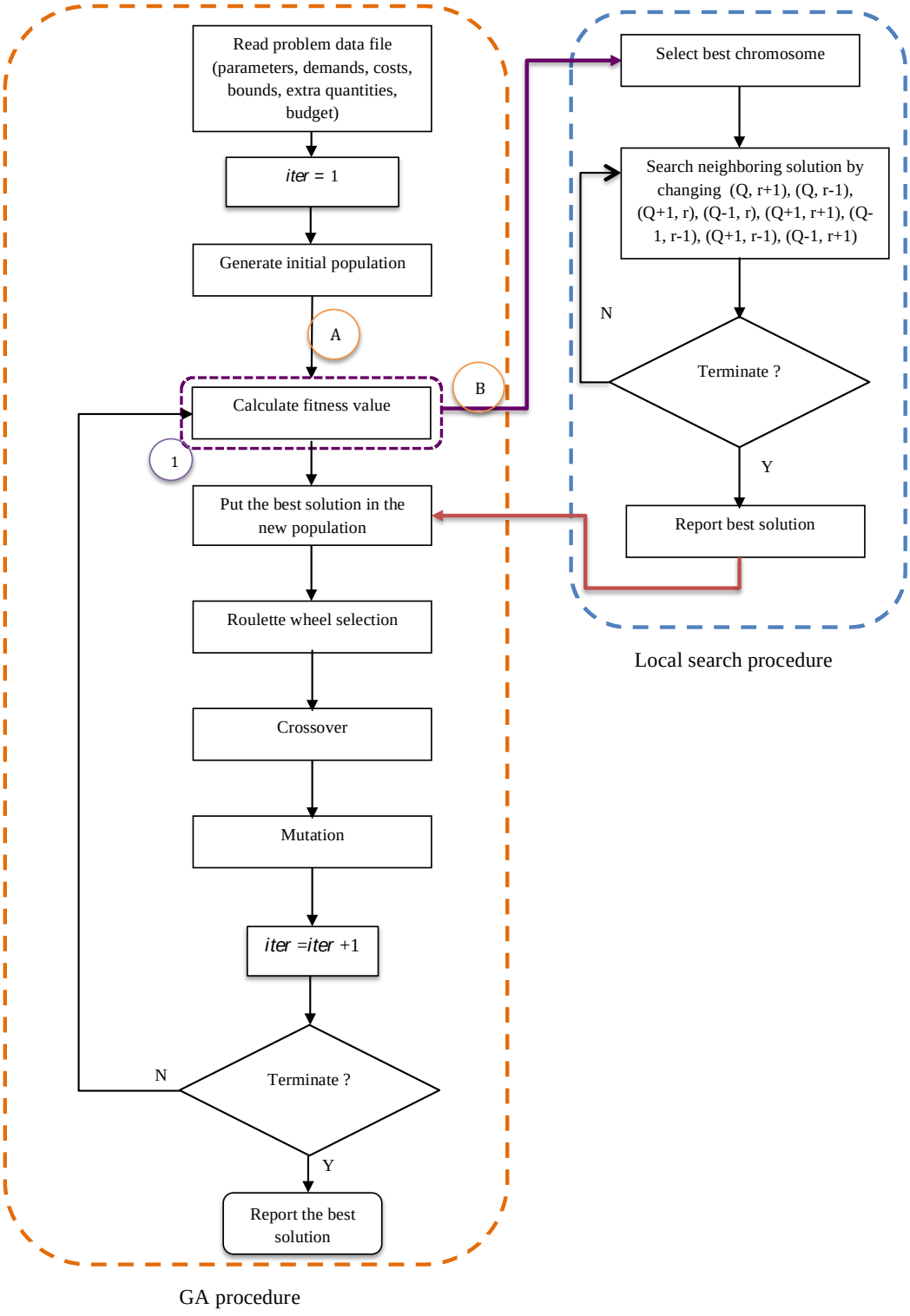



Figure 4.4 General structure of the GA with local search

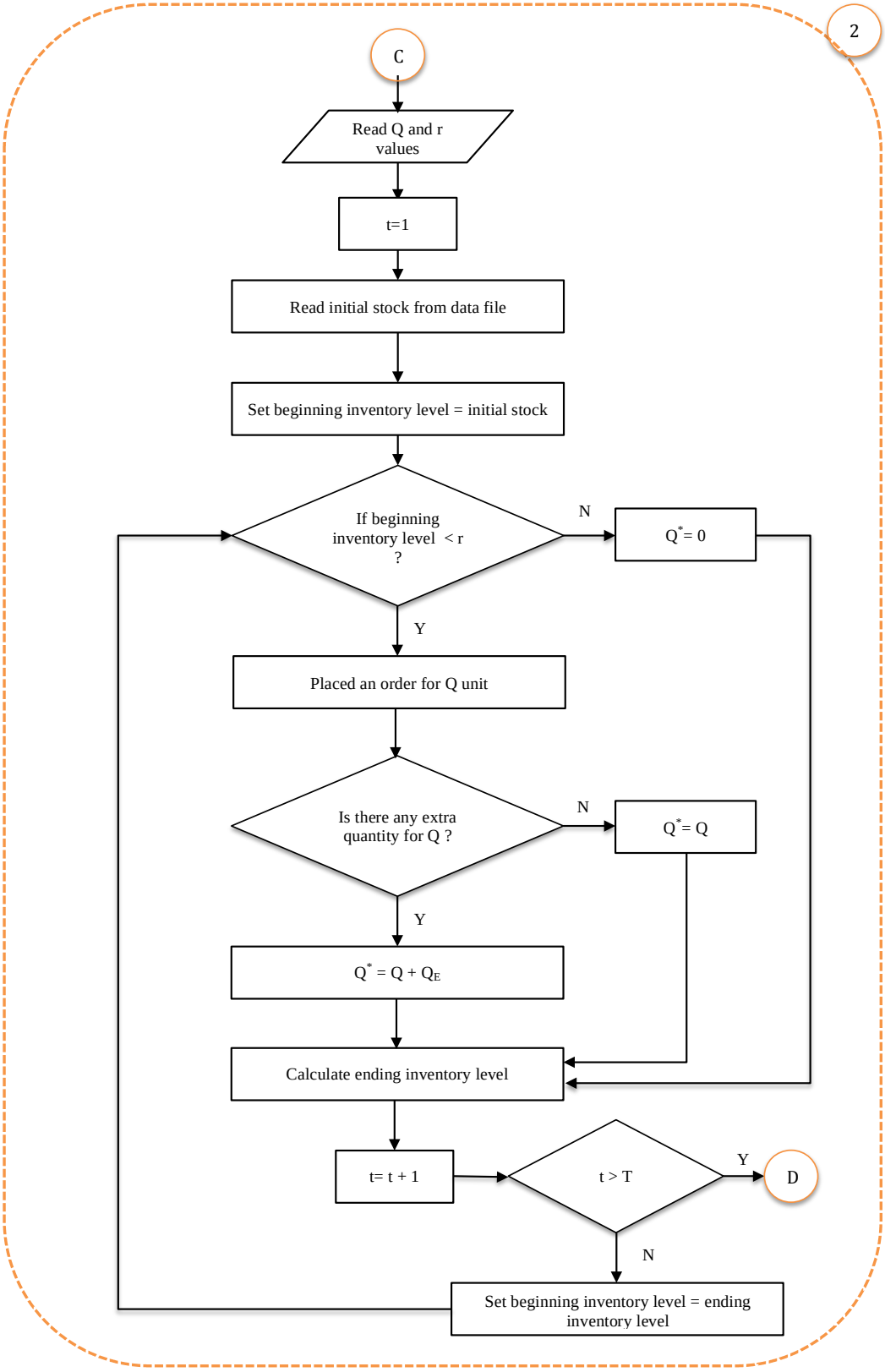



Figure 4.5 Calculate inventory level matrix for the deterministic demand case

```
Calculate fitness value for deterministic demand

1. Set fitnesspop ==0
2.  For i=1 to numofchrom (number of chromosome) do
3.     For j=1 to numofproduct (number of product) do
4.       Read Q and R
5.          Calculate inventory matrix according to
         deterministic    demand
6.        Calculate profit
7.        Evaluate storage area constraints
8.        If required storage area > storage area capacity
9.        Calculate penalty term for storage area constraint
10.       Else End
11.       Evaluate lifetime constraints
12.       If calculated lifetime > required lifetime
13.       Calculate penalty term for lifetime constraint
14.       Else End
15.Calculate total purchasing cost
16.If calculated total purchase > budget
17.Calculate penalty term for budget constraint
18.Else End
19.Calculate fitnessvalue = profit - penalty value
20.    End
21. End
22.Set fitnessvalue= max(fitnessvalue)
23.Set best chromosome
```

Figure 4.6 Pseudocode for calculating fitness value for deterministic demand

#### *4.4.1.4 Proposed Local Search*

For avoiding local optimum during the GA process, it is combined with different procedures and near optimum or optimum solution is tried to be obtained. In this thesis, a local search approach searching neighbour values is applied with genetic algorithm. The general structure of the GA running with the local search is defined as in Figure 4.7.

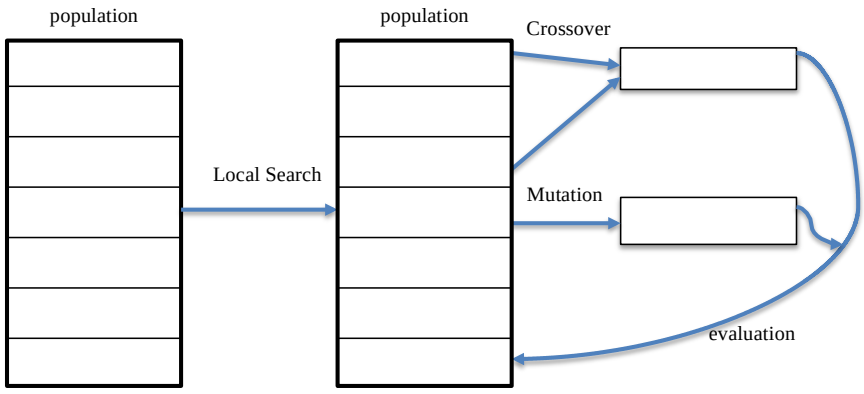



Figure 4.7 The GA procedure with local search

The local search approach proposed in this study is used by Zhao et al. (2012) to find neighbour solutions with local search on the current (Q, r) solution generated by iterative procedure. The neighbour solution of the current solution is defined as a solution obtained by increasing or decreasing $Q$ and/or $r$ one by one. So, for the relevant product $n$, the eight neighbour solution of (Q, r) is obtained by changing $Q$ and $r$ as in the example, i.e., $(Q_n, r_n)$ to $(Q_n, r_n + 1), (Q_n, r_n - 1), (Q_n + 1, r_n), (Q_n - 1, r_n), (Q_n + 1, r_n + 1), (Q_n - 1, r_n - 1), (Q_n + 1, r_n - 1), (Q_n - 1, r_n + 1)$. If a chromosome having a better fitness value is obtained in each iteration at the local search, that chromosome is included in the population. This procedure is iterated as much as the determined iteration number. The local search procedure is given in Figure 4.8.

```
   Procedure for local search algorithm
1.   Procedure (Localsearch)
2.    For i=1 to numofchrom do (second row does not change)
3.       Select best chromosome from the population and assign to
         tabu
4.       For z=1 to reach lower bound do
5.          Decrase gene value by one
6.          Calculate fitness value
7.          If fitnessvalue >= bestvalue then
8.          Set best chromosome
9.          Set best fitnessvalue
10.         Else
11.          Break
12.         End if
```

Figure 4.8 Pseudocode for local search

```
   Procedure for local search algorithm (cont.)
13.       End for
14.    End for
15.    For i=1 to numofchrom do (second row does not change)
16.       Select best chromosome from the population and assign to
          tabu
17.       For z=1 to reach upper bound do
18.          Increase gene value by one
19.          Calculate fitness value
20.          If fitnessvalue >= bestvalue then
21.          Set best chromosome
22.          Set best fitnessvalue
23.          Else
24.           Break
25.          End if
26.       End for
27.    End for
28.    For i=1 to numofchrom do
29.       Select best chromosome from the population and assign to
          tabu
30.       For z=1 to reach upper or lower bound do
31.          Increase gene value by one, decrease gene value by one
32.          Calculate fitness value
33.          If fitnessvalue >= bestvalue then
34.          Set best chromosome
35.          Set best fitnessvalue
36.          Else
37.           Break
38.          End if
39.       End for
40.     End for
41.    End for
42.    For i=1 to numofchrom do
43.       Select best chromosome from the population and assign to
          tabu
44.       For z=1 to reach upper or lower bound do
45.          Increase gene value by one, increase gene value by one
46.          Calculate fitness value
47.          If fitnessvalue >= bestvalue then
48.          Set best chromosome
49.          Set best fitnessvalue
50.          Else
51.           Break
52.          End if
53.       End for
54.     End for
55.    End for
```

Figure 4.8 Pseudocode for local search (cont.)

```
  Procedure for local search algorithm (cont.)
56.  For i=1 to numofchrom do
57.       Select best chromosome from the population and assign to
         tabu
58.       For z=1 to reach upper or lower bound do
59.         Decrease gene value by one, decrease gene value by one
60.         Calculate fitness value
61.         If fitnessvalue >= bestvalue then
62.         Set best chromosome
63.         Set best fitnessvalue
64.         Else
65.          Break
66.         End if
67.       End for
68.    End for
69.   End for
70.  For i=1 to numofchrom do (first row does not change)
71.       Select best chromosome from the population and assign to
         tabu
72.       For z=1 to reach lower bound do
73.         Decrease gene value by one
74.         Calculate fitness value
75.         If fitnessvalue >= bestvalue then
76.         Set best chromosome
77.         Set best fitnessvalue
78.         Else
79.          Break
80.         End if
81.       End for
82.   End for
83.  For i=1 to numofchrom do (first row does not change)
84.       Select best chromosome from the population and assign to
         tabu
85.       For z=1 to reach upper bound do
86.         Increase gene value by one
87.         Calculate fitness value
88.         If fitnessvalue >= bestvalue then
89.         Set best chromosome
90.         Set best fitnessvalue
91.         Else
92.          Break
93.         End if
94.       End for
95.   End for
```

Figure 4.8 Pseudocode for local search (cont.)

*4.4.1.5 Selection*

In this study, roulette wheel selection, which is a commonly used selection method, is used. Roulette wheel selection method, which is a proportional selection method, is a probabilistic selection approach depending on the performance of the individuals. Fitness values $fitnessvalue(v_j)$ of each chromosome in the population and the total fitness value $F = \sum_{j=1}^{pop_size} fitnessvalue\ (v_j)$ for the population are calculated, where *pop_size* is the number of chromosomes in the population. Probability value $p_j = fitnessvalue\ (v_j)/F$ for each chromosome and cumulative probability value $q_j = \sum_{j=1}^{pop_size} p_j$ are found. A random number (rand) is derived from the uniform [0, 1] range and the chromosome in the range which includes that random number is selected to produce new individuals. This process is shown by an example in Table 4.5.

Roulette wheel is spin 5 times as shown in Figure 4.9 and a chromosome is selected for a new population at each time. In order to perform this calculation, five numbers are derived randomly and a new population is generated by selecting chromosomes in the range which includes these values. For instance, if our randomly selected values are {0.432, 0.657, 0.187, 0.855, 0.864}, then the new population will include {*v2*, *v3*, *v1*, *v4*, *v4*} chromosomes. Gen & Cheng (1997) discussed this procedure step by step with a detailed example in their book.

Table 4.5 Selection probabilities of individuals

| Chromosome number | Fitness values | Probability | Cumulative probability | Probability range | |
|---|---|---|---|---|---|
| $v_1$ | 10.748,52 | 0,233 | 0,233 | 0,000 | 0,232 |
| $v_2$ | 12.761,08 | 0,277 | 0,511 | 0,233 | 0,510 |
| $v_3$ | 8.814,71 | 0,191 | 0,702 | 0,511 | 0,701 |
| $v_4$ | 12.060,10 | 0,262 | 0,964 | 0,702 | 0,963 |
| $v_5$ | 1.662,06 | 0,036 | 1,000 | 0,964 | 0,999 |
| Total | 46.046,46 | 1 | | | |

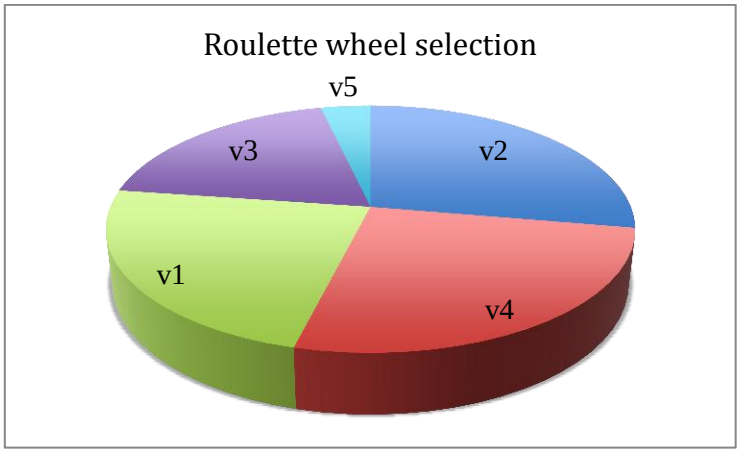



Figure 4.9 Representation of the roulette wheel selection probabilities

*4.4.1.5 Genetic Operators -Crossover*

The design of the genetic operators has an important effect on stabilizing the performance of GA. Genetic operators are grouped into two types as crossover and mutation. Crossover performs the mating process of the individuals that will produce new offsprings for the future new population. Mutation operator is the process of randomly changing the genetic value in the chromosome. In crossover, the number of new individuals obtained is equal to the size of the population and a group of current and new individuals are transferred to the new offsprings. This group consists of individuals having the best function value and it is at the size of the population. Three different crossover operators are applied on randomly selected parents thus obtaining new individuals at the size of population. Classical crossover rate is not applied and the number of new individuals that are expected to be obtained is determined. Thus, new individuals are generated at the size of the population and the individuals having the best fitness value from all obtained solutions are selected. In the proposed GA approach, since our chromosome structure is a 2x$N$ matrix, crossover operator is applied in three different ways. Information is given about the application of the proposed crossover procedure related to our example problem as follows:

1. *Changing just one row:* If the random number we have derived in [0 1] range is smaller than the first crossover type rate $P_{c,1}$ that we have determined, then a new offspring is generated by entirely changing the randomly selected row with the row of the other parent. This crossover type is given in Figure 4.10.

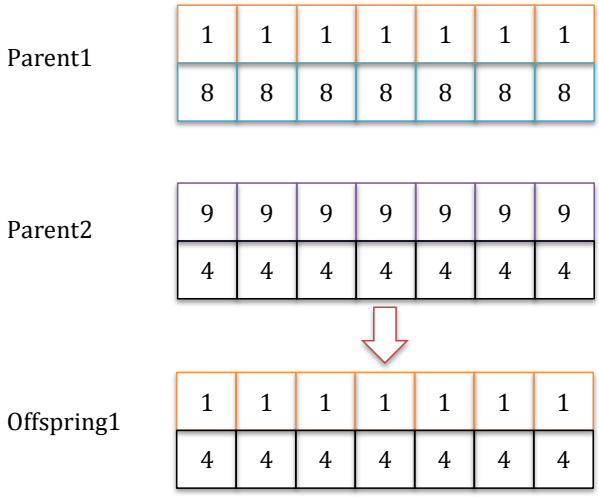



Figure 4.10 Only one row applied for crossover

2. *Changing the right or left hand side of both rows from the determined point:* If the derived random number is smaller than the determined second crossover type rate $P_{c,2}$, then both rows are replaced together. It is decided whether a crossover will be made from the right or left hand side of the column with 50% probability by selecting a random number (crosspoint) in [1 *N*-1] range. This crossover type is given in Figure 4.11.

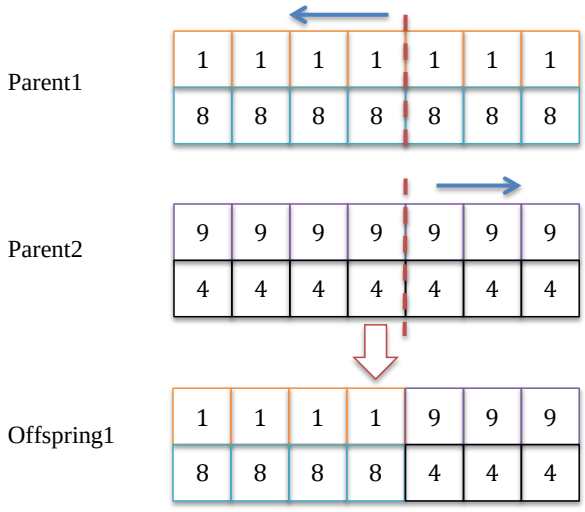



Figure 4.11 Crossover applied on the column

3. *Crossovering the right or left hand side of a row selected:* If the derived random number is smaller than the determined third crossover type rate, then a new offspring is obtained by replacing the right or left hand side of the selected row with 50% probability. This crossover type is given in Figure 4.12.

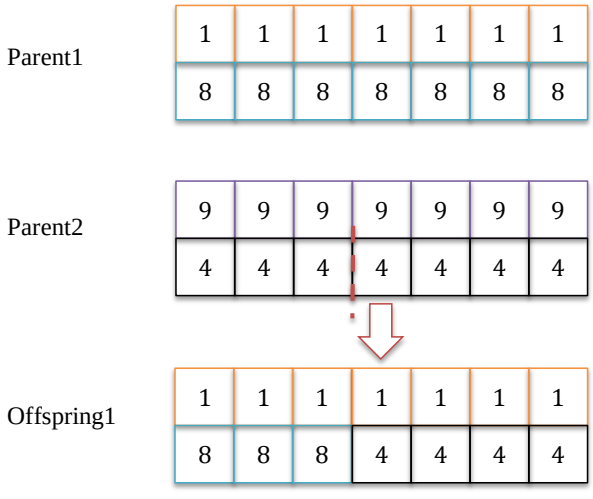



Figure 4.12 Crossover applied both row and column

The demonstration of the applied crossover operator as Pseudocode is given in Figure 4.13.

```
   Procedure for crossover operator
1.     Procedure (Crossover)
2.   For i=1 to N do
3.         Select mother and father chromosome randomly
4.         Select crosspoint randomly
5.         If random [0 1] < Pc,1 then
6.            Change only one row; 1st row or 2nd row randomly
7.            Set newpop
8.         Elseif random [0 1] < Pc,2 then
9.            If random [0 1] < 0.5 then
10.              Change right side of the chromosome from crosspoint
11.              Set newpop
12.           Else
13.              Change left side of the chromosome from crosspoint
14.              Set newpop
15.           End if
16.        Else
17.           Select only one row; 1st row or 2nd row randomly
18.           If random [0 1] < 0.5 then
19.              Change right side of the selected row from crosspoint
20.              Set newpop
21.           Else
22.              Change left side of the selected row from crosspoint
23.              Set newpop
24.           End if
25.        End if
26.   End for
```

Figure 4.13 Pseudocode for the crossover operator

*4.4.1.6 Genetic Operators -Mutation*

This operator arbitrarily selects genes to counteract premature convergence and to maintain enough diversity in the population. Mutation is employed to maintain the diversity of the entire population by changing individuals bit by bit with a small probability $P_m \in [0, 1]$. Mutation prevents the algorithm to escape from local minima trap. Crossover and mutation operators ensure that almost all children are different from their parents. The genes selected for the mutation operator applied in this study are changed randomly between the lower and upper limits. The application of mutation operator and the pseudocode information are discussed by Eiben & Smith (2007) and Goldberg (1989).

*4.4.1.7 Termination Criteria*

Generally two methods are used for terminating the genetic algorithm. In one of these methods, the algorithm can be terminated after a certain number of generations (*iter_num*) defined previously and in the other method, algorithm can be terminated when no further development can be obtained in the solution after a predefined number of successive generations. In the proposed GA approach, we used the method of terminating the algorithm when the predefined generation number is reached.

## 4.5 Parameter Settings for Efficient GA

We have presented general information about parameter optimization in Chapter 3.6. Here we have focused on applying this method on our numerical example. We have already mentioned that generation number (*iter_size*), population number (*pop_size*), crossover type rate ($P_{c,1}, P_{c,2}$) and mutation rate ($P_m$) are the most important parameters of GA. In the studies, it is seen that mostly the method of choosing the parameter optimization's parameter set which gives the best fitness value (*fitnessvalue*) is used. In recent years, statistical analysis methods such as factorial design, regression analysis and Taguchi design are started to be used for parameter optimization. In Taguchi method, orthogonal arrays are used instead of testing all the possible combinations like in the factorial design. This allows for the collection of the necessary data to determine which factors most affect product quality with a minimum amount of experimentation, thus saving time and resources. Therefore, our aim in this study is to decide on the best parameter set by using Taguchi method. Firstly, we decided on the parameter set through our small sized numerical example and then, we ran our large size problem with the same parameter set.

Taguchi method is applied to decide on the best combination of design factors and decrease signal-to-noise factors causing variation. Signal-to-noise ratio (S/N ratio) shows the current variation amount in response variables. *Signal* term shows the

mean response variable and *noise* term shows the standard deviation. The aim is to maximize S/N ratio. Maximization increases the S/N ratio but decreases the variation on the other hand. Taguchi categorizes the objective function in 3 groups: the smaller-the-better type, the larger-the-better type, and nominal-is-best type. Larger-the-better type S/N ratio is chosen since our objective function is maximizing the profit and this ratio is calculated as follows:

$$S/N = -10 * log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{(objective\ function)_i^2}\right) \tag{4.41}$$

where $n$ is the replication number, $objective\ function_i$ is the results of experiment in replication $i$.

Four design factors that are important in GA performance are defined for [3P_8T] type problem: (1) iteration size (*iter_size*), (2) population size (*pop_size*), (3) crossover type rate ($P_{c,1}, P_{c,2}$) and (4) mutation rate ($P_m$). The possible three levels are considered for each design factor as shown in Table 4.6. While $3^4$ = 81 experiments should be made for 4 factors with 3 levels, Taguchi analysis can enable us to obtain parameter optimization with just 27 experiments. When we make 81 experiments and 10 replications it means 810 experiments and it would take very long. Therefore, when we make 27 experiments with 10 replications by choosing L27 (313) orthogonal array (OA) with Taguchi method, we can perform parameter optimization with 270 experiments.

When we run experiment sets for [3P_8T] problem type, we obtain average, minimum and maximum fitness values for each experiment as shown in Table 4.7.

Table 4.6 Experimential factors with their levels

| Factors | Iteration Number | Population Size | Mutation Rate | Crossover Type Rate |
|---|---|---|---|---|
| Coding | iter_num | pop_size | $P_m$ | ($P_{c,1}$, $P_{c,2}$) |
| Level 1 | 200 | 20 | 0.05 | (0.25, 0.50) |
| Level 2 | 500 | 50 | 0.15 | (0.33, 0.66) |
| Level 3 | 1000 | 100 | 0.4 | (0.50, 0.75) |

Table 4.7 The experimental results for [3P_8T] problem

| ex.no. | parameters | min. | avg. | max. | s.d. |
|---|---|---|---|---|---|
| 1 | (200,20,0.05,0.25) | 20388.92 | 21651.27 | 22663.83 | 733.48 |
| 2 | (200,20,0.15,0.33) | 19808.20 | 22239.00 | 23242.63 | 1317.90 |
| 3 | (200,20,0.4,0.5) | 19744.89 | 21587.94 | 22739.08 | 918.99 |
| 4 | (200,50,0.05,0.33) | 21629.43 | 22792.78 | 23242.63 | 460.53 |
| 5 | (200,50,0.15,0.5) | 20180.30 | 22043.01 | 23242.63 | 1061.14 |
| 6 | (200,50,0.4,0.25) | 20262.09 | 21520.86 | 22296.95 | 768.81 |
| 7 | (200,100,0.05,0.5) | 20392.44 | 22032.12 | 23242.63 | 849.64 |
| 8 | (200,100,0.15,0.25) | 19610.18 | 21559.99 | 22286.35 | 1060.57 |
| 9 | (200,100,0.4,0.33) | 20226.77 | 21408.15 | 22296.95 | 697.29 |
| 10 | (500,20,0.05,0.25) | 20392.44 | 21949.09 | 23242.63 | 754.37 |
| 11 | (500,20,0.15,0.33) | 20962.56 | 22521.99 | 23531.93 | 757.45 |
| 12 | (500,20,0.4,0.5) | 20392.44 | 21510.23 | 22296.95 | 749.31 |
| 13 | (500,50,0.05,0.33) | 20937.96 | 22465.82 | 22953.23 | 737.88 |
| 14 | (500,50,0.15,0.5) | 20378.32 | 21920.03 | 22953.23 | 986.32 |
| 15 | (500,50,0.4,0.25) | 20950.35 | 21799.75 | 22265.35 | 398.65 |
| 16 | (500,100,0.05,0.5) | 21716.23 | 22465.50 | 22953.23 | 475.82 |
| 17 | (500,100,0.15,0.25) | 20620.22 | 21989.24 | 22672.51 | 577.23 |
| 18 | (500,100,0.4,0.33) | 21566.56 | 22033.55 | 22655.65 | 360.04 |
| 19 | (1000,20,0.05,0.25) | 20962.56 | 22362.77 | 23242.63 | 685.47 |
| 20 | (1000,20,0.15,0.33) | 20378.32 | 21843.38 | 22953.23 | 921.38 |
| 21 | (1000,20,0.4,0.5) | 20939.76 | 21854.20 | 22286.35 | 428.38 |
| 22 | (1000,50,0.05,0.33) | 20962.56 | 22432.46 | 23242.63 | 809.51 |
| 23 | (1000,50,0.15,0.5) | 20501.51 | 22256.38 | 22953.23 | 786.48 |
| 24 | (1000,50,0.4,0.25) | 20943.24 | 22206.10 | 22867.07 | 533.03 |
| 25 | (1000,100,0.05,0.5) | 20973.15 | 22308.88 | 23242.63 | 810.02 |
| 26 | (1000,100,0.15,0.25) | 21696.91 | 22065.44 | 22842.47 | 423.46 |
| 27 | (1000,100,0.4,0.33) | 22166.60 | 22370.30 | 22867.07 | 279.08 |

Statistical variance analysis (ANOVA) is performed for the problem to decide whether which control parameter is effective and the results are shown in Table 4.8. As seen in Table 4.8, factors with $p \leq 0.05$ at 95% confidence level, i.e. iteration number and mutation rate, are effective on the results. Also, it is seen that the interaction between the four factors does not cause a significant difference in the results. Moreover, as seen in Table 4.8, the most effective factor is the mutation rate having the highest value in terms of F value. We can obtain the information about which factor is more effective with Taguchi analysis as shown in Table 4.9.

Table 4.8 ANOVA table of means for [3P_8T] problem type.

| Source | DF | Seq SS | Adj SS | Adj MS | F | P |
|---|---|---|---|---|---|---|
| Main Effects | 4 | 1492494 | 1802638 | 450660 | 6.58 | 0.002 |
| iter_num | 1 | 455951 | 455951 | 455951 | 6.66 | 0.019(*) |
| pop_size | 1 | 28267 | 36512 | 36512 | 0.53 | 0.475 |
| $P_m$ | 1 | 965861 | 1183356 | 1183356 | 17.28 | 0.001(*) |
| $P_c$ | 1 | 42416 | 146676 | 146676 | 2.14 | 0.162 |
| 2-Way Interactions | 4 | 522674 | 316515 | 79129 | 1.16 | 0.365 |
| Iter_num x pop_size | 1 | 112564 | 112564 | 112564 | 1.64 | 0.217 |
| Iter_num x $P_m$ | 1 | 137754 | 137754 | 137754 | 2.01 | 0.174 |
| Pop_size x $P_m$ | 1 | 27378 | 15521 | 15521 | 0.23 | 0.640 |
| $P_m$ x $P_c$ | 1 | 244978 | 66176 | 66176 | 0.97 | 0.339 |
| 3-Way Interactions | 1 | 227173 | 227173 | 227173 | 3.32 | 0.086 |
| Pop_size x $P_m$ x $P_c$ | 1 | 227173 | 227173 | 227173 | 3.32 | 0.086 |
| Residual Error | 17 | 1164360 | 1164360 | 68492 | | |
| Total | 26 | 3406701 | | | | |

(*): Factors effective at 95% confidence level

Table 4.9 Response table for signal to noise ratios (larger is better)

| Level | Iteration *iter_num* | Population *pop_size* | Mutation $P_m$ | Crossover Type $P_{c,1}$,$P_{c,2}$ |
|---|---|---|---|---|
| 1 | 86.77 | 86.81 | 86.94 | 86.80 |
| 2 | 86.86 | 86.90 | 86.85 | 86.92 |
| 3 | 86.91 | 86.85 | 86.76 | 86.83 |
| Delta | 0.14 | 0.09 | 0.18 | 0.13 |
| Rank | 2 | 4 | 1 | 3 |

Response table shows the average of response characteristics (*S/N* ratios) for each level of all factors. Table 4.9 includes the ranks based on Delta statistics comparing the relevant sizes of the effects. Delta statistics is the difference between the greatest and the smallest average values of each factor. *S/N* ratio and mean tables have the same rank values. It is pointed out that the mutation parameter has the greatest effect on both *S/N* ratio and the average. Iteration number is the next greatest effect, and crossover and population follows it. As seen in Table 4.9, the levels with *S/N* ratio that gives the greatest value indicate the levels effective on the solution. So, the first level for the mutation, the third level for the iteration, the second level for the crossover and the second level for the population are the most effective levels. Finally, the parameter values that we should use are as follows: {$P_m$ = 0.05, *iter_num*

= 1000, $P_{c,1}$, $P_{c,2}$= (0.33,0.66) and *pop_size* = 50}. The effect of each parameter level on the average and S/N ratio of the results are shown on the graph in Figure 4.14 and Figure 4.15.

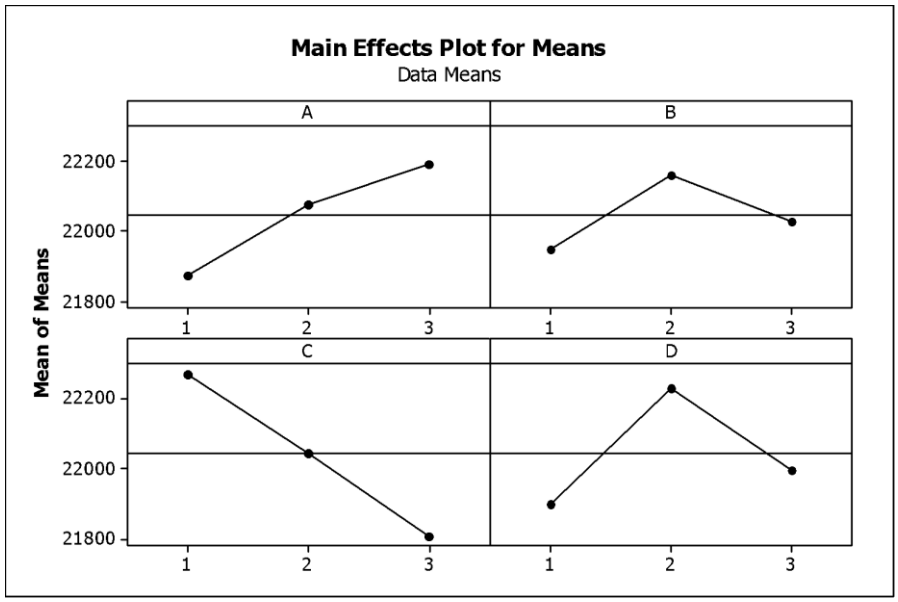



Figure 4.14 Main effects plot for means

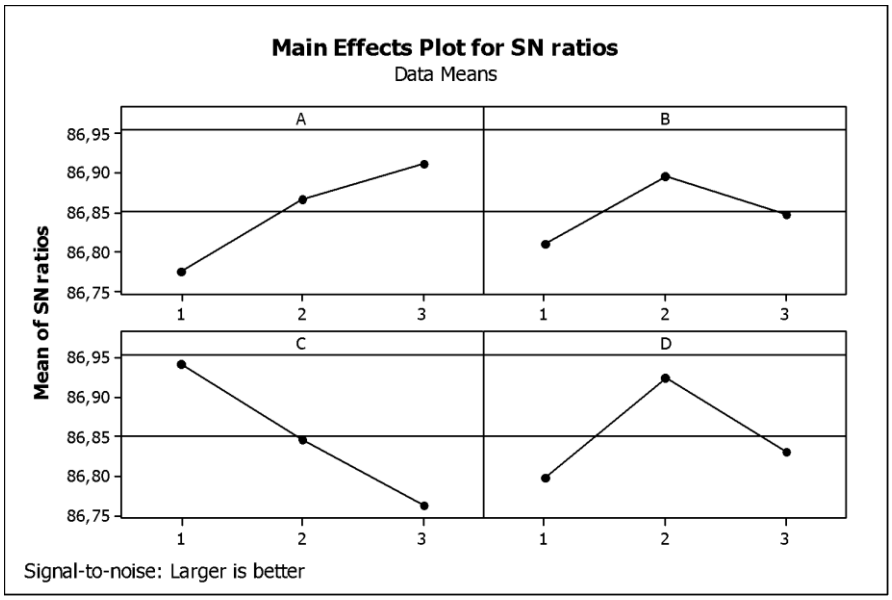



Figure 4.15 Main effects plot for S/N ratios

## 4.6 Computational Study

In this section, different test problems are solved in terms of different number of products and periods in order to evaluate the performance of the proposed GA and they are compared with the results of ILP model. ILP model is solved with LINGO 9.0 software and the execution of the model is terminated after 5 hours. The optimality of the solutions in the rows marked with an asterisk is not confirmed since LINGO could not complete its search for an optimum solution within the specified time limit. GA solutions are obtained from MATLAB 7.11. All test problems are solved on an Intel® Core ™ Duo PC with CPU P8700@ 2.53GHz and 4.00 GB RAM.

Table 4.10 shows the total profit value and CPU time (sec) results of ILP model and proposed GA. Seventh and eighth columns show the required CPU time for each approach. Fourth columns shows the optimum solution from ILP and seventh column shows the best value of GA results. Proposed GA is terminated when the iteration number reaches 1000, and 20 replications are applied in order to find the best value of the profit. The change of the solution values for [3P_8T] problem by iteration number are shown in Figure 4.16.

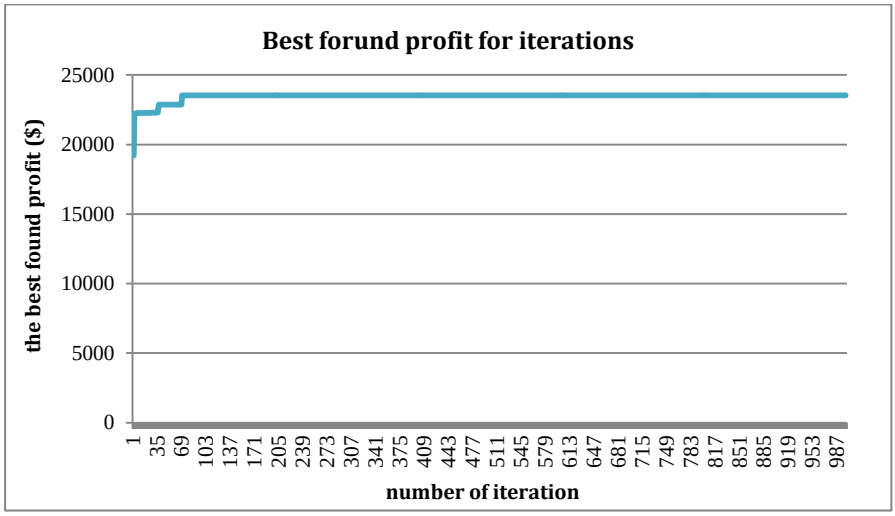



Figure 4.16 Best found profit for 1000 iteration

GAP (%) values are calculated by using Equation (4.42) for clearly comparing the results obtained with ILP model and GA approach and the computed values are shown in Table 4.10.

$$\text{GAP}\ (\%) = \frac{(\text{GA Solution} - \text{ILP Solution})}{\text{ILP Solution}} \text{ x } 100 \tag{4.42}$$

Table 4.10 Results obtained by proposed GA method and ILP

| Problem type | Product no | Period no | Total profit ($) | | | Computational time CPU (sec) | | |
|---|---|---|---|---|---|---|---|---|
| | | | ILP | Proposed GA-best | GAP (%) | ILP | Proposed GA | GAP (%) |
| 2P_4T | 2 | 4 | 9792.86 | 9624.49 | -1.72 | 5.00 | 22.80 | 356.00 |
| 2P_8T | 2 | 8 | 19510.75 | 19221.80 | -1.48 | 69.00 | 29.87 | -56.71 |
| 2P_10T | 2 | 10 | 27606.49 | 27606.49 | 0.00 | 756.00 | 32.51 | -95.70 |
| 2P_11T | 2 | 11 | 30467.76 | 30466.99 | 0.00 | 1457.00 | 35.27 | -97.58 |
| 2P_12T | 2 | 12 | 33870.36 | 33630.79 | -0.71 | 13698.00 | 35.45 | -99.74 |
| 2P_14T | 2 | 14 | 40200.92 | 40200.92 | 0.00 | 18000.00* | 39.00 | -99.78 |
| 2P_15T | 2 | 15 | 43840.68 | 43858.74 | 0.04 | 18000.00* | 40.73 | -99.77 |
| 3P_4T | 3 | 4 | 10566.50 | 10566.50 | 0.00 | 26.00 | 36.44 | 40.15 |
| 3P_8T | 3 | 8 | 23531.93 | 23531.93 | 0.00 | 17259.00 | 41.82 | -99.76 |
| 3P_10T | 3 | 10 | 29122.90 | 32004.85 | 9.90 | 18000.00* | 53.77 | -99.70 |
| 4P_4T | 4 | 4 | 10987.09 | 10385.54 | -5.48 | 54.00 | 44.45 | -17.69 |
| 4P_8T | 4 | 8 | 22621.60 | 23029.78 | 1.80 | 18000.00* | 57.43 | -99.68 |
| 5P_4T | 5 | 4 | 10984.86 | 10947.90 | -0.34 | 162.00 | 55.31 | -65.86 |
| 5P_8T | 5 | 8 | 24034.04 | 23828.11 | -0.86 | 18000.00* | 83.67 | -99.53 |
| 6P_4T | 6 | 4 | 11115.78 | 11071.98 | -0.39 | 596.00 | 68.17 | -88.56 |
| 6P_8T | 6 | 8 | 22237.50 | 23852.91 | 7.26 | 18000.00* | 98.73 | -99.45 |
| 7P_4T | 7 | 4 | 11146.60 | 11146.60 | 0.00 | 2812.00 | 83.64 | -97.03 |
| 7P_8T | 7 | 8 | 21534.83 | 23493.55 | 9.10 | 18000.00* | 111.51 | -99.38 |
| 8P_4T | 8 | 4 | 11162.11 | 11149.30 | -0.11 | 18000.00* | 104.92 | -99.42 |

(*): Interrupt after 5h the solution is may not be optimal

While an optimum solution for 2 product 12 period [2P_12T] problem can be obtained in 3 hours and 48 minutes by the ILP model, only a 0.71% worse solution is reached in 35.45 seconds by the proposed GA as shown in Figure 4.17. As the number of products and periods increases, reaching an optimum solution with ILP becomes more difficult. Problem types that we could reach the optimum solution with ILP model and the values of the (*Q*, *r*) decision variables providing the best profit that we obtained with GA are shown in Table 4.11. It is seen that the solutions obtained with GA are the same with the optimal solutions or very near to them. Feasible solutions of which the optimality is not confirmed and obtained with ILP by terminating the model execution after 5 hours for larger size problems and the values of the (*Q*, *r*) variables providing the best profit that we obtained with GA are shown in Table 4.12. This table shows that better results can be obtained with GA in much shorter computation times.

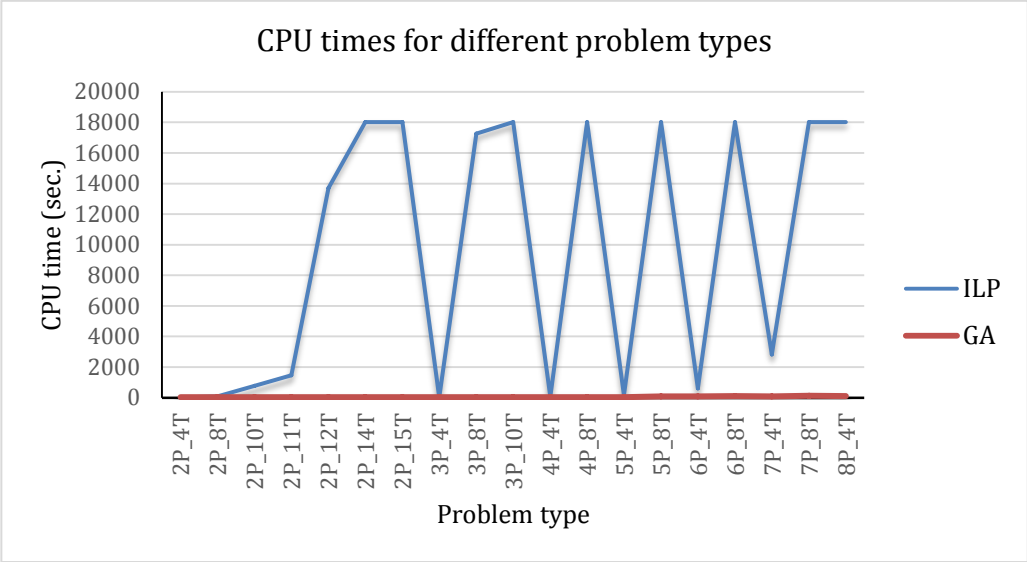



Figure 4.17 CPU times (sec.) for different problem types with ILP and GA

Table 4.11 (Q, r) values found for small size problems by ILP and GA

| Problem type | ILP | | | Proposed GA | | | %GAP |
|---|---|---|---|---|---|---|---|
| | Optimal Solution | Q | R | Best Solution | Q | R | |
| 2P_4T | 9792.86 | 49 | 120 | 9624.49 | 50 | 14 | -1.72% |
| | | 9 | 1 | | 9 | 6 | |
| 2P_8T | 19510.75 | 56 | 44 | 19221.80 | 57 | 49 | -1.48% |
| | | 20 | 51 | | 20 | 60 | |
| 2P_10T | 27606.49 | 51 | 79 | 27606.46 | 51 | 74 | 0.00% |
| | | 25 | 92 | | 25 | 92 | |
| 2P_11T | 30467.76 | 50 | 77 | 30466.99 | 50 | 80 | 0.00% |
| | | 22 | 75 | | 21 | 72 | |
| 2P_12T | 33870.36 | 100 | 79 | 33630.79 | 50 | 54 | -0.71% |
| | | 62 | 91 | | 77 | 84 | |
| 3P_4T | 10566.50 | 100 | 79 | 10566.50 | 100 | 32 | 0.00% |
| | | 10 | 30 | | 9 | 5 | |
| | | 46 | 5 | | 46 | 4 | |
| 3P_8T | 23531.93 | 56 | 44 | 23531.93 | 56 | 43 | 0.00% |
| | | 20 | 60 | | 20 | 56 | |
| | | 100 | 81 | | 100 | 70 | |
| 4P_4T | 10987.09 | 100 | 79 | 10385.54 | 50 | 10 | -5.48% |
| | | 10 | 1 | | 0 | 40 | |
| | | 46 | 5 | | 46 | 3 | |
| | | 125 | 94 | | 110 | 63 | |
| 5P_4T | 10984.86 | 100 | 24 | 10947.90 | 100 | 59 | -0.34% |
| | | 10 | 1 | | 10 | 10 | |
| | | 46 | 1 | | 46 | 5 | |
| | | 125 | 55 | | 125 | 89 | |
| | | 6 | 1 | | 6 | 2 | |
| 6P_4T | 11115.78 | 100 | 24 | 11071.98 | 100 | 63 | -0.39% |
| | | 10 | 1 | | 10 | 10 | |
| | | 46 | 5 | | 46 | 5 | |
| | | 125 | 94 | | 125 | 89 | |
| | | 6 | 1 | | 6 | 2 | |
| | | 3 | 3 | | 3 | 3 | |
| 7P_4T | 11146.60 | 100 | 79 | 11146.60 | 100 | 32 | 0.00% |
| | | 10 | 1 | | 10 | 22 | |
| | | 46 | 5 | | 46 | 5 | |
| | | 125 | 55 | | 125 | 55 | |
| | | 6 | 1 | | 6 | 2 | |
| | | 3 | 3 | | 3 | 3 | |
| | | 23 | 1 | | 23 | 2 | |

Table 4.12 (Q, r) values found for large size problems by ILP and GA

| Problem type | ILP | | | Proposed GA | | | %GAP |
|---|---|---|---|---|---|---|---|
| | Feasible Solution | Q | R | Best Solution | Q | R | |
| 2P_14T | 40200.92 | 53 | 57 | 40200.92 | 53 | 54 | 0.00% |
| | | 55 | 10 | | 54 | 75 | |
| 2P_15T | 43840.68 | 50 | 82 | 43858.74 | 50 | 79 | 0.04% |
| | | 50 | 67 | | 50 | 78 | |
| 3P_10T | 29122.90 | 51 | 70 | 32004.85 | 50 | 47 | 9.90% |
| | | 18 | 22 | | 21 | 71 | |
| | | 51 | 16 | | 10 | 68 | |
| 4P_8T | 22621.60 | 56 | 44 | 23029.78 | 58 | 53 | 1.80% |
| | | 10 | 60 | | 10 | 87 | |
| | | 50 | 12 | | 10 | 78 | |
| | | 62 | 11 | | 62 | 27 | |
| 5P_8T | 24034.04 | 56 | 44 | 23828.11 | 57 | 49 | -0.86% |
| | | 10 | 51 | | 20 | 52 | |
| | | 100 | 60 | | 10 | 61 | |
| | | 54 | 35 | | 10 | 19 | |
| | | 21 | 18 | | 11 | 16 | |
| 6P_8T | 22237.50 | 56 | 43 | 23852.91 | 56 | 43 | 7.26% |
| | | 38 | 69 | | 20 | 51 | |
| | | 100 | 81 | | 10 | 79 | |
| | | 50 | 35 | | 10 | 19 | |
| | | 11 | 12 | | 12 | 20 | |
| | | 0 | 1 | | 0 | 18 | |
| 7P_8T | 21534.83 | 58 | 54 | 23493.55 | 57 | 46 | 9.10% |
| | | 10 | 60 | | 20 | 51 | |
| | | 100 | 84 | | 10 | 79 | |
| | | 28 | 35 | | 10 | 18 | |
| | | 21 | 1 | | 21 | 1 | |
| | | 1 | 1 | | 0 | 0 | |
| | | 50 | 1 | | 34 | 4 | |
| 8P_4T | 11162.11 | 100 | 79 | 11149.30 | 10 | 59 | -0.11% |
| | | 8 | 1 | | 0 | 22 | |
| | | 46 | 5 | | 46 | 5 | |
| | | 125 | 94 | | 12 | 10 | |
| | | 6 | 1 | | 0 | 57 | |
| | | 3 | 2 | | 3 | 2 | |
| | | 23 | 1 | | 50 | 30 | |
| | | 26 | 1 | | 26 | 1 | |

## 4.7 Chapter Conclusions

In this section, we created an ILP model for a multi-product, multi-period (Q, r) inventory policy. The aim of the problem is to maximize the profit under the constraints such as storage area, budget, lifetime and promotion. We have run the model for problems with different number of products and periods. We have reached a conclusion that as the number of products and periods increase, an optimum solution cannot be reached with ILP in acceptable computational time. We have

proposed a GA approach since the case to be addressed in our study is a larger sized problem.

We have presented the parameter optimization and general information about how GA approach is used in our problem. After this phase, ILP model and GA are applied on different problem instances and their results are compared. We have showed that near optimum or optimum results can be reached in acceptable computational time for large size problems.

In the next section, the solution phases of an inventory problem of a real company are shown step by step and the solution with GA is presented.

# CHAPTER FIVE
# A CASE STUDY FOR PHARMACEUTICAL DISTRIBUTOR

## 5.1 Introduction

Until now, we have addressed the multi-product, multi-period inventory problem mathematically and proposed a GA approach for its solution. In this section, we have addressed an inventory management problem of a pharmaceutical distributor from real life and explained the solution process step by step.

In the previous chapter, we have modeled the multi-product, multi-period (Q, r) inventory policy as an ILP model. With this model, we tried to reach a mathematical solution for the products having deterministic demand. We have seen that as the number of products and periods increases, an optimum solution cannot be reached in acceptable computational time. From this point of view, we proposed a GA based solution approach. We have compared the results obtained both from the ILP model and the proposed GA and showed that optimum or near optimum solutions can be reached in shorter time by using GA.

In this chapter, the size of the inventory problem for the real life case grows much larger. It should be also noted that for the computational study of the ILP model and the proposed GA in section 4.2, we have only considered those products with deterministic demand into account and therefore, run GA for the products with deterministic demand as well. However, in this chapter, we present how GA is run to reach a solution with products that have variable demand structures. Firstly, we explain the general information about the pharmaceutical sector and then the proposed GA approach is evaluated for the real case.

## 5.2 Pharmaceutical Market

Research and development (R&D) studies in pharmaceutical industry are demanding, long and expensive. R&D process includes approximately 12-15 years of

development process from the discovery of a drug molecule to introducing this discovery to medical field, and an average of 880 million dollars investment for this molecule. 15.3% of the total R&D expenses in the world belong to the pharmaceutical industry (Ministry of Industry, 2013). Pharmaceutical industry is the sector which spends most for R&D activities.

The main aim of developing a drug is to change people's lives in a positive manner. The impact obtained after the use of each drug or vaccine developed should be protective, curative or should reduce symptoms and indications of the illness. Pharmaceutical industry needs both reference and generic drugs to increase people's quality of life. Reference drug is the first product developed and introduced to the market under the patent protection by the innovator company. Generic drugs are the products proved to have the same features with the reference drugs and therefore can have the same treatment on patients by the help of scientific studies and introduced for sale after the protection duration of the reference drugs is completed.

According to the report that Pharmaceutical Manufacturer Association of Turkey (IEIS) (2012) has received from Intercontinental Marketing Services [IMS] Health, World drug market has reached 961 billion dollars in 2012. IMS Health is a company that provides information, services and technology for the healthcare industry. 80% of the market is comprised of North American, European and Japanese markets. However now, these markets are losing their main player position that triggers the growth. The importance of other countries and their impact on growth increase as the national income grows, access to health services increases and quality rises.

By the year of 2012, Turkey has the 6th biggest market in Europe and the 15th biggest market in the world as seen in Figure 5.1. According to the data from IEIS, there are about 300 pharmaceutical companies operating in Turkey. About 30000 people are employed in pharmaceutical industry and more than 50% have higher education. In pharmaceutical industry, employment of personnel having higher education compliant with advanced technology increases and technical knowledge

level rises accordingly. There are about 5000 drugs and 23500 pharmacies in the market.

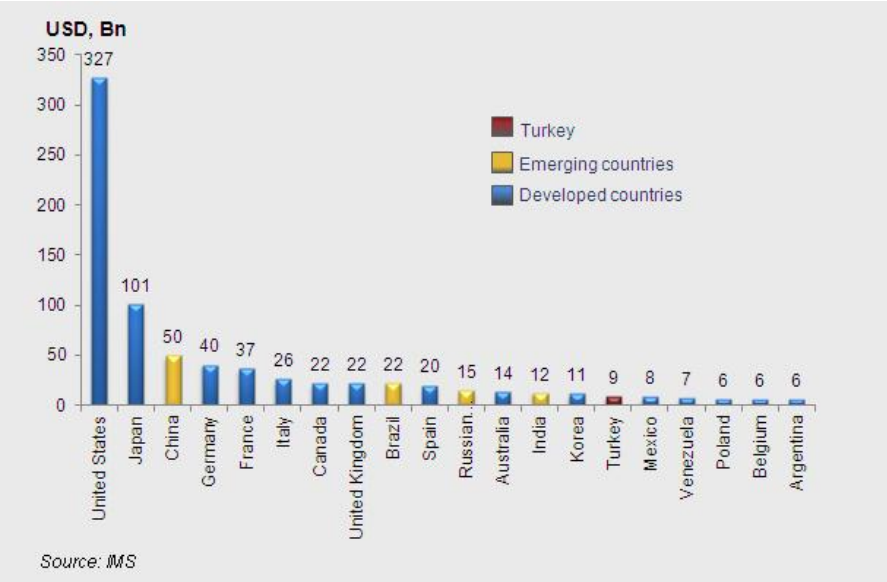



Figure 5.1 World pharmaceutical market (2012)

While Turkey prescription drug market volume was 1 billion 556 million boxes in 2011, it increased by 2.8% in 2012 and reached 1 billion 600 million boxes. In 2012, 78.1% of 1 billion 600 million boxes, in other words about 1 billion 249 million boxes are produced in Turkey. In Turkey, drug consumption per person is very low. While drug consumption per person was 123 dollars in Turkey in 2011, it decreased by 13.7% and became about 106.2 dollars in 2012. In Turkey, boxes-based drug consumption per person increased by 1.6% in 2012 with respect to the previous year and became 23.4 boxes.

### 5.3 Pharmaceutical Supply Chain

Big players in drug supply chain are drug manufacturers, wholesalers and retailers. In Turkey, operation system of the distribution channels in the drug distribution industry is similar to the structure in developed countries. Industry players are comprised of foreign-financed companies, national companies, regional companies and cooperatives in our country. Distribution activities in pharmaceutical

industry are performed in two levels as wholesale and retail. There are pharmaceutical distributors at the level of wholesale and pharmacies at the level of retail. Production and distribution industries that take part in drug supply chain are shown in Figure 5.2.




Figure 5.2 The players in pharmaceutical supply chain

Distributors/wholesalers making whole-distribution can provide service with more than one storage as central and regional in order to provide faster service. In recent years, the consolidation encountered in world pharmaceutical industry affected the drug distribution channels and the market share of the first three distributors increased over 70% in countries having heavy drug consumption rates. On the other hand, in Turkey, while the market share of the first three players was 37% in 1995, it rose rapidly and reached 83% in 2008. Currently the number of companies having remarkable market share in the market is about 10. The access level of the industry is too high due to the typical dynamics of the industry.

Distribution channels are the intermediary figures between the manufacturers and the pharmacy. Pharmaceutical distributor purchases the drugs from the manufacturers and mainly performs inventory management, storage and replenishment, and distribution of the drugs. Pharmaceutical distributors operate in two different sales branches. First one is conveying the drugs received from the manufacturers to the free pharmacies. These distributors compete in the areas of term, discount and extra quantities for pharmacies and also in other service areas. On the other hand, tendering pharmaceutical distributors participate in the tenders initiated by the hospitals and supply the products according to the conditions and durations determined in the specifications. Although it is seen that the distributors operating for free pharmacy and hospital markets are separate from each other, there are also storages operating in both areas. These pharmaceutical distributors perform drug sales for hospitals, pharmacies and other medical activities. There is a big

competition in pharmaceutical industry and this competition is also seen intensely in distribution industry According to the data from Selçuk Pharmaceutical Distributor (2014), there are approximately 300 pharmaceutical distributors operating in Turkey.

The factors determining the competition in drug distribution channels are price and service quality. While the price competition is based on the discount rates offered to the pharmacies, the service quality competition is prominently based on distribution frequency and reliability as well as efficient distribution timing and stock sufficiency.

Retail sales are generally performed by pharmacies in pharmaceutical industry. In some countries, big retail stores also operate in this area as well as the pharmacies. The region that the pharmacy is located, its relationship with the patients and rapid supply of the drugs can be counted among the most important factors affecting the sales of the pharmacies. According to the information obtained from the meetings performed, a pharmacy can work with 2 or 3 distributors. The effective factors in distributor choice are price and service quality. Distributors can offer price advantage by providing various promotions (convenience in extra quantity and payments terms) since the prices are determined by the government.

Doctors have an important place among the factors that determine the demand in pharmaceutical industry. On the contrary to the other players, doctors choose the drug substance/drug among an extensive treatment group as he/she finds most suitable for the patient's condition. The factors affecting the drug choice of doctors can be counted as patient feedbacks, congresses, articles, drug promotion seminars, etc.

Product and demand flow in pharmaceutical industry supply chain are presented in Appendix A.

## 5.4 The Pharmaceutical Distributor Case

The addressed company in this thesis is a pharmaceutical distributor performing drug distribution with a single regional warehouse. Firstly, current state analysis is presented and problem evaluation is made through meetings with company managers. The studies performed in the current state analysis are given in below articles:

1. Receiving the necessary information from the database
2. Arranging the data
3. Classifying the products
4. Determining the demand structure of the products
5. Researching the seasonality of the products
6. Finding the demand distributions of the products
7. Reviewing the current stock quantities and values
8. Finding the stock turnover rate

### *5.4.1 ABC analysis for Classification Products*

Inventory management requires thousands or millions of individual transactions each year. Inventory managers have to focus on important topics by avoiding unnecessary details in order to perform the inventory related tasks efficiently. Providing a more efficient inventory management can be possible through classifying the inventories and deciding which inventory will be counted more frequently and needs close follow-up. Since the pharmaceutical distributor stores about 5000 products and makes distribution to 900 pharmacies, the products are classified with ABC analysis that is frequently used in inventory management. The distributor distributes 7700 boxes from 800 different products to 150 pharmacies daily on average. The most intense business section of the distributor is its drug distribution operation from storage to the pharmacies.

The information like the names, barcodes, sales quantities, storage inputs, unit prices and dates of the drugs that are sold by the pharmaceutical distributor are stored in the company database. The data set obtained from the database and the drugs sold from the pharmaceutical storage are classified in terms of their unit price, amount of usage and total usage value by using MS Office Excel Program. Products that are classified in terms of total usage value are ordered from the most valuable to the least valuable. Few high value A-class products (constitutes the significant segment of the usage values), medium value B-class products and many low value C-class products are determined by this order. An example is given in Table 5.1 from A-class drugs.

Table 5.1 An ABC inventory classification example for class A

| Barcode of Drug | Annual Demand | Price ($) | Annual Volume ($) | Cumulative Annual Volume ($) | Percent of Total Volume |
|---|---|---|---|---|---|
| 8699532090027 | 10429 | 55.39 | 577662.31 | 577662.31 | 1.41% |
| 8699504070033 | 267 | 1209.79 | 323013.93 | 900676.24 | 2.20% |
| 8699693550019 | 5234 | 59.7 | 312469.80 | 1213146.04 | 2.96% |
| 8699777790140 | 1228 | 253.04 | 310733.12 | 1523879.16 | 3.72% |
| 8699786550124 | 3710 | 77.21 | 286449.10 | 1810328.26 | 4.42% |
| 8699522553266 | 3073 | 89.73 | 275740.29 | 2086068.55 | 5.10% |
| 8699676950706 | 4371 | 60.01 | 262303.71 | 2348372.26 | 5.74% |
| 8697621750029 | 2705 | 91.31 | 246993.55 | 2595365.81 | 6.34% |
| 8699504340037 | 41048 | 5.41 | 222069.68 | 2817435.49 | 6.88% |
| 8697928020085 | 23471 | 9.15 | 214759.65 | 3032195.14 | 7.41% |
| 8699532090027 | 4844 | 40.64 | 196860.16 | 3229055.30 | 7.89% |
| 8699504070033 | 3833 | 50.78 | 194639.74 | 3423695.04 | 8.37% |

ABC analysis is made through reviewing the amount of usage, unit price and total cost of 5000 drugs used in pharmaceutical distributor. Materials constituting 19.02% of the amount of usage and 80% of the total value were evaluated as class A, materials constituting 23.94% of the amount of usage and 15% of the total value were evaluated as class B, and materials constituting 57.05% of the amount of usage and 5% of the total value were evaluated as class C. Data are shown in Table 5.2 and Figure 5.3.

Table 5.2 The items into the ABC classification

| **Class** | Item number | Percentage of Items | Total Volume (*1000 dollars) | Percentage of Dollar Usage |
|---|---|---|---|---|
| **A** | 958 | %19.02 | 32737.98 | %80 |
| **B** | 1206 | %23.94 | 6137.58 | %15 |
| **C** | 2874 | %57.05 | 2047.01 | %5 |
| **Total** | 5038 | %100 | 40922.86 | %100 |

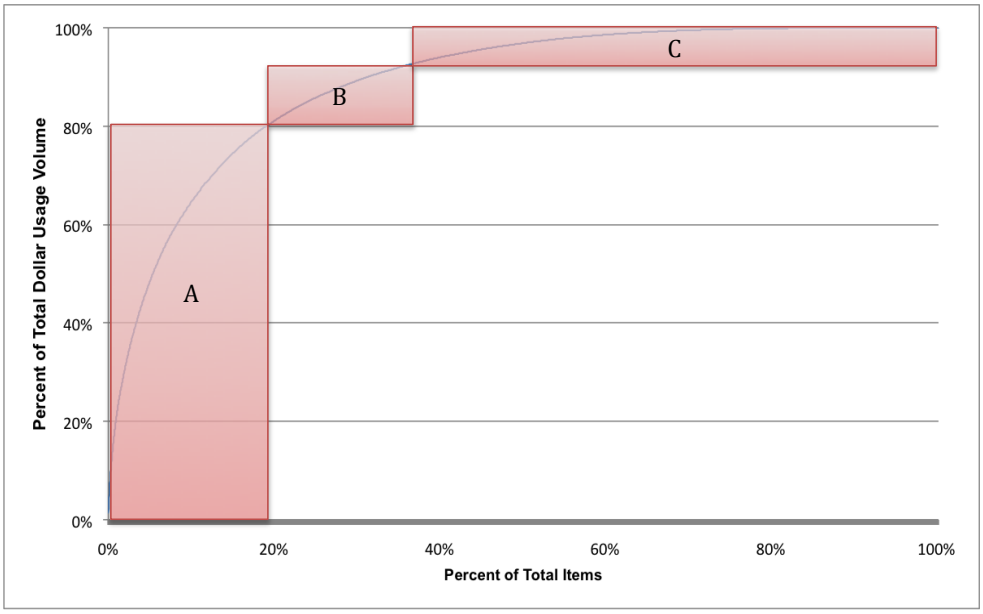



Figure 5.3 Typical ABC inventory analysis

In Tersine (1994), it is summarized how the inventory management of the products classified by ABC analysis will be handled as in Table 5.3:

Table 5.3 Comparison of A, B and C classes

| Class | Degree of Control | Type of Records | Lot Sizes | Frequency of Review | Size of Safety Stocks |
|---|---|---|---|---|---|
| A | Tight | Accurate and complete | Low | Continuous | Small |
| B | Moderate | Good | Premium | Occasional | Moderate |
| C | Loose | Simple | Large | Infrequent | Large |

### *5.4.2 Financial Inventory Performance Measures*

From the financial point of view, inventory is an asset and represents money that is tied up and cannot be used for other purposes. Holding the inventory too much causes an increase in inventory holding cost and investment of money into the inventory. On the other hand, the needs of the customers will not be met if the inventory is below the required level. The balance should be very well established. The ABC class-based average inventory values of the pharmaceutical distributor addressed in this study are summarized in Table 5.4.

Table 5.4 The current value of inventory level

| Class | Average inventory value (*1000 dollars) |
|---|---|
| A | 3505 |
| B | 1197 |
| C | 748 |
| Total | 5450 |

Inventory turnover is a measure of the number of times the average inventory investment is consumed during a year and calculated with the following formula:

$$Inventory\ turnover\ rate = \frac{annual\ cost\ of\ goods\ sold}{average\ inventory\ in\ dollars} \tag{5.1}$$

Inventory turnover rate for the pharmaceutical distributor is calculated as 5.7 and this value shows us that the inventory stays in the stock for 9 weeks on average. Ideally, inventory turns are expected to be 12. This shows that in order to reach this degree, inventory value should be brought to the level of 2608 dollars or an inventory holding which meets 4.3 weeks need is required.

### *5.4.3 Demand Type*

Since the company is newly established, the demand structure of the products is decided based on Turkey’s top selling first 100 products' quarterly sale quantities of

the recent four years taken from IMS Health company. Coefficient of variation (V) value is calculated as in Equation (2.1). The products having V values below 20% are considered as deterministic products and the ones having V values higher 20% are reviewed within the context of variable products.

A quarter-based comparison is made for the products that are determined as having variable demand structure and it is researched whether they are seasonal. At the end of these researches, nine products in total are chosen such that they represent the A, B and C classes and have deterministic, seasonal and variable demand structures. The products chosen and their drug groups are given in Table 5.5.

Table 5.5 Products selected to represent the various groups along with their descriptions

| Product name | Product description |
|---|---|
| Pro1 | Helps treat asthma |
| Pro2 | Treats many different infections caused by bacteria |
| Pro3 | Relaxes muscles and increases blood flow to particular areas of the body |
| Pro4 | Antibiotics for surgery infection |
| Pro5 | Breaks disulfide bonds in mucus and liquefies it |
| Pro6 | Inhibits gastric acid secretion |
| Pro7 | Treats chronically elevated levels of iron in the blood |
| Pro8 | Alleviates itching pain related to insect sting and simple skin irritations |
| Pro9 | Alleviates nausea |

For seven products among these nine products, V values calculated using IMS data are shown in Table 5.6. The eighth and ninth products are chosen from the company data since they are not in first 100 products of IMS. We can say that Pro1, Pro4 and Pro7 have a deterministic structure since their V values for all quarters are below 20% according to the Table 5.6. It is seen that V values of Pro2, Pro3, Pro5 and Pro6 are higher than 20% and there are differences between the quarters. Since

QTR1 and QTR4 for Pro2 have close values, it should be analyzed whether there is any difference between the quarters.

Table 5.6 Quarterly demands of the selected products

| | | Demands for products | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Year | Quarter | Prod1 | Prod2 | Prod3 | Prod4 | Prod5 | Prod6 | Prod7 (*) |
| 2009 | QRT1 | 452614 | 2689770 | 224394 | 298824 | 2690116 | 158069 | 0 |
| 2009 | QRT2 | 450903 | 2334484 | 268849 | 271505 | 2017571 | 365736 | 32 |
| 2009 | QRT3 | 399016 | 1074184 | 203148 | 221802 | 1371878 | 301147 | 34 |
| 2009 | QRT4 | 492081 | 1655217 | 230016 | 301712 | 3252905 | 601659 | 44 |
| 2010 | QRT1 | 555905 | 2268883 | 194027 | 319250 | 3987010 | 677204 | 448 |
| 2010 | QRT2 | 541630 | 1529575 | 334231 | 300522 | 1354556 | 677842 | 8511 |
| 2010 | QRT3 | 456175 | 1482003 | 195044 | 249541 | 954779 | 684421 | 9518 |
| 2010 | QRT4 | 566555 | 1205948 | 250240 | 334464 | 3293199 | 810057 | 11077 |
| 2011 | QRT1 | 660348 | 1866547 | 267909 | 377353 | 3760117 | 681763 | 11679 |
| 2011 | QRT2 | 533406 | 1238532 | 180226 | 344646 | 1451307 | 606167 | 12517 |
| 2011 | QRT3 | 468453 | 1306177 | 193773 | 246244 | 1148716 | 522081 | 13459 |
| 2011 | QRT4 | 620745 | 2241529 | 195225 | 348478 | 4068341 | 263294 | 14504 |
| 2012 | QRT1 | 623554 | 2639800 | 187649 | 396267 | 2777489 | 425769 | 14405 |
| 2012 | QRT2 | 601780 | 1489400 | 550333 | 400221 | 1305628 | 481027 | 16203 |
| 2012 | QRT3 | 550904 | 1661016 | 227628 | 288644 | 2079200 | 530708 | 16709 |
| 2012 | QRT4 | 628767 | 2412548 | 177715 | 383.002 | 1898796 | 709492 | 18457 |
| QTR1 | MEAN | 573105 | 2366250 | 218495 | 347924 | 3303683 | 485701 | 13042 |
| | STDDEV | 91233 | 382395 | 36637 | 46318 | 665485 | 249030 | 1928 |
| | **V (%)** | **16** | **16** | **17** | **13** | **20** | **51** | **15** |
| QTR2 | MEAN | 531930 | 1647998 | 333410 | 329224 | 1532266 | 532693 | 14360 |
| | STDDEV | 62023 | 475431 | 157786 | 56076 | 329151 | 137853 | 2606 |
| | **V (%)** | **12** | **29** | **47** | **17** | **21** | **26** | **18** |
| QTR3 | MEAN | 468637 | 1380845 | 204898 | 251558 | 1388643 | 509589 | 15084 |
| | STDDEV | 62637 | 250563 | 15712 | 27647 | 490902 | 157709 | 2298 |
| | **V (%)** | **13** | **18** | **8** | **11** | **35** | **31** | **15** |
| QTR4 | MEAN | 577037 | 1878811 | 213299 | 341914 | 3128310 | 596126 | 16481 |
| | STDDEV | 63018 | 553528 | 32848 | 33680 | 901494 | 237645 | 2795 |
| | **V (%)** | **11** | **29** | **15** | **10** | **29** | **40** | **17** |
| TOTAL | MEAN | 537677 | 1818476 | 242525 | 317655 | 2338226 | 531027 | 14742 |
| | STDDEV | 77722 | 536745 | 92059 | 55133 | 1072230 | 185134 | 2266 |
| | **V (%)** | **14** | **30** | **38** | **17** | **46** | **35** | **15** |

(*): The years 2009 and 2010 are considered since it is a new product.

To find out whether there is any difference for the demand of a product between the quarters, variance analysis (ANOVA) is performed. ANOVA will help us to find whether there is any significant difference between the quarters according to the hypothesis that we have established as below.

$H_0$: there is no difference between the quarters.

$H_1$: there is a significant difference between the quarters.

The ANOVA results showing whether there is any difference between the quarters for Pro2 are obtained by using Minitab 16 statistical software at 95% confidence level and these results are given in Table 5.7. ANOVA results for PRO5 are presented in Appendix B.

As it is seen in Table 5.7, there is a statistically significant difference between the quarters and $H_0$ hypothesis is rejected. It is considered which quarters have difference by using Tukey' s test and the result of this analysis is given in Table 5.8. The result shows that QTR1 and QTR3 is different than the other quarters, QTR2 and QTR4.

Table 5.7 ANOVA table for Pro2

| Source | Degrees of freedom | Sum of squares | Mean square | F-ratio | P–value |
|---|---|---|---|---|---|
| Between groups | 3 | 2.09712E+12 | 6.99040E+11 | 3.77 | 0.041 |
| Within groups | 12 | 2.22430E+12 | 1.85359E+11 | | |
| Total | 15 | 4.32143E+12 | | | |

Table 5.8 Tukey's test results for Pro2

| Quarter | No of observation | Mean | Grouping |
|---|---|---|---|
| QTR1 | 4 | 2366250 | A |
| QTR4 | 4 | 1878811 | A B |
| QTR2 | 4 | 1647998 | A B |
| QTR3 | 4 | 1380845 | B |

ABC class and demand structure of nine chosen products and whether they are original or generic are presented in Table 5.9.

Table 5.9 The characteristics of the selected products

| Products | ABC class | Product type | Demand type |
|---|---|---|---|
| Pro1 | A | Reference | Deterministic |
| Pro2 | A | Generic | Seasonal |
| Pro3 | A | Generic | Variable |
| Pro4 | B | Generic | Deterministic |
| Pro5 | B | Generic | Seasonal |
| Pro6 | B | Generic | Variable |
| Pro7 | C | Reference | Deterministic |
| Pro8 | C | Generic | Seasonal |
| Pro9 | C | Generic | Variable |

### *5.4.4 Determination of Demand Distribution*

In the previous section, we have reviewed whether the demand structure of the product is variable. In this section, we will have a distribution decision for the products having variable demand structure. We have used the demand data obtained from the company for the distribution decision. The steps below should be implemented in order to convert the obtained values into a probability density function (pdf) of a theoretical distribution.

1. Summarize the raw data in a suitable frequency histogram form and decide for the appropriate empirical pdf.
2. Use goodness-of-fit test to obtain a known theoretical pdf result from the empirical pdf.

"Arena 10.0 Input Analyzer" is used to determine the distribution which the reviewed data conform. The Chi-Square test and the Kolmogorov–Smirnov test (Banks, Carson, Nelson, & Nicol, 2001) are the most widely used tests for goodness-of-fit of a distribution to sample data. In this study, the most suitable theoretical distribution is chosen by using Chi-Square test. The Chi-Square statistics are obtained from the following formula:

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e} = chi - square \tag{5.2}$$

where

$$f_0 = \text{observed frequencies}$$
$$f_e = \text{expected frequencies}$$

Weekly demand data for Pro3 with variable demand structure obtained from the company are shown in Table 5.10.

Table 5.10 Demand data for Pro3

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 43 | 1 | 5 | 22 | 178 | 57 | 44 | 118 | 6 | 11 | 15 | 9 |
| 24 | 1513 | 1180 | 304 | 1145 | 122 | 92 | 96 | 80 | 333 | 105 | 119 | 32 |
| 1119 | 10 | 100 | 19 | 675 | 30 | 38 | 6 | 87 | 176 | 7 | 5 | 7 |
| 6 | 8 | 3 | 5 | 10 | 10 | 0 | 0 | 5 | 236 | 22 | 1 | 4 |

The difference between the minimum and maximum values of the demand data given in Table 5.10 is 1513. Although it is not a very strict rule, in order to calculate optimal bin width, 5 to 10 classes can be taken for 52 observations according to Banks (2001). Class interval is chosen as 5 in order to obtain the form of empirical distribution according to data. The histogram obtained for the determined intervals is shown in Figure 5.4. The following hypotheses are established assuming that the distribution conforms to Weibull distribution:

$H_0$= the random variable is Weibull distributed

$H_1$= the random variable is not Weibull distributed

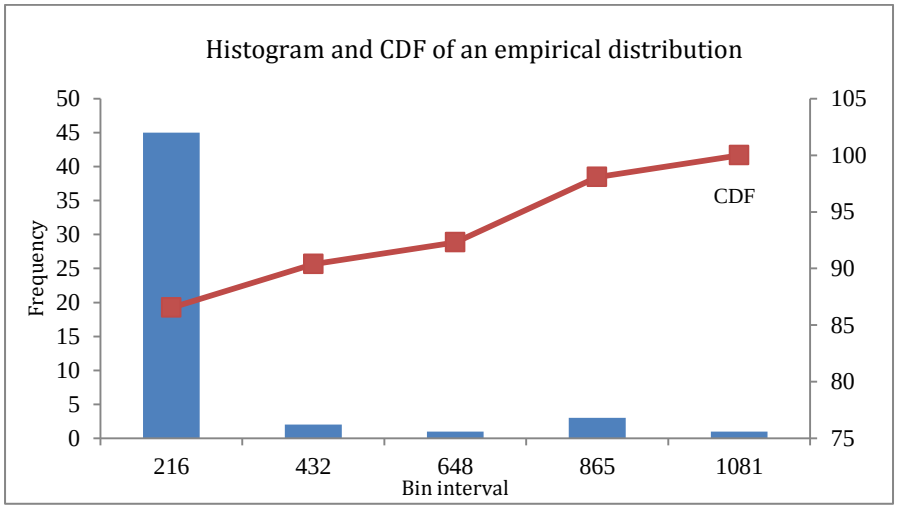



Figure 5.4 Histogram and CDF of an empirical distribution

The observed frequency values for the bin width and expected frequency values that hypothesized probability associated with Weibull distribution are shown in Table 5.11. The two parameters of the Weibull distribution are scale parameter, $\propto$, and shape parameter, $\beta$. Since the number of parameters is 2 and the number of thousand intervals is 5, Chi-square table value is found as $\chi^2_{0.05,2} = 5.99$ at a significance level of 0.05, where degree of freedom is k= 5-2-1= 2. The calculated chi-square value (5.59) is accepted as $H_0$ hypothesis since it is less than the table value. Thus, our demand data can be expressed in terms of Weibull distribution. The distribution analysis result for the other Pro6 and Pro9 is given in Appendix B. The best distribution that will best represent demand values is determined with Arena 10.0 Input analyzer and distribution summary results are shown in Table 5.12. For Pro3, it is found that the demand best fitted to Weibull distribution and distribution parameter values are calculated as (0.459, 66.5).

Table 5. 11 Chi-square-of-fit test for Pro3

| Bin interval | $f_0$ | $f_e$ | $\chi^2$ |
|---|---|---|---|
| [0, 216) | 45 | 43 | 0.09 |
| [217, 432) | 2 | 4 | 1.00 |
| [433, 648) | 1 | 2 | 0.50 |
| [649, 865) | 3 | 1 | 4.00 |
| [866, 1081) | 1 | 1 | 0.00 |
| Total | 52 | 52 | 5.59 |

Table 5.12 Distribution summary for Pro3

| Function | Sq Error |
|---|---|
| Weibull | 0.00559 |
| Exponential | 0.034 |
| Erlang | 0.034 |
| Beta | 0.0486 |
| Gamma | 0.111 |
| Lognormal | 0.147 |
| Normal | 0.406 |
| Triangular | 0.51 |
| Uniform | 0.581 |

### *5.4.5 Calculation of the Inventory Costs*

We have classified the products, determined demand structure and made product selection in the previous section. After this stage, we need to calculate the cost values required to apply the (Q, r) inventory policy for the selected products. The following costs are used in inventory management decisions.

- Item cost
- Inventory holding cost
- Ordering cost
- Stockout cost

We have presented preliminary information about inventory costs in Chapter 2. In this section, it is shown how to estimate cost values with real life values.

The sales prices of drugs and profit margins of distributors and pharmacies are determined by the government. The manufacturers, distributors and pharmacies cannot make any change on the prices. Profit margins that are determined by the government based on drug prices are given in Table 5.13. Pro2's sales price from the manufacturer to distributor is $17.26. The sales price of the same drug from the distributor to the pharmacy is 17.26x1.08= $18.64. The sales price from the pharmacy to the consumer will be 18.64x1.25= $23.30 (exclusive VAT).

Table 5.13 Drugs price strategy

| Manufacturer price | Distributor Mark-up | Pharmacy Mark-up |
|---|---|---|
| 0-10 | %9 | %25 |
| 10-50 | %8 | %25 |
| 50-100 | %7 | %25 |
| 100-200 | %4 | %16 |
| 200 and more | %2 | %12 |

Table 5.14 shows how all the inventory costs elements can be combined to make a percentage that can be used to estimate future inventory costs. This percentage helps management to evaluate the effect of a certain level of inventory on the profit. For the distributor, percentage ranges suitable for the structure of the company are selected from the percentage ranges given by Greene (1997). We would assess a charge of 20.09 cents for every dollar that we have invested in inventory during a one-year period. Inventory holding cost is decided to use of 18.51 percent because of the extra quantities that given from manufacturer to distributor as a promotion.

Table 5.14 The percentage of the cost elements

| | Range of percent (%) | | Distributor |
|---|---|---|---|
| Cost elements | Low | High | selection |
| Interest | 10 | 14 | 5.5 |
| Insurance and taxes | 3 | 5 | 4 |
| Damage and shrinkage | 0.5 | 3 | 1.5 |
| Obsolescence and shelf life | 1.5 | 4 | 3 |
| Storage and handling | 5 | 10 | 6.10 |
| Percentage of holding inventory (%) | 20 | 36 | 20.09 |
| Calculation for storage and handling for the distributor | | | |
| Warehouse rent | $170000 | Average inventory = $7875000 | |
| Personnel | 190000 | | |
| Maintenance | 60000 | | |
| Utilities | 60000 | | |
| Total | $480000 | $\frac{480000}{7875000}$ x 100 = 6.10 percent | |

Ordering cost depends on the number of orders placed in a planning period. This cost may be reduced by placing fewer orders through placing higher volume order at a time. The pharmaceutical distributor's cost of placing an order is given in Table 5.15.

Table 5.15 Ordering cost calculation

| Cost elements (Annually) | Annual value ($) |
|---|---|
| Purchasing department cost | 60000 |
| Transportation cost (*) | 0 |
| Products receipt, acceptance and control | 240000 |
| Orders placed a year | 30000 |
| Ordering cost ($cs_i$) | $10 |

(*): This cost is taken as zero since the delivery of drugs to the pharmaceutical distributor is provided by the manufacturer company.

Stockout cost is the most difficult cost type to estimate. In many cases, stockout cost includes intangible components such as loss of goodwill and potential delays in the other parts of the system. Stockout cost is a type of cost which is considered as loss of goodwill and loss of customer, which originate from the failure to meet the demand during the lead time.

The pharmaceutical distributor receives request from an average of 150 pharmacies in a day. A pharmacy can request one or more drugs from the distributor. If the pharmaceutical distributor is out of stock for one of these drugs, the pharmacy meets its need from the other distributor that it cooperates. It is not possible for the pharmacy to wait for the order due to the intense competition in the industry. The pharmacy is subject to loss when its order cannot be met for five times. Nahmias (1993) showed how Type I and Type II service levels are used in stockout cost calculation. Type I service level is the probability of not stocking out in the lead time specified as α. Type II service level is to calculate the rate, β, which measures the proportion of demands that are met from the stock. We tried to estimate the stockout cost by using Type I service level in this study. A one-year planning horizon is handled as 52 periods (weeks) in our study. The lead time of drugs from the manufacturing company is known as one week on average. If a customer is lost when it is rejected for 5 times by a pharmacy throughout the planning period, it is expected that the demand is met for 52-5= 47 times. Therefore, α is calculated as α= 47/52= 0.90. This value corresponds to the $F(r)$ value given in Equation (2.7). As a result, stockout cost is imputed as in Equation (5.3).

$$sc_i = Q_i h_i / [(1 - F(r))X_i] \quad (5.3)$$

The stockout cost is calculated as $sc_2$= (190)(3.45)/[(1-0.90)(6228)]= $1.09 for Pro2. Stockout cost can be reduced by holding extra inventory to protect against those items when the demand during lead time is greater than forecast. Extra inventory is called as safety stock in literature. Inventory manager holds a safety stock in order to protect against uncertainty in demand. If a large amount of safety stock is carried, the cost of carrying inventory will be quite high, whereas stockout cost will increase when a smaller amount of safety stock is carried. The inventory manager should find the reorder point which will balance the costs of carrying extra inventory against the cost of a stockout.

Safety stock can be obtained through multiplying a certain service factor by the standard deviation within the lead time. Safety factor points out the probability of not stocking out in the lead time, and factors for different service levels are given in Table 5.16. If the pharmaceutical distributor wants to provide service at 90% service level for Pro2, he/she must find the safety stock level through multiplying the safety factor (1.28) by the standard deviation of demand during lead time (138.35). For Pro2, the safety stock required to be hold at 90% service level is = (1.28)(138.35)= 177 units. Input data calculated for all products are summarized in Table 5.17.

Table 5.16 Safety factors for the service levels

| Service levels | Safety factors |
|---|---|
| 0.50 | 0 |
| 0.75 | 0.68 |
| 0.80 | 0.84 |
| 0.85 | 1.04 |
| 0.90 | 1.28 |
| 0.95 | 1.65 |
| 0.98 | 2.05 |
| 0.99 | 2.33 |
| 0.995 | 2.58 |
| 0.999 | 3.08 |
| 0.9999 | 3.72 |

Table 5.17 Input data for the products

| Product i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Lead Time ($l_i$) (week) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Economic Order Quantity ($EOQ_i$) | 102 | 206 | 132 | 462 | 159 | 95 | 5 | 294 | 308 |
| Purchasing Cost ($pp_i$) ($) | 59.70 | 17.26 | 55.39 | 3.40 | 8.05 | 17.30 | 1209.79 | 3.40 | 2.72 |
| Sales Price ($sp_i$) ($) | 63.88 | 18.64 | 59.27 | 3.70 | 8.80 | 18.70 | 1233.99 | 3.70 | 3.00 |
| Inventory Holding Cost ($h_i$) ($) | 0.21 | 0.06 | 0.20 | 0.01 | 0.03 | 0.06 | 4.31 | 0.01 | 0.01 |
| Ordering Cost ($cs_i$) ($) | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Shortage Cost ($sc_i$) ($) | 2.14 | 1.09 | 1.64 | 0.47 | 0.66 | 2.27 | 43.14 | 0.74 | 0.71 |
| Interest rate ($I_e$) (%) | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| Payment Term for Distributor ($M_i$) (week) | 12 | 16 | 17 | 3 | 12 | 17 | 12 | 17 | 8 |
| Payment Term for Pharmacy ($N_i$) (week) | 8 | 12 | 10 | 3 | 8 | 12 | 8 | 12 | 4 |
| Demand Type ($D_i$) | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| Shelf life ($shf_i$) (week) | 12 | 12 | 24 | 16 | 10 | 36 | 12 | 36 | 36 |
| Storage limit ($w_i$) (unit) | 0 | 0 | 0 | 0 | 1500 | 0 | 0 | 0 | 0 |
| Initial invenory ($I0_i$) (unit) | 156 | 186 | 412 | 187 | 753 | 26 | 2 | 49 | 43 |

### *5.4.6 Promotion on the Drugs*

Manufacturing companies cannot make any discount on the price for pharmaceutical distributors due to the intense competition in the pharmaceutical industry and hence may offer extra quantity or various promotions that will provide convenience for the payment period. Extra quantity can be offered especially for equivalent and generic drugs. Extra quantities provided for the selected nine drugs are shown in Table 5.18. These promotions vary for each product and manufacturer. There is also hard competition between pharmaceutical distributors. Pharmaceutical distributors can obtain customer satisfaction by providing requested products and through a good communication as well as reliability. Payment term convenience that will be provided to the pharmacy is one of the important factors in this selection. The payment terms offered by the manufacturing company to the distributor and by the distributor to the pharmacy are shown separately in $M_i$ and $N_i$ rows of Table 5.17.

### *5.4.7 Lifetime of the Drugs*

The lifetime of a drug starts once its production is complete and the expiration date is indicated on the package. The packaged drugs are firstly stored in the warehouse of manufacturing company. The imported drugs are dispatched from the company storage to big international storages in other countries. The amount of time passed during this process is unknown. Besides, domestic production drugs are

directly sold to the pharmaceutical distributor. Especially, it is observed that import drugs coming to the pharmaceutical distributor typically have a short lifetime like six months. And pharmacies do not want to buy the products that have a short lifetime left. Products with lifetime equal to or less than one year are categorized as short lifetime products. In this study, the lifetimes of the drugs are taken as ¼ of their real lifetimes thus guaranteeing that they are sold before the lifetime expires. Lifetimes are specified in Table 5.17 in $shf_i$ row where input data is shown.

FIFO rule is applied for the delivery of drugs in the storage of pharmaceutical distributor. Thus, drugs with shorter lifetime left are first issued and sold before their lifetime expires. Also it is possible to see some drugs with lifetime expired at the stocks due to the errors such as incorrectly placing the drugs on the shelves and not implementing the FIFO rule. These drugs are periodically collected by medical waste disposal companies.

Table 5.18 Extra promotional quantities offered with some products

| Pro2 | | | Pro3 | | | Pro4 | | | Pro5 | | | Pro6 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lower Limits | Upper Limits | Extra quantities | Lower Limits | Upper Limits | Extra quantities | Lower Limits | Upper Limits | Extra quantities | Lower Limits | Upper Limits | Extra quantities | Lower Limits | Upper Limits | Extra quantities |
| 10 | 49 | 0 | 0 | 4 | 0 | 0 | 4 | 0 | 0 | 4 | 0 | 0 | 4 | 0 |
| 50 | 99 | 4 | 5 | 9 | 1 | 5 | 9 | 5 | 5 | 9 | 0 | 5 | 8 | 1 |
| 100 | 149 | 5 | 10 | 19 | 3 | 10 | 49 | 15 | 10 | 49 | 5 | 9 | 15 | 2 |
| 150 | 199 | 10 | 20 | 49 | 8 | 50 | 99 | 90 | 50 | 99 | 10 | 16 | 25 | 4 |
| 200 | 249 | 15 | 50 | 249 | 30 | 100 | 249 | 240 | 100 | 249 | 100 | 26 | 62 | 6 |
| 250 | 399 | 18 | 250 | 499 | 250 | 250 | 499 | 750 | 250 | 499 | 100 | 63 | 95 | 10 |
| 400 | 599 | 39 | 500 | 999 | 500 | 500 | 749 | 1500 | 500 | and more | 200 | 96 | 160 | 24 |
| 600 | 1199 | 119 | 1000 | 1499 | 1000 | 750 | 999 | 2250 | | | | 161 | 241 | 34 |
| 1200 | and more | 500 | 1500 | 2999 | 1500 | 1000 | 1999 | 3000 | | | | 242 | 338 | 54 |
| | | | 3000 | and more | 3000 | 2000 | 2499 | 6000 | | | | 339 | and more | 78 |
| | | | | | | 2500 | and more | 7500 | | | | | | |

### *5.4.8 Problem Definition of the Pharmaceutical Distributor*

Until this section, we generated necessary data for implementing the multi-product, multi-period (Q, r) inventory policy on a pharmaceutical distributor. In this section, the applicability of our proposed ILP model on real data is tested where the ILP model aims to find the optimum (Q, r) policy. It is aimed to decide on the optimum order quantity and order point in order to maximize its profit under budget, storage area, shelf life and various promotions of the pharmaceutical distributor whose problem is being handled. In Chapter 4, we had carried out performance measurement and parameter analysis on a small size numeric example with deterministic demand structure on the real-case study.

Each drug that is sold by the pharmaceutical distributor has a different demand structure. We have explained the relevant study in Section 5.4.3. In the next section, it is explained how products with variable demand structure can be included in the genetic algorithm.

### *5.4.9 Application of Variable Demand in Genetic Algorithm*

We have selected $i$ = {1,2,…,9} products for the problem of pharmaceutical distributor. We have taken our planning horizon as one year. We have modeled the problem as multi-period where $t$ = {1,2,…,52} since the demand has a variable structure. The products Pro1, Pro4, Pro7 have a deterministic structure and we defined their demand structure as $D_i = 0$ in the algorithm. The demand structure for the products Pro2, Pro3, Pro5, Pro6, Pro8 and Pro9 varies and hence we have defined their demand structure as $D_i = 1$.

We formed a single demand set for the products with deterministic structure (Pro1, Pro4 and Pro7). We generated 10 different demand sets, $J = 10$, for products with variable demand structure (Pro2, Pro3, Pro5, Pro6, Pro8 and Pro9). Since the demand structure of Pro3, Pro6 and Pro9 conform to a certain distribution, we have generated 10 different demand types for these products conforming to the

distribution using Arena 10.0 Input Analyzer. Demand sets are generated for Pro3, Pro6 and Pro9 by using distribution of Weibull (0.459, 66.5), Beta (0.702, 1.51, 0, 87) and Normal (42.1, 30.2), respectively. Since Pro2, Pro5 and Pro8 are seasonal products, varying periods are evaluated separately; we have generated 10 different demand sets according to normal distribution that conforms to the average and standard deviation for each period. Demand data calculated for all products are summarized in Appendix B.

The [3P_8T] problem type given in Chapter 4 was solved with the assumption that all the three products have deterministic demand. In this section, products with deterministic, variable and seasonal demand structure are solved in a single model. Inventory level calculation procedure for products with deterministic demand structure, $D_i = 0$, were shown in Figure 4.5. Inventory level calculation procedure for products with variable and seasonal demand structure, $D_i = 1$, is shown in Figure 5.5.

As seen in Figure 5.5, inventory level calculation procedure for products with variable demand structure works different from deterministic structure. For each product *i*, every demand set *j* is defined as the expected demand $\varepsilon_{i,j,t}$ in each period. Fitness value calculation procedure for products with variable and seasonal structure is shown in Figure 5.5. If $D_i = 0$, there is only one demand set and the calculation will be performed with the procedure given in Figure 4.5, and set *j*=1. If $D_i = 1$, *J* number ($j = 1,\ldots,J$) of demand sets are generated and fitness value for each demand set will be found according to Figure 5.6. Fitness values for all products are calculated as shown in Figure 5.7.

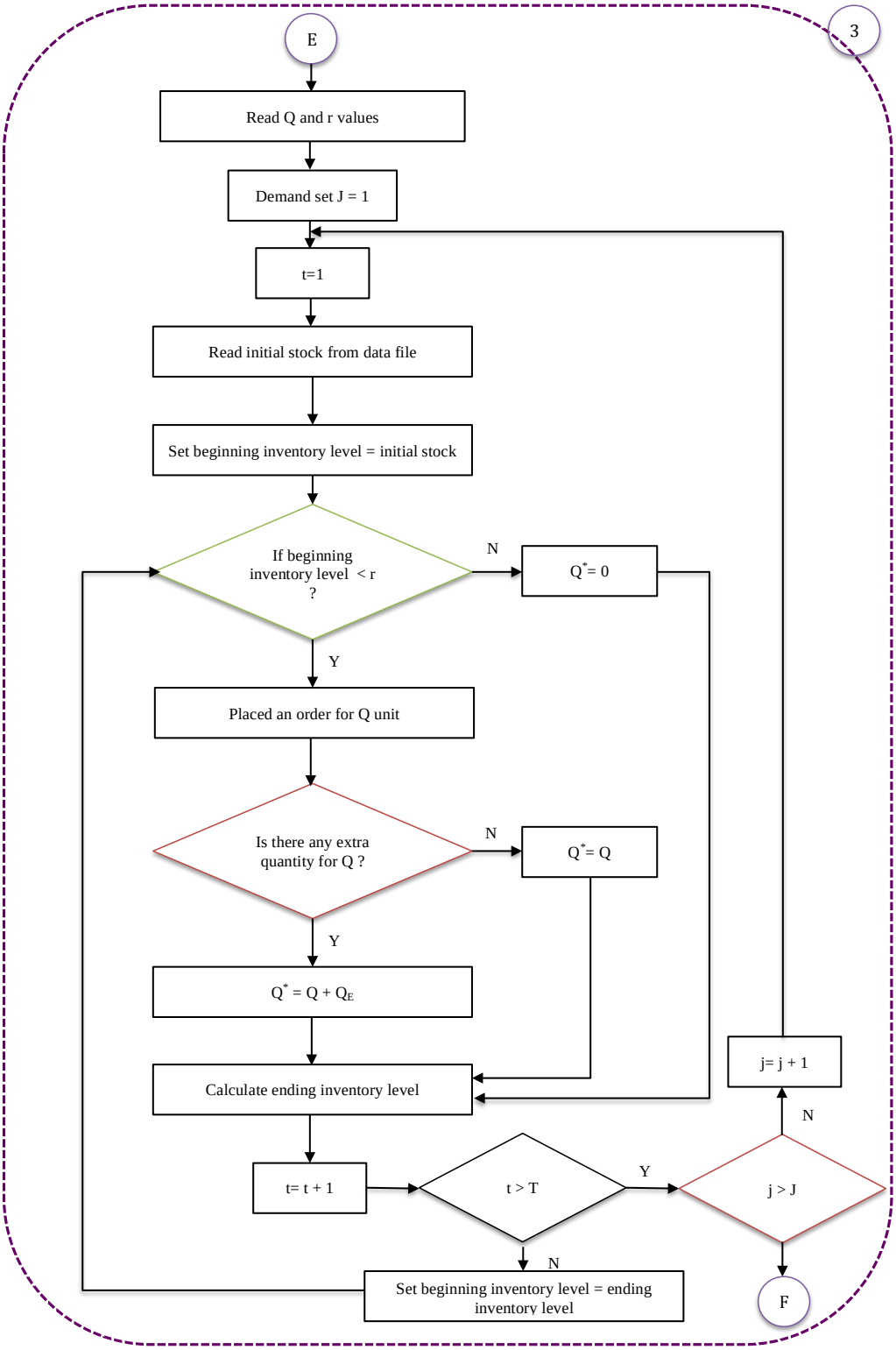



Figure 5.5 Calculate inventory matrix for variable demand

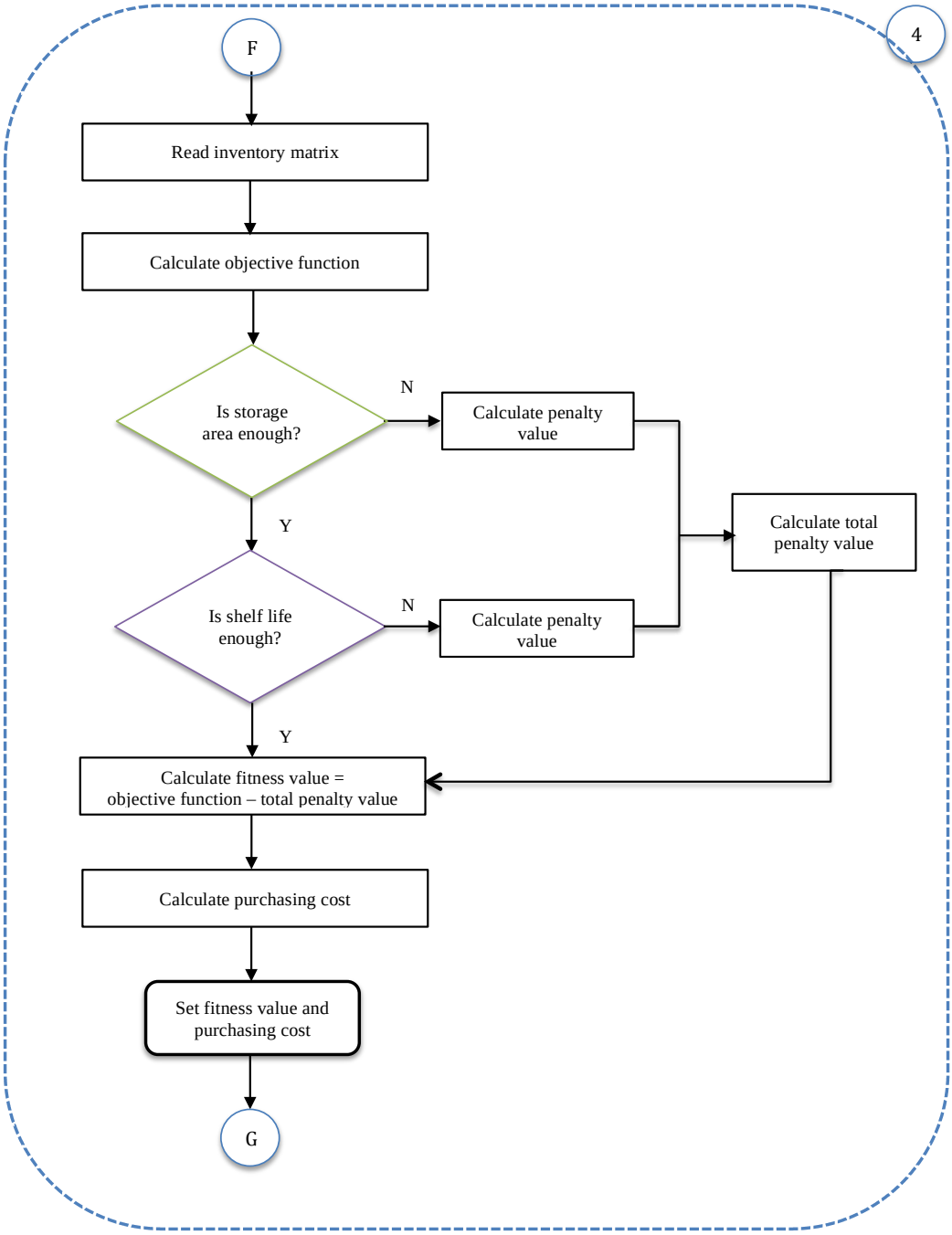



Figure 5.6 Calculate fitness value for one demand set

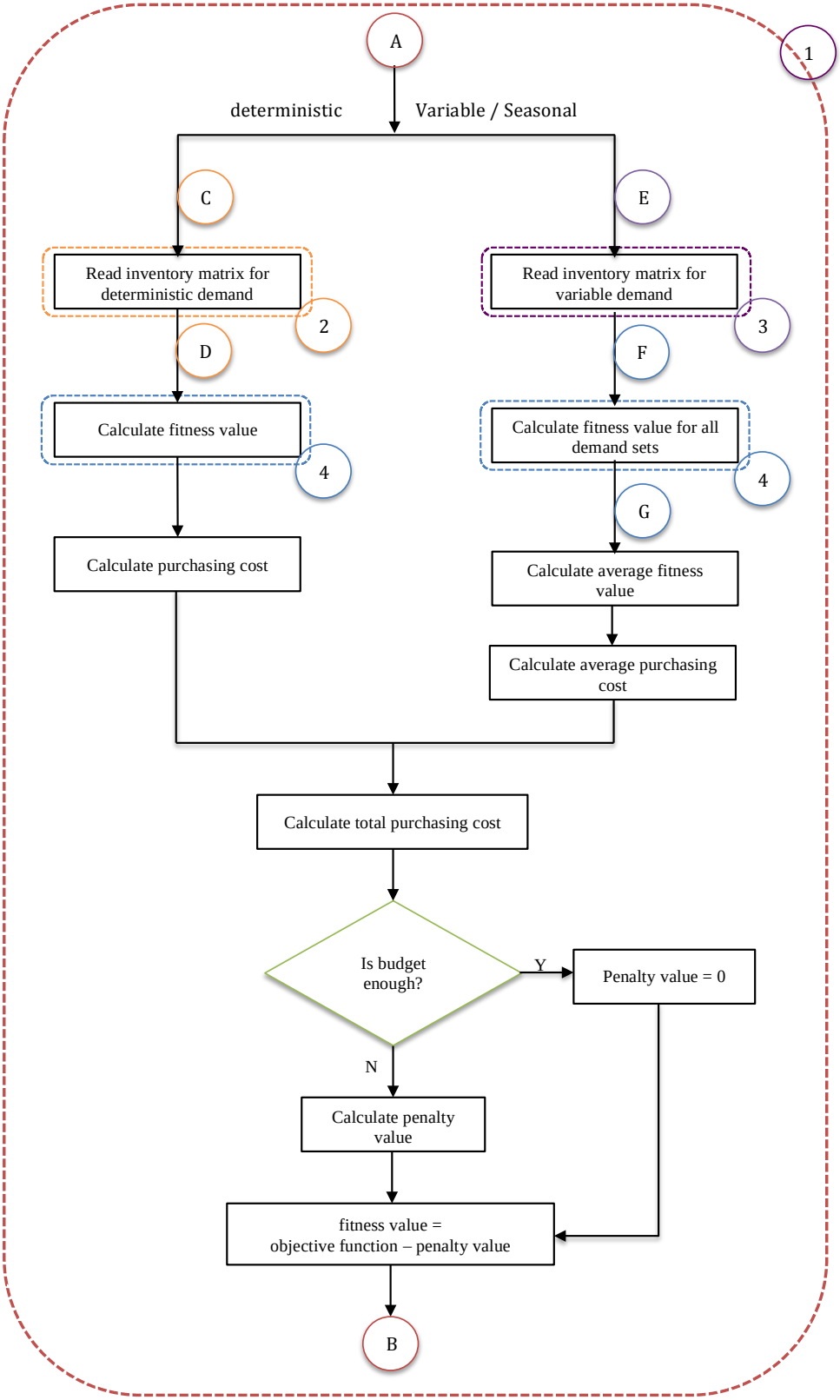



Figure 5.7 Calculate fitness value for all products

For (Q, r) variables that are derived within randomly determined lower and upper bounds, the fitness value for each demand set, ETPRj, is calculated. The average expected best profit value ($ETAPR_i$) is calculated with the (Q, r) values which are selected by taking the average of fitness values for each demand set according to the below formula:

$$ETAPR_i = \frac{\sum_{j=1}^{J} ETPR_j}{J} \qquad i = 1, \dots, N \qquad (5.4)$$

For instance, the fitness values that will be obtained for each demand set against selected ($Q_3$, $r_3$) = (250, 540) values for Pro3 are summarized in Table 5.19.

Table 5.19 Example for calculation of the ETPR

| Period (t) | $\varepsilon_{3,1}$ | $\varepsilon_{3,2}$ | … | $\varepsilon_{3,10}$ |
|---|---|---|---|---|
| 1 | 0 | 19 | … | 20 |
| 2 | 43 | 16 | … | 0 |
| . | . | . | | . |
| . | . | . | | . |
| . | . | . | | . |
| 52 | 4 | 253 | … | 5 |
| $ETPR_j$ | \$162150.36 | \$163218.56 | | \$180058.71 |

$\sum_{j=1}^{10} ETPR_j =$ \$1662800.17 , $EATPR_3$= 1662800.17/10 = \$166280.02

The pseudocode for calculating fitness value for variable or seasonal demand is shown as in Figure 5.8.

```
Calculate fitness value for variable or seasonal demand
    1.    For j=1 to numofproduct (number of product) do
    2.      Read Q and R for variable demand
    3.        For s= 1 to demandset do
    4.          Calculate inventory matrix
    5.          Calculate objective function
    6.          Evaluate constraints if constraints is not
                feasible then calculate penalty function
    7.          Fitnessvalue = objective function - penalty
                value
    8.        End for
    9.          Calculate average fitness value by dividing the
                number of demand sets
    10.         Calculate average purchasing cost
    11.         Calculate total purchasing cost for all
                numofproduct
    12.         Evaluate budget constraint if constraints is
                not feasible then calculate penalty function
    13.         fitnessvalue = fitness value - penalty value
    14.       Select and set bestfitnessvalue= max(fitnessvalue)
    15.       Set bestchrom that gives the maximum fitnessvalue
    16.   End for
```

Figure 5.8 Pseudocode to calculate fitness value for variable or seasonal demand

### *5.4.10 Computational Results*

We have performed parameter analysis by using small size problem data in Chapter 4. For our [9P_52T] problem type, we have used the parameter values same as the best parameter values selected after parameter analysis: *iter_size* = 1000, *pop_size* = 50, $P_m$ = 0.05 and $P_{c,1}$, $P_{c,2}$ = (0.33,0.66). We have prepared all the necessary data to solve our problem with GA. We must decide on the lower and upper bound values for our (Q, r) decision variables. Our chromosome structure represents (Q, r) decision variables and consists of 2x9 matrix. For instance, if we assume that the genes in the chromosome are $EOQ_i$ and the $r_i$ values that shows the mean demand during the lead time and there is no safety stock, our chromosome data will be as follows:

$$\begin{matrix} Q_i \\ r_i \end{matrix} = \begin{bmatrix} 93 & 190 & 122 & 428 & 302 & 88 & 5 & 271 & 284 \\ 100 & 120 & 158 & 120 & 241 & 26 & 5 & 48 & 42 \end{bmatrix}$$

Upper bound value ($upper_Q_i$) for $Q_i$ decision variable is four times the calculated $EOQ_i$ value; lower bound value ($lower_Q_i$) is taken as zero. Upper bound value ($upper_r_i$) of $r_i$ decision variable is thought to be as much as the demand that will occur during the lead time and four times this value is taken; lower bound value ($lower_r_i$) is selected as zero. In the solution with GA approach, the solution space is kept large by selecting the boundary values from a wide range. In the other hand, there would be extra quantities for some products, so upper bound for these products increased according to their extra quantities in given Table 5.18. Thus, Pro2, Pro3, Pro4, Pro5 and Pro6 upper bound value for $Q_i$ decision variable are increased to 5000 units. In the later sections, lower and upper bound levels are tested for $r$ variable according to different service levels. Therefore, lower and upper bound values are taken as in Table 5.20.

Table 5.20 The lower and upper bounds for (Q, r) decision variables

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 |
|---|---|---|---|---|---|---|---|---|---|
| lower_$Q_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| upper_$Q_i$ | 372 | 760 | 488 | 1712 | 1208 | 352 | 20 | 1084 | 1136 |
| lower_$r_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| upper_$r_i$ | 400 | 480 | 632 | 480 | 964 | 104 | 20 | 192 | 168 |

The pharmaceutical distributor has a limited budget from the bank which is called letter of credit. Its budget is \$1200000 for the selected 9 products. Some products require being stored under cold storage conditions. Pro5 is a drug that should be handled within this context. The storage capacity for this drug is taken as 1500 boxes per period. Since the storage cost is higher for the products that should be stored at cold storage conditions, inventory holding cost percentage is taken 10% higher. The lifetimes of selected products are given in Table 5.17 and their extra quantities are given in Table 5.18.

With \$1200000 budget and all other constraints, the total profit that the pharmaceutical distributor will obtain at the end of the planning period is estimated as \$334642.19. This profit level can be reached by giving order at $Q_i^*$ = {241, 600, 500, 121, 100, 96, 6, 133, 207} order quantities and $r_i^*$= {101, 203, 392, 182, 237,

30, 5, 59, 73} inventory levels. 10 independent replications are applied for the proposed GA approach and the details of the best result obtained are presented in Table 5.21.

The results are obtained at 90% service level and with inventory holding, ordering and stockout cost data as given in Table 5.17. The computational time for an average replication is 2737.07 seconds.

Table 5.21 Results for the [9P_52T] problem type

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 241 | 600 | 500 | 121 | 100 | 96 | 6 | 133 | 207 | |
| $r^*$ | 101 | 203 | 392 | 182 | 237 | 30 | 5 | 59 | 73 | |
| Expected Total Sales Revenue (ETSR) ($) | 332170.8 | 127178.72 | 383785.40 | 23125.44 | 134714.72 | 23087.81 | 320834.80 | 8230.65 | 6919.84 | 1360048.20 |
| Total Earning from Payment Term TPFS ($) | 10514.53 | 4564.92 | 5541.76 | 60.84 | 668.43 | 809.14 | 13577.47 | 360.89 | 150.21 | 36248.23 |
| Total Purchasing Cost (TPC) ($) | 302141.70 | 98382.00 | 191095.5 | 6993.80 | 57624.00 | 17438.40 | 312125.82 | 7777.84 | 6474.96 | 1000054.02 |
| Total Losing from Payment Term (TLPFC) ($) | 7706.36 | 4425.81 | 11129.77 | 201.19 | 1562.69 | 803.45 | 13956.31 | 286.42 | 80.27 | 40152.30 |
| Total Holding Cost (THC) ($) | 1309.56 | 1517.20 | 8155.28 | 100.63 | 497.00 | 216.43 | 568.92 | 43.39 | 68.19 | 12476.62 |
| Total Ordering Cost (TOC) ($) | 210 | 95 | 69 | 170 | 420 | 105 | 430 | 172 | 115 | 1786.00 |
| Expected Total Shortage Cost (ETSC) ($) | 0 | 389.57 | 3089.64 | 0 | 3500.53 | 53.13 | 0 | 151.18 | 1.224 | 7185.28 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 0 | 282.3 | 796.3 | 0 | 3182.3 | 38.5 | 0 | 487.7 | 5.1 | 4792.2 |
| Expected Total Profit (ETPR) ($) | 31317.70 | 26934.04 | 175787.97 | 15720.66 | 71778.93 | 5280.54 | 7331.21 | 160.69 | 330.41 | 334642.19 |

### *5.4.11 Sensitivity Analysis*

In this section, because inventory holding cost, ordering cost, and stockout cost are difficult to decide, sensitivity analysis is performed in order to see the effect of these costs on the profit value. The sensitivity analysis is shown by increasing or decreasing cost parameters, taking one at a time and keeping the others at their own values. In each case, the best value of optimal profit is found.

In accordance with actual company values, the inventory holding cost was taken as 18.51% of one unit purchasing cost. As seen in Table 5.10, we have made estimates for some items in order to calculate the percentage of inventory holding. For this reason, we researched how our expected maximum profit value will change if there is for a -/+ 40% change in the inventory holding cost. The results are summarized in Table 5.22.

Table 5.22 Percentage change in holding cost versus percentage change in profit

| % change on holding rate | Holding Cost ($) | Ordering Cost ($) | Shortage Cost ($) | Best Found Profit ($) | % change on profit |
|---|---|---|---|---|---|
| -40 | 7549.46 | 1786.00 | 7173.84 | 339573.78 | 1.47% |
| -20 | 10127.40 | 1786.00 | 7185.29 | 336991.42 | 0.70% |
| 0 | 12476.63 | 1786.00 | 7185.29 | 334642.19 | 0.00% |
| +20 | 13006,12 | 1968,00 | 8979,00 | 331154,37 | -1.04% |
| +40 | 16314,52 | 1598,00 | 8837,60 | 320028,78 | -4,37% |

As seen in Table 5.22, there is an inverse relationsip between the inventory holding cost and its effect on the profit. For example, if the inventory holding cost is decreased by 40%, then it is observed that the expected maximum profit value increases by 1.47%. If the inventory holding cost is increased by 40%, then the expected maximum profit value is observed to decrease by 4.37%. The effect of inventory holding cost change on the expected maximum profit value is depicted Figure 5.9.

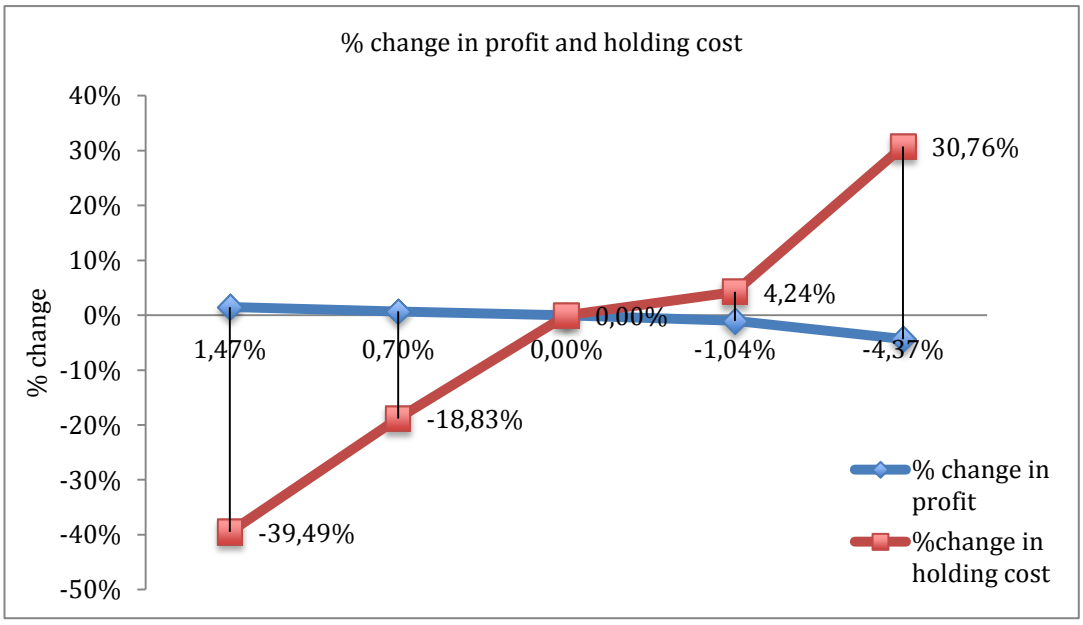



Figure 5.9 Profit versus inventory holding cost % change

We have estimated ordering cost at $10 per order. When the ordering cost is 5, 10, 15 and 20 dollars, the possible change in our expected maximum profit value is given in Table 5.23. According to this table, it is observed that there is an inverse relationship between the ordering cost and the expected maximum profit. Also, it is seen that the change in ordering cost has a very low effect on the profit. For instance, when the ordering cost decreases by 50%, the change in the expected maximum profit value is only 0.69%. And when the ordering cost increases by 50%, there is a 0.52% decrease in the profit value.

Table 5.23 Percentage change in ordering cost versus percentage change in profit

| Ordering cost ($) | Holding Cost ($) | Ordering Cost ($) | Shortage Cost ($) | Best Found Profit ($) | % Change on Profit |
|---|---|---|---|---|---|
| 20 | 12940.48 | 4038.00 | 7183.39 | 332301.35 | -0.70% |
| 15 | 11994.87 | 2433.00 | 7468.94 | 332894.92 | -0.52% |
| 10 | 12476.63 | 1786.00 | 7185.29 | 334642.19 | 0.00% |
| 5 | 12308.20 | 1022.50 | 7295.95 | 336965.81 | 0.69% |

The change between the ordering cost and the expected maximum profit value is shown in Figure 5.10.

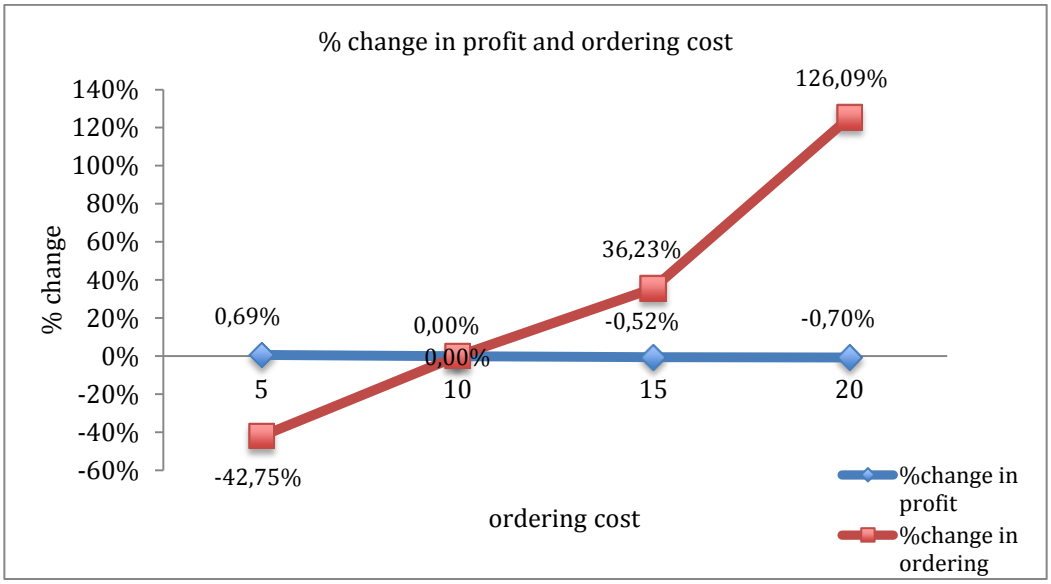



Figure 5.10 Profit versus ordering cost % change

In accordance with actual company values, we assumed stockout cost as a loss from the profit. The obtained stockout cost corresponds to the 90% customer service level (CSL). Table 5.24 shows the effect of stockout cost on the expected maximum profit value when CSL is 99.5%, 99% and 80%. If CSL is chosen as 99%, the profit value will decrease by 12.16%, whereas if CSL is chosen as 99.5%, the profit value will decrease by 20.07%. Thus, we can assert that CSL has an important effect on the profit value. Inventory holding cost increases as the service level increases. The effect of shortage cost change on the expected maximum profit value is shown in Figure 5.11.

Table 5.24 Percentage change in shortage cost versus percentage in profit

| Shortage cost (*) | Holding Cost ($) | Ordering Cost ($) | Shortage Cost ($) | Best Found Profit ($) | % Change on Profit |
|---|---|---|---|---|---|
| 99,50% | 15165.14 | 1815.00 | 59192.25 | 267476.85 | -20.07% |
| 99,00% | 15904.25 | 2094.00 | 27363.84 | 293949.56 | -12.16% |
| Loss of profit | 12476.63 | 1786.00 | 7185.29 | 334642.19 | 0.00% |
| 80% | 14286.22 | 1858.00 | 1863.39 | 334741.71 | 0.03% |

(*): Shortage cost imputed according to the CSL

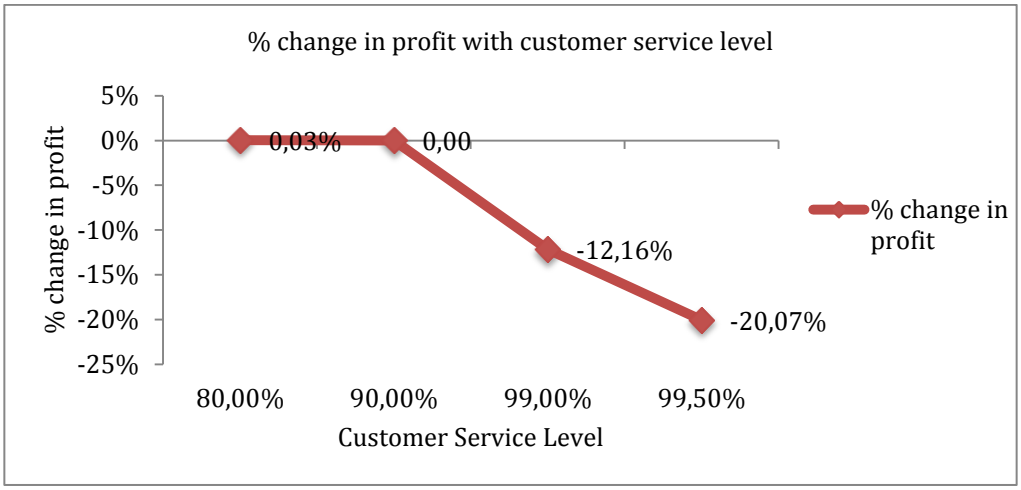



Figure 5.11 The percentage change in profit for corresponding customer service level

### *5.4.12 Different Scenarios for the Company*

In the previous section, we have run our inventory problem for a pharmaceutical distributor with $1200000 budget and under the constraints of storage area, shelf life and various discount constraints. In the solution of inventory problems, it is required to find the best scenario and decide on a policy that is suitable for the company structure. In this section, we applied different scenarios on our problem and summarized the scenario results in Table 5.25 to help inventory manager and company directors to make decision.

In Scenario A, when our budget is $800000 our profit becomes $316986.69 with storage area and shelf life constraints, and various discounts. However, stockout quantity increases when the budget decreases and the expected CSL will become 80%. CSL being 80% means that a distributor cannot meet a pharmacy's demand for about 10 times throughout the planning period. This result will cause the distributor to lose some of its customers.

In Scenario B, it is assumed that there no storage or shelf life constraint and the the budget is $800000. In this case, the expected maximum profit value would be $317995.26. There is not an important difference between Scenario A and Scenario B compared to the profit value of $334642.19 found for the original problem. In

summary, decrease of budget and absence constraints do not have much effect on the result.

Table 5.25 The results of different scenarios

| Scenarios | Budget | Constraint | Holding Cost ($) | Ordering Cost ($) | Shortage Cost ($) | Best Found Profit ($) | % Change on Profit |
|---|---|---|---|---|---|---|---|
| A | 800000 | with constraints | 8955.99 | 3072.00 | 13536.19 | 316986.69 | -5,28% |
| B | 800000 | no constraints | 15990.24 | 2074.00 | 12767.61 | 317995.26 | -4,97% |
| C | 1200000 | with constraints | 12476.63 | 1786.00 | 7185.29 | 334642.19 | 0.00% |
| D | 1200000 | no constraints | 20439.58 | 1805.00 | 4544.77 | 347511.53 | 3,85% |
| E | 1200000 | with constraints &deterministic | 27877.45 | 1830.00 | 1677.90 | 369194.50 | 10.33% |
| F | 1200000 | %95 CSL with constraint | 15488.67 | 2689.00 | 3770.34 | 317749.89 | -5.05% |
| G | 1200000 | %99 CSL with constraints | 22746.72 | 2243.00 | 2764.61 | 312368.89 | -6.66% |
| H | 1200000 | %99.99 CSL with constraint | 23161.99 | 2460.00 | 1596.64 | 278757.41 | -12.27% |
| I | 3000000 | no constraint & no extra quantity | 7610.65 | 2757.00 | 17715.90 | 76774.88 | -77.06% |

Scenario C shows the scenario studied with real company data. With $1200000 budget and constraints, our maximum profit value is expected to be $334642.19.

In Scenario D, if storage area and shelf life constraints were ignored with $1200000 budget, the profit value would be $347511.53. Here, it is seen that constraints have 3.85% effect on the profit when the budget is sufficient. In this scenario, a 93% CSL level can be obtained.

According to scenario E, if all the products in this problem were quantified as the deterministic demand rate not seasonal or variable, the profit would be $369194.5 under the constraints. In systems where demand is uncertain, the expected profit level can decrease around 16.5% as a result of more holding inventory and/or experiencing stockout due to uncertainty.

In Scenario F, if a 95% service level service is wanted to be provided with $1200000 budget, our expected maximum profit value becomes $317749.89. The lower bound value for reorder point decision value consists of safety stock (SS)

values that will ensure a service level of 95%. For instance, for Pro2, the standard deviation in lead time is 138.35 and the safety factor value corresponding to 95% CSL is 1.28; as a result, SS is calculated as SS= (1.28)(138.35)= 177. The determined lower and upper bound values of 95% service level are given in Appendix D.

In Scenario G, if a 99% service level is aimed to reach with $1200000 budget, our expected maximum profit value becomes $312368.89. In Scenario H, if a 99.99% service level is aimed to reach with the same budget, our expected maximum profit value becomes $278757.41.

In Scenario I, if our budget was $3000000, the storage area and shelf life constraints were ignored and no extra quantity was offered, the profit value of the distributor would be expected to be $76774.88. The extra quantities offered by manufacturing companies have a quite high effect (77.06%) on profitability since drug prices and profit margins are determined by the government and profit margins are low. It is seen that there is around 4% increase or decrease in the profitability value of Scenario C and the other A, B and D Scenarios. Nevertheless, it is observed that Scenario I is totally different. Thus, the effect of extra quantities on profit can be clearly seen. The expected change in profit values in scenarios is presented in Figure 5.12. Scenario results are shown in Appendix D.

The expected stockout levels in the result of applied scenarios are given in Figure 5.13. In the result of restricted budget, customer demand cannot be met at the desired level. If it is wanted to increase the percentage of meeting the customer demand, this means that it is required to increase the stock of drugs selected in the analysis and consequently to increase the budget.

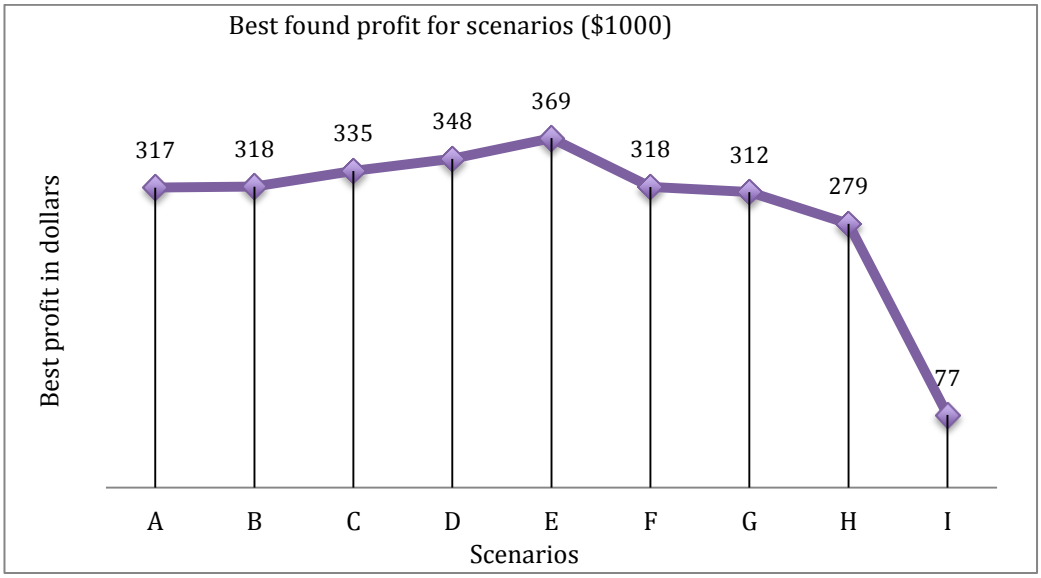



Figure 5.12 Best found profit for scenarios ($1000)

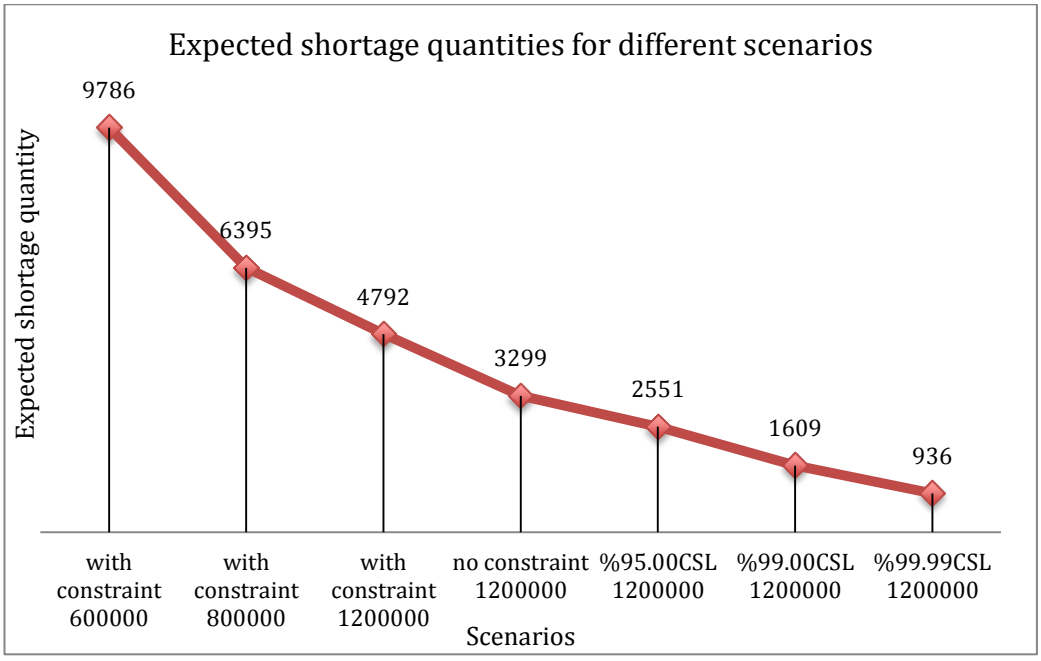



Figure 5.13 Expected shortage quantities for different scenarios

## 5.5 Chapter Conclusions

In this chapter, we have solved a real life pharmaceutical distributor inventory problem. We have measured the performance of our model by applying the multi-item multi-period (Q, r) inventory policy that we modeled in Chapter 3 on this company's problem. Since the inventory holding cost, ordering cost and stockout

cost, which affect the total inventory cost, are calculated with some assumptions, we have studied the effect of costs on the results of our model by performing a sensitivity analysis.

In the current situation, we aimed to use the budget correctly ensuring that order is given at the required amount when required by checking stocks continuously in the drug distribution industry which has a very dynamic structure. The aim is to maximize profit and when doing this, we ensured that shelf lives and storage constraints are taken into account. In addition, we also tried to get the most out of extra quantity promotions that are provided by manufacturing companies from time to time. It is observed that extra quantities are quite effective on profitability. Also, it is seen that budget is another important factor affecting profitability.

In conclusion, the scenarios applied here will help company directors decide fast in terms of customer satisfaction and from the financial perspective.

In the next chapter, comments and considerations about the performed study and planned studies for the future are presented.

# CHAPTER SIX
# CONCLUSION AND FUTURE RESEARCH DIRECTIONS

## 6.1 Conclusion

The importance of inventory management problems gradually increases in today's economic and hard competitive conditions. Due to the increasing product range and growing alternative supply channels, reaching the desired product at desired time is one of the most important factors in obtaining customer satisfaction. Optimization studies of inventory problems have started when Harris (1915) found the EOQ formula. Many studies have been performed on inventory problems since then. Chapter 2 includes literature studies on inventory problems. In the result of the literature survey performed, it is seen that multi-product, multi-period (Q, r) inventory policy is not applied.

The main purpose of this thesis is to solve multi-product, multi-period (Q, r) inventory policy under the budget, storage area, shelf life constraint and various promotions depending on the quantity ordered. In Chapter 3, we have formulated an ILP model for maximizing the profit along with the constraints. Since the computational complexity of the proposed mathematical model will increase as the size of the problem increases, it becomes unfeasible to find an optimum solution in acceptable time. Therefore, GA approach with local search is proposed for solving this problem.

In chapter 4, our ILP model was tested on a small size problem and was compared with the results obtained from GA approach. It is observed that optimum or near optimum results are obtained in acceptable time with GA. In chapter 5, the inventory problem of the pharmaceutical distributor is addressed and the proposed GA approach is applied with the real company data. Firstly, our ILP model was run for the products having deterministic demand. In our problem taken from real life, some products can have seasonal and some of them can have variable demand. We have

proposed a GA approach that considers the products having variable demand and we have obtained feasible solutions in acceptable time.

Inventory holding cost, ordering cost and stockout cost are among the effective factors in the solution of inventory problems. We have examined the effects of the costs on profitability by performing a sensitivity analysis since we have estimated some costs.

We have proposed some scenarios for helping inventory manager and company directors on how to decide the methods about the inventory management.

### 6.2 Future Research Directions

This thesis is focused on developing a model which aims to decide optimum order quantity and reorder point that will maximize the profit under budget constraint, storage area, shelf life constraints and various promotions. Since the problem is applied on a case from real life, planned studies for the future are called for in terms of the method and applied policy. Future research may extend into two directions, i.e. the perspective of problem and the solution method.

From the perspective of problem, some of the future research directions can be stated as follows:

- In our model, we have considered proposed promotions as extra quantity given by the producers to the distributors. In a future study, the extra quantity given by distributors to the pharmacies can also be included in the model.

- We have established our model by the assumption that the payment term convenience provided by producers to the distributors and provided by distributors to the pharmacies does not depend on the quantity ordered. In

a future study, payment term convenience offered may be dependent on the quantity ordered.

- In problem solution, seasonal demand is also addressed as variable demand. A different method can be researched for seasonal demand.

- Since there are lots of products, the problem was solved by choosing a few products. By making a good categorization for the products, a study comprising all the products can be performed.

From the perspective of solution method, some of the future research directions can be stated as follows:

- It is observed that the determined lower and upper bound values for the decision variables affect the solution. An optimization study can be performed to determine the optimum levels of these bound values.

- In order to solve the problem, simulation method can be used and the results can be compared with the proposed GA.

- The problem was solved by assuming that the shelf life is constant and the products in the store have the same shelf life. However, the shelf lives of the products in shelves can expire in different times. Therefore, a model considering different expiration dates can be considered.

- We have created our chromosome structure from the order quantity and reorder point which are two decision variables. In the future study, safety stock level or safety factor can be included in the chromosome structure as a decision variable in order to decide on the most feasible safety stock.

- We have applied our problem for a distributor that has one warehouse. The problem can be extended as a multi-echelon inventory problem.

- The distribution cost from distributor to the pharmacy can be considered in the future study.

Finally, the performance of the proposed GA approach can be examined with other metaheuristic methods and other policies such as (s, S), (Q, r, T) policies etc. rather than (Q, r) inventory policy can be applied on the problem.

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# APPENDICES

## APPENDIX A

## PRODUCT AND DEMAND FLOW IN PHARMACEUTICAL INDUSTRY SUPPLY CHAIN

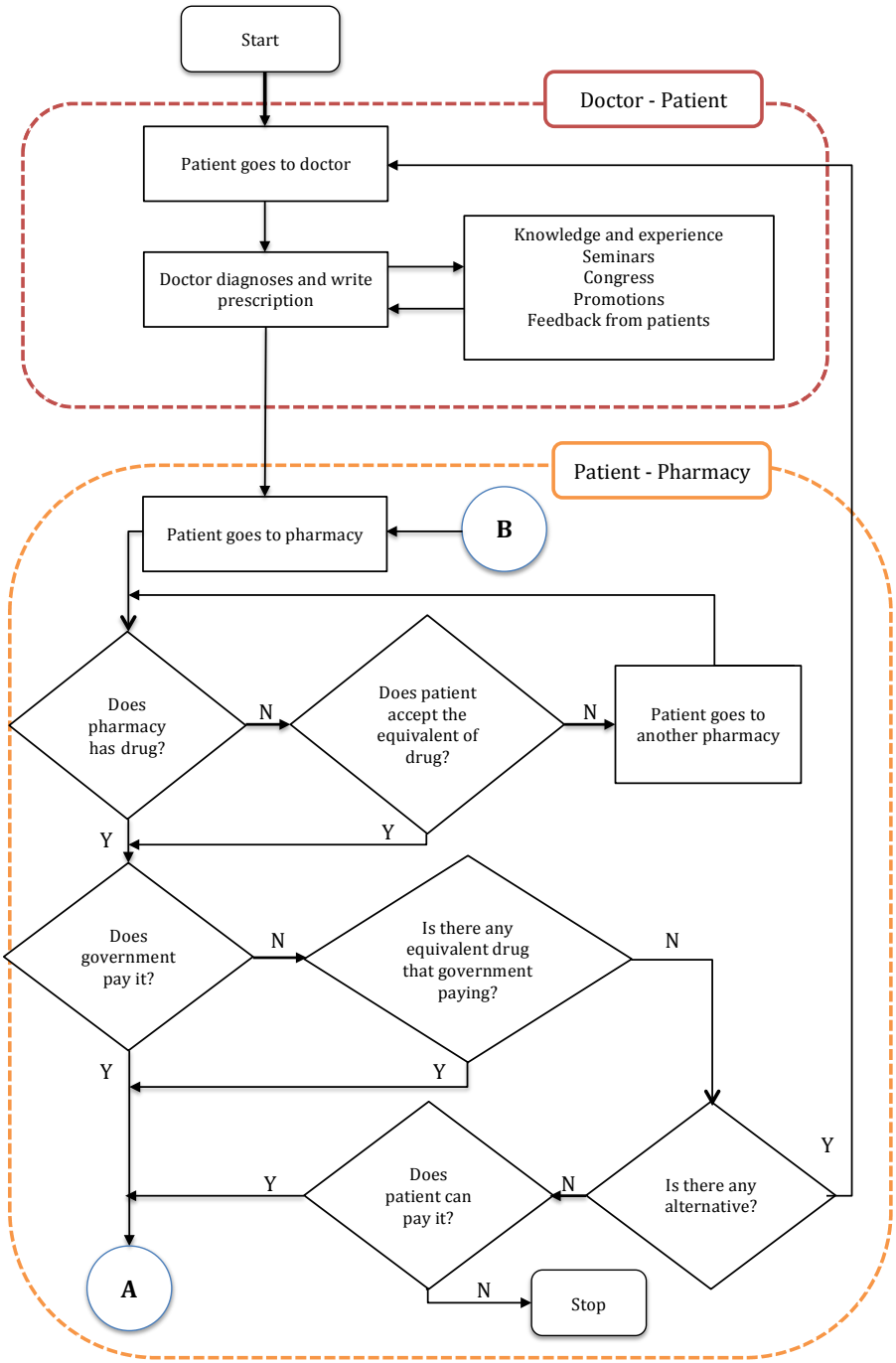



Figure A.1 Demand generation in the pharmaceutical industry: patient – pharmacy

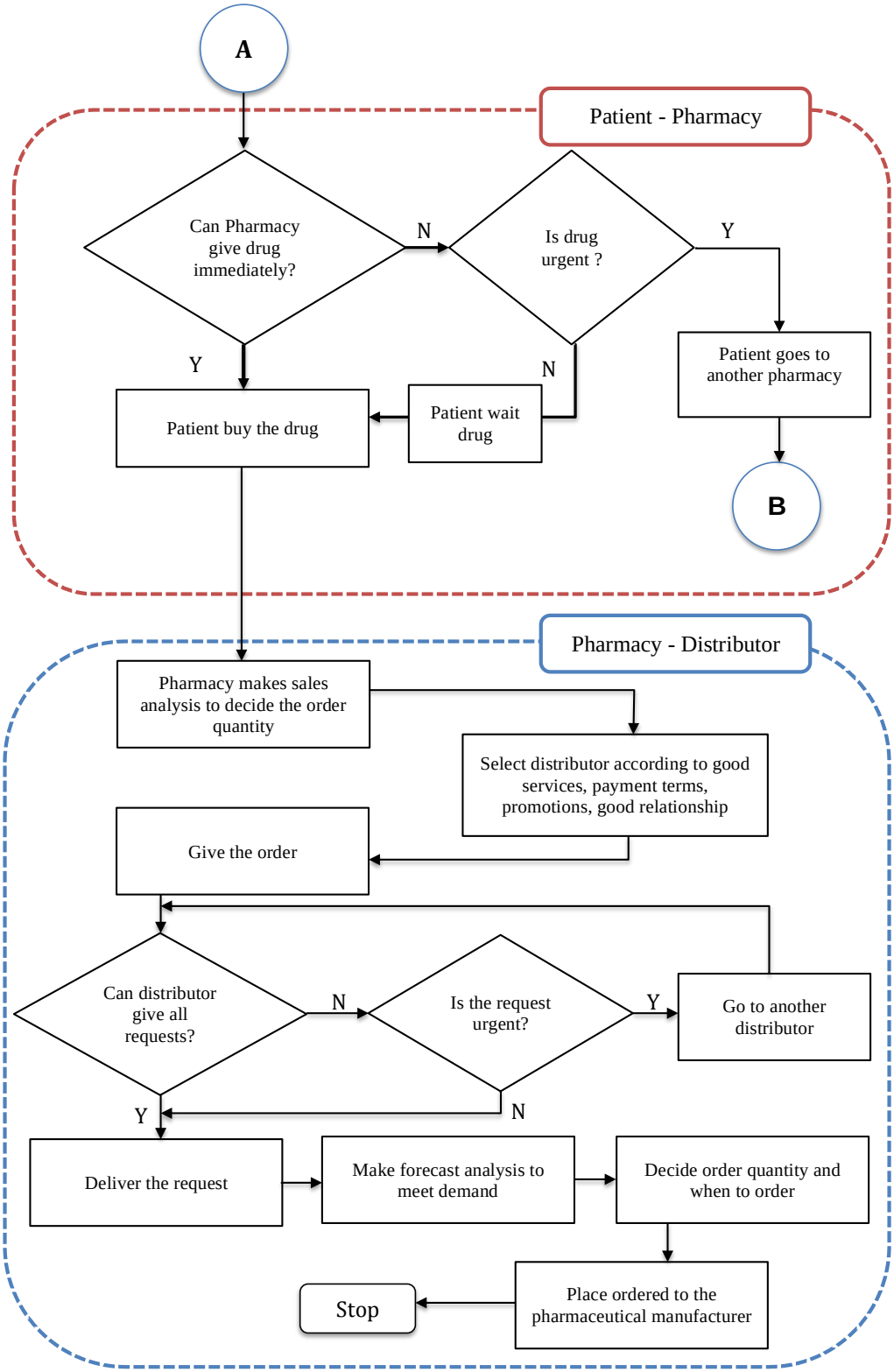



Figure A.2 Demand generation in the pharmaceutical industry: pharmacy - distributor and distributor - manufacturer

# APPENDIX B

# ANOVA RESULTS FOR PRO5

Table B.1 ANOVA table for Pro5

| Source | Degrees of freedom | Sum of squares | Mean square | F-ratio | P–value |
|---|---|---|---|---|---|
| Between groups | 3 | 1.24305E+13 | 4.14349E+12 | 10.33 | 0.001 |
| Within groups | 12 | 4.81466E+12 | 4.01222E+11 | | |
| Total | 15 | 1.72451E+13 | | | |

Table B.2 Tukey's test results for Pro5

| Quarter | No of observation | Mean | Grouping |
|---|---|---|---|
| QTR1 | 4 | 3303683 | A |
| QTR4 | 4 | 3128310 | A |
| QTR2 | 4 | 1532266 | B |
| QTR3 | 4 | 1388643 | B |

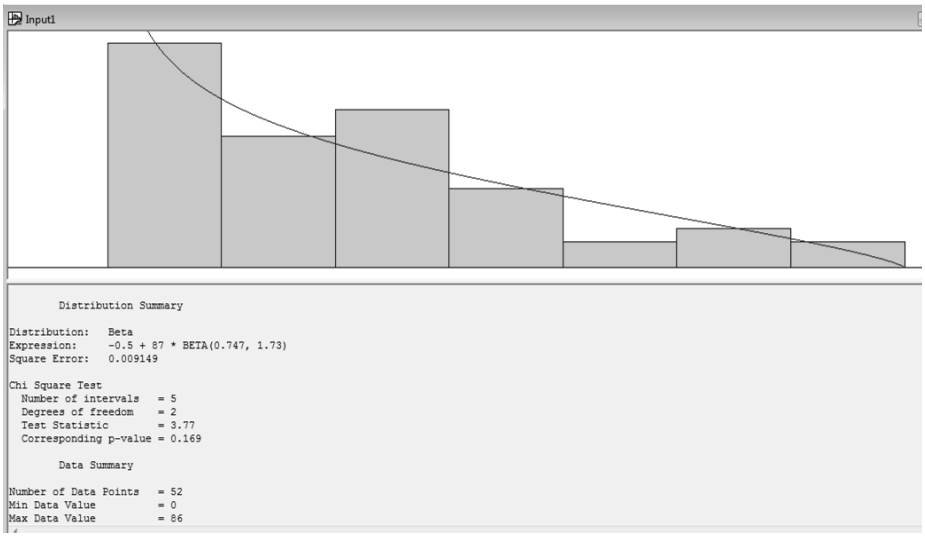



Figure B.1 Distribution fit graph and summary for Pro6

Table B.3 Distribution summary results for Pro6

| Function | Sq Error |
|---|---|
| Beta | 0.0147 |
| Erlang | 0.0168 |
| Exponential | 0.0168 |
| Gamma | 0.0169 |
| Weibull | 0.0173 |
| Triangular | 0.0179 |
| Normal | 0.0201 |
| Lognormal | 0.0213 |
| Uniform | 0.0225 |
| Poisson | 0.0561 |

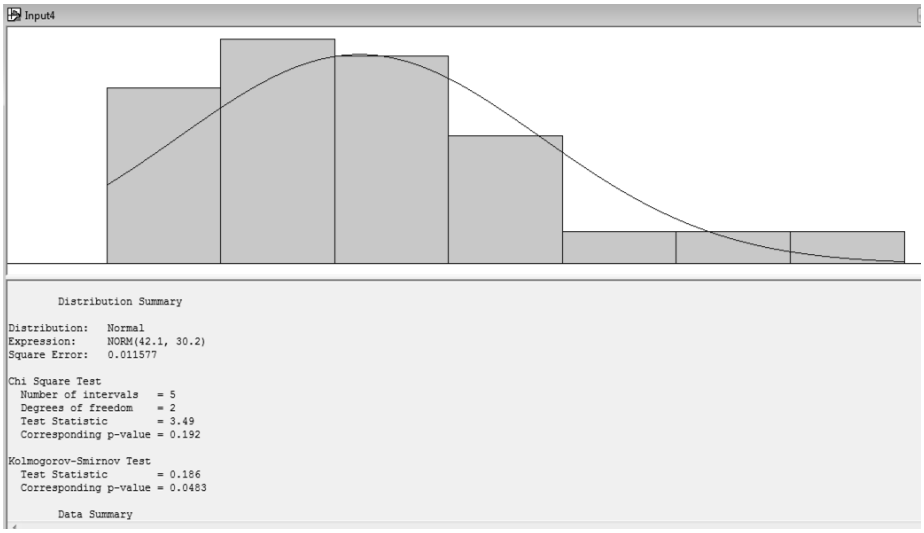



Figure B.2 Distribution fit graph and summary for Pro9

Table B.4 Distribution summary results for Pro9

| Function | Sq Error |
|---|---|
| Normal | 0.0116 |
| Beta | 0.0153 |
| Triangular | 0.0248 |
| Erlang | 0.0393 |
| Exponential | 0.0393 |
| Weibull | 0.0462 |
| Gamma | 0.0606 |
| Uniform | 0.065 |
| Lognormal | 0.128 |

# APPENDIX C

# DEMAND DATA FOR REAL CASE PROBLEM

Table C.1 The demands data for the deterministic products

| Period | Demand Sets | | |
|---|---|---|---|
| | Pro1 | Pro4 | Pro7 |
| 1 | 100 | 120 | 5 |
| 2 | 100 | 120 | 5 |
| 3 | 100 | 120 | 5 |
| 4 | 100 | 120 | 5 |
| 5 | 100 | 120 | 5 |
| 6 | 100 | 120 | 5 |
| 7 | 100 | 120 | 5 |
| 8 | 100 | 120 | 5 |
| 9 | 100 | 120 | 5 |
| 10 | 100 | 120 | 5 |
| 11 | 100 | 120 | 5 |
| 12 | 100 | 120 | 5 |
| 13 | 100 | 120 | 5 |
| 14 | 100 | 120 | 5 |
| 15 | 100 | 120 | 5 |
| 16 | 100 | 120 | 5 |
| 17 | 100 | 120 | 5 |
| 18 | 100 | 120 | 5 |
| 19 | 100 | 120 | 5 |
| 20 | 100 | 120 | 5 |
| 21 | 100 | 120 | 5 |
| 22 | 100 | 120 | 5 |
| 23 | 100 | 120 | 5 |
| 24 | 100 | 120 | 5 |
| 25 | 100 | 120 | 5 |
| 26 | 100 | 120 | 5 |
| 27 | 100 | 120 | 5 |
| 28 | 100 | 120 | 5 |
| 29 | 100 | 120 | 5 |
| 30 | 100 | 120 | 5 |
| 31 | 100 | 120 | 5 |
| 32 | 100 | 120 | 5 |
| 33 | 100 | 120 | 5 |
| 34 | 100 | 120 | 5 |
| 35 | 100 | 120 | 5 |
| 36 | 100 | 120 | 5 |
| 37 | 100 | 120 | 5 |
| 38 | 100 | 120 | 5 |
| 39 | 100 | 120 | 5 |
| 40 | 100 | 120 | 5 |
| 41 | 100 | 120 | 5 |
| 42 | 100 | 120 | 5 |
| 43 | 100 | 120 | 5 |
| 44 | 100 | 120 | 5 |
| 45 | 100 | 120 | 5 |
| 46 | 100 | 120 | 5 |
| 47 | 100 | 120 | 5 |
| 48 | 100 | 120 | 5 |
| 49 | 100 | 120 | 5 |
| 50 | 100 | 120 | 5 |
| 51 | 100 | 120 | 5 |
| 52 | 100 | 120 | 5 |

Table C.2 The demand sets data for the seasonal demand – Pro2

| Period | Demand Sets for Pro2 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 0 | 9 | 9 | 3 | 6 | 9 | 1 | 9 | 0 | 1 |
| 2 | 0 | 5 | 3 | 2 | 0 | 1 | 5 | 5 | 5 | 8 |
| 3 | 0 | 1 | 0 | 5 | 5 | 9 | 1 | 7 | 10 | 8 |
| 4 | 10 | 9 | 6 | 2 | 0 | 10 | 6 | 5 | 9 | 2 |
| 5 | 77 | 77 | 59 | 51 | 14 | 40 | 38 | 53 | 52 | 14 |
| 6 | 24 | 52 | 68 | 28 | 40 | 11 | 30 | 17 | 22 | 50 |
| 7 | 8 | 39 | 37 | 35 | 68 | 57 | 37 | 73 | 62 | 29 |
| 8 | 55 | 17 | 51 | 21 | 47 | 30 | 61 | 45 | 53 | 18 |
| 9 | 143 | 80 | 10 | 83 | 20 | 14 | 15 | 18 | 14 | 12 |
| 10 | 78 | 140 | 10 | 16 | 99 | 14 | 98 | 65 | 17 | 77 |
| 11 | 59 | 75 | 15 | 15 | 11 | 14 | 80 | 67 | 20 | 96 |
| 12 | 206 | 130 | 16 | 16 | 12 | 89 | 88 | 63 | 15 | 73 |
| 13 | 193 | 177 | 19 | 85 | 16 | 81 | 18 | 20 | 12 | 61 |
| 14 | 68 | 63 | 63 | 52 | 47 | 52 | 54 | 62 | 50 | 56 |
| 15 | 48 | 54 | 61 | 66 | 50 | 52 | 65 | 66 | 48 | 67 |
| 16 | 65 | 65 | 63 | 52 | 56 | 49 | 56 | 47 | 45 | 55 |
| 17 | 45 | 56 | 47 | 50 | 66 | 68 | 63 | 65 | 57 | 45 |
| 18 | 108 | 125 | 11 | 11 | 10 | 13 | 10 | 10 | 97 | 12 |
| 19 | 92 | 101 | 10 | 98 | 13 | 10 | 13 | 11 | 14 | 11 |
| 20 | 99 | 149 | 98 | 12 | 13 | 12 | 12 | 12 | 11 | 12 |
| 21 | 150 | 99 | 13 | 93 | 12 | 12 | 13 | 13 | 10 | 14 |
| 22 | 60 | 191 | 13 | 15 | 30 | 94 | 24 | 10 | 17 | 15 |
| 23 | 109 | 152 | 11 | 60 | 13 | 30 | 14 | 25 | 14 | 20 |
| 24 | 81 | 252 | 89 | 22 | 15 | 19 | 20 | 28 | 21 | 13 |
| 25 | 312 | 145 | 24 | 28 | 13 | 28 | 15 | 29 | 86 | 75 |
| 26 | 0 | 186 | 15 | 10 | 5 | 5 | 18 | 19 | 42 | 16 |
| 27 | 198 | 135 | 16 | 17 | 22 | 95 | 14 | 87 | 17 | 11 |
| 28 | 78 | 36 | 19 | 85 | 11 | 2 | 16 | 96 | 17 | 42 |
| 29 | 121 | 104 | 17 | 16 | 17 | 51 | 19 | 11 | 12 | 87 |
| 30 | 93 | 103 | 10 | 92 | 10 | 85 | 10 | 10 | 83 | 98 |
| 31 | 110 | 109 | 82 | 92 | 94 | 78 | 82 | 76 | 10 | 90 |
| 32 | 74 | 85 | 10 | 93 | 10 | 90 | 10 | 10 | 88 | 88 |
| 33 | 77 | 94 | 78 | 77 | 75 | 10 | 86 | 97 | 10 | 98 |
| 34 | 89 | 81 | 77 | 94 | 81 | 77 | 10 | 10 | 90 | 91 |
| 35 | 314 | 93 | 48 | 27 | 12 | 27 | 23 | 15 | 41 | 13 |
| 36 | 159 | 72 | 17 | 23 | 16 | 88 | 30 | 14 | 27 | 73 |
| 37 | 0 | 168 | 41 | 35 | 10 | 60 | 11 | 77 | 10 | 22 |
| 38 | 0 | 178 | 26 | 19 | 25 | 29 | 28 | 20 | 48 | 23 |
| 39 | 70 | 124 | 73 | 63 | 85 | 85 | 89 | 10 | 60 | 11 |
| 40 | 52 | 120 | 97 | 61 | 82 | 95 | 54 | 68 | 75 | 11 |
| 41 | 126 | 116 | 63 | 10 | 12 | 99 | 62 | 63 | 10 | 70 |
| 42 | 114 | 120 | 11 | 97 | 11 | 54 | 10 | 88 | 61 | 80 |
| 43 | 193 | 217 | 23 | 25 | 16 | 14 | 65 | 61 | 43 | 66 |
| 44 | 355 | 233 | 37 | 33 | 75 | 13 | 31 | 38 | 46 | 29 |
| 45 | 36 | 198 | 20 | 23 | 21 | 20 | 32 | 34 | 14 | 19 |
| 46 | 127 | 175 | 25 | 34 | 26 | 74 | 25 | 58 | 11 | 22 |
| 47 | 54 | 267 | 30 | 31 | 16 | 35 | 23 | 12 | 16 | 11 |
| 48 | 351 | 368 | 12 | 51 | 49 | 33 | 72 | 50 | 44 | 78 |
| 49 | 855 | 366 | 75 | 38 | 35 | 48 | 38 | 75 | 83 | 98 |
| 50 | 68 | 372 | 62 | 83 | 57 | 22 | 35 | 47 | 22 | 83 |
| 51 | 100 | 835 | 33 | 47 | 63 | 57 | 34 | 11 | 21 | 50 |
| 52 | 324 | 520 | 37 | 95 | 36 | 34 | 43 | 38 | 13 | 11 |
| Mean demand | 120 | 149 | 144 | 149 | 143 | 132 | 143 | 133 | 118 | 135 |

Table C.3 The demand sets data for the variable demand – Pro3

| Period | Demand Sets for Pro3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 0 | 19 | 14 | 37 | 2 | 28 | 58 | 2 | 15 | 20 |
| 2 | 43 | 16 | 85 | 39 | 9 | 58 | 88 | 0 | 19 | 0 |
| 3 | 1 | 2 | 13 | 0 | 0 | 0 | 24 | 3 | 11 | 0 |
| 4 | 5 | 33 | 17 | 22 | 0 | 27 | 0 | 1 | 26 | 71 |
| 5 | 22 | 2 | 2 | 57 | 0 | 24 | 44 | 0 | 0 | 17 |
| 6 | 178 | 194 | 10 | 94 | 13 | 49 | 3 | 11 | 20 | 2 |
| 7 | 57 | 19 | 65 | 25 | 55 | 22 | 24 | 4 | 26 | 0 |
| 8 | 44 | 59 | 62 | 12 | 11 | 11 | 39 | 0 | 0 | 91 |
| 9 | 118 | 1 | 64 | 0 | 74 | 32 | 1 | 17 | 25 | 17 |
| 10 | 6 | 172 | 11 | 0 | 10 | 7 | 15 | 36 | 94 | 3 |
| 11 | 11 | 28 | 26 | 7 | 12 | 26 | 5 | 73 | 2 | 13 |
| 12 | 15 | 53 | 11 | 12 | 7 | 9 | 14 | 47 | 1 | 38 |
| 13 | 9 | 82 | 11 | 93 | 38 | 3 | 0 | 3 | 87 | 22 |
| 14 | 24 | 4 | 11 | 12 | 48 | 57 | 3 | 40 | 69 | 99 |
| 15 | 151 | 45 | 23 | 48 | 38 | 76 | 25 | 6 | 66 | 9 |
| 16 | 118 | 0 | 4 | 28 | 47 | 23 | 36 | 0 | 19 | 8 |
| 17 | 304 | 350 | 26 | 2 | 14 | 9 | 10 | 1 | 47 | 77 |
| 18 | 114 | 1 | 28 | 52 | 0 | 13 | 1 | 0 | 32 | 21 |
| 19 | 122 | 0 | 3 | 0 | 69 | 14 | 1 | 9 | 20 | 0 |
| 20 | 92 | 5 | 15 | 30 | 0 | 1 | 36 | 10 | 58 | 40 |
| 21 | 96 | 3 | 26 | 22 | 10 | 8 | 75 | 37 | 8 | 2 |
| 22 | 80 | 127 | 2 | 53 | 1 | 32 | 1 | 1 | 4 | 0 |
| 23 | 333 | 156 | 79 | 68 | 14 | 19 | 54 | 28 | 2 | 54 |
| 24 | 105 | 55 | 0 | 88 | 3 | 14 | 7 | 20 | 81 | 12 |
| 25 | 119 | 233 | 3 | 18 | 87 | 12 | 0 | 19 | 28 | 31 |
| 26 | 32 | 388 | 0 | 71 | 73 | 9 | 15 | 85 | 94 | 18 |
| 27 | 111 | 10 | 65 | 27 | 0 | 4 | 1 | 1 | 0 | 1 |
| 28 | 10 | 120 | 9 | 28 | 19 | 34 | 50 | 16 | 11 | 3 |
| 29 | 100 | 34 | 0 | 0 | 28 | 11 | 14 | 80 | 5 | 10 |
| 30 | 19 | 308 | 79 | 0 | 54 | 4 | 3 | 69 | 12 | 0 |
| 31 | 675 | 1 | 85 | 10 | 17 | 15 | 12 | 64 | 28 | 18 |
| 32 | 30 | 49 | 55 | 10 | 3 | 56 | 33 | 2 | 38 | 24 |
| 33 | 38 | 0 | 0 | 86 | 3 | 5 | 27 | 22 | 3 | 11 |
| 34 | 6 | 67 | 60 | 89 | 16 | 64 | 34 | 17 | 34 | 1 |
| 35 | 87 | 82 | 22 | 17 | 3 | 27 | 39 | 19 | 18 | 22 |
| 36 | 176 | 164 | 13 | 1 | 22 | 22 | 7 | 40 | 25 | 6 |
| 37 | 7 | 20 | 0 | 12 | 71 | 53 | 33 | 0 | 24 | 68 |
| 38 | 5 | 0 | 40 | 8 | 14 | 41 | 90 | 4 | 17 | 21 |
| 39 | 7 | 216 | 88 | 26 | 0 | 0 | 0 | 15 | 21 | 0 |
| 40 | 6 | 767 | 13 | 76 | 0 | 21 | 67 | 8 | 1 | 55 |
| 41 | 8 | 82 | 25 | 90 | 15 | 61 | 0 | 50 | 0 | 3 |
| 42 | 3 | 138 | 19 | 0 | 23 | 69 | 0 | 10 | 0 | 40 |
| 43 | 5 | 17 | 0 | 4 | 20 | 9 | 40 | 9 | 15 | 17 |
| 44 | 10 | 1 | 58 | 87 | 10 | 11 | 0 | 75 | 1 | 71 |
| 45 | 10 | 100 | 0 | 11 | 28 | 50 | 21 | 0 | 27 | 22 |
| 46 | 0 | 2 | 74 | 17 | 12 | 23 | 10 | 69 | 85 | 24 |
| 47 | 0 | 17 | 28 | 2 | 24 | 6 | 10 | 1 | 0 | 29 |
| 48 | 5 | 42 | 0 | 24 | 1 | 0 | 87 | 16 | 18 | 10 |
| 49 | 236 | 5 | 19 | 12 | 12 | 0 | 56 | 3 | 1 | 19 |
| 50 | 22 | 16 | 91 | 43 | 61 | 12 | 0 | 3 | 19 | 0 |
| 51 | 1 | 865 | 5 | 3 | 13 | 79 | 9 | 34 | 8 | 0 |
| 52 | 4 | 253 | 0 | 37 | 20 | 22 | 2 | 10 | 21 | 5 |
| Mean demand | 159 | 138 | 140 | 159 | 122 | 115 | 123 | 143 | 129 | 171 |

Table C.4 The demand sets data for the seasonal demand – Pro5

| Period | Demand Sets for Pro5 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 279 | 254 | 33 | 33 | 19 | 27 | 14 | 17 | 30 | 23 |
| 2 | 207 | 249 | 15 | 29 | 77 | 31 | 19 | 18 | 13 | 87 |
| 3 | 292 | 226 | 10 | 12 | 28 | 26 | 21 | 28 | 19 | 43 |
| 4 | 185 | 58 | 76 | 11 | 16 | 20 | 83 | 29 | 27 | 28 |
| 5 | 264 | 240 | 19 | 40 | 35 | 96 | 26 | 14 | 23 | 32 |
| 6 | 276 | 89 | 15 | 16 | 16 | 74 | 60 | 16 | 18 | 21 |
| 7 | 189 | 15 | 74 | 14 | 24 | 17 | 18 | 18 | 19 | 25 |
| 8 | 273 | 394 | 21 | 27 | 15 | 30 | 33 | 27 | 83 | 13 |
| 9 | 303 | 288 | 21 | 19 | 32 | 26 | 23 | 29 | 25 | 20 |
| 10 | 278 | 193 | 30 | 23 | 31 | 18 | 20 | 31 | 28 | 11 |
| 11 | 269 | 206 | 25 | 29 | 17 | 23 | 22 | 25 | 28 | 26 |
| 12 | 104 | 173 | 16 | 49 | 20 | 95 | 95 | 13 | 25 | 21 |
| 13 | 139 | 140 | 15 | 40 | 12 | 63 | 31 | 19 | 24 | 22 |
| 14 | 317 | 136 | 24 | 18 | 23 | 31 | 17 | 15 | 51 | 57 |
| 15 | 87 | 287 | 16 | 18 | 14 | 49 | 22 | 12 | 27 | 10 |
| 16 | 62 | 249 | 24 | 16 | 20 | 11 | 17 | 10 | 13 | 34 |
| 17 | 371 | 77 | 12 | 17 | 15 | 71 | 37 | 13 | 13 | 27 |
| 18 | 340 | 55 | 24 | 66 | 18 | 11 | 18 | 56 | 24 | 11 |
| 19 | 112 | 372 | 21 | 18 | 21 | 81 | 28 | 14 | 31 | 53 |
| 20 | 99 | 231 | 32 | 22 | 19 | 17 | 21 | 22 | 10 | 13 |
| 21 | 69 | 46 | 23 | 24 | 11 | 71 | 4 | 13 | 23 | 14 |
| 22 | 116 | 120 | 24 | 20 | 23 | 27 | 15 | 22 | 21 | 11 |
| 23 | 47 | 268 | 25 | 15 | 13 | 82 | 24 | 20 | 32 | 22 |
| 24 | 229 | 83 | 15 | 16 | 81 | 29 | 10 | 17 | 14 | 15 |
| 25 | 199 | 123 | 56 | 19 | 50 | 11 | 28 | 21 | 11 | 16 |
| 26 | 187 | 174 | 12 | 31 | 12 | 15 | 18 | 54 | 24 | 23 |
| 27 | 0 | 192 | 35 | 13 | 91 | 1 | 19 | 17 | 22 | 24 |
| 28 | 322 | 110 | 19 | 11 | 19 | 79 | 13 | 35 | 12 | 27 |
| 29 | 224 | 147 | 14 | 20 | 35 | 52 | 20 | 10 | 90 | 13 |
| 30 | 375 | 57 | 17 | 85 | 11 | 82 | 23 | 28 | 11 | 4 |
| 31 | 148 | 51 | 16 | 22 | 25 | 24 | 14 | 17 | 94 | 92 |
| 32 | 170 | 117 | 16 | 21 | 69 | 11 | 15 | 15 | 1 | 38 |
| 33 | 210 | 264 | 19 | 11 | 20 | 24 | 28 | 17 | 24 | 17 |
| 34 | 200 | 166 | 28 | 70 | 75 | 10 | 13 | 12 | 95 | 53 |
| 35 | 89 | 187 | 14 | 15 | 95 | 23 | 19 | 42 | 12 | 26 |
| 36 | 170 | 90 | 22 | 35 | 20 | 46 | 44 | 18 | 63 | 33 |
| 37 | 151 | 45 | 28 | 20 | 12 | 21 | 32 | 16 | 32 | 12 |
| 38 | 192 | 183 | 13 | 24 | 16 | 90 | 11 | 80 | 17 | 14 |
| 39 | 143 | 92 | 14 | 13 | 15 | 26 | 28 | 12 | 18 | 67 |
| 40 | 100 | 119 | 27 | 27 | 70 | 15 | 53 | 60 | 75 | 28 |
| 41 | 725 | 101 | 41 | 84 | 99 | 20 | 50 | 24 | 47 | 70 |
| 42 | 209 | 275 | 25 | 35 | 11 | 56 | 75 | 59 | 10 | 18 |
| 43 | 498 | 234 | 10 | 65 | 66 | 57 | 19 | 39 | 74 | 22 |
| 44 | 196 | 573 | 14 | 59 | 90 | 67 | 10 | 56 | 63 | 59 |
| 45 | 865 | 104 | 72 | 34 | 36 | 59 | 14 | 32 | 62 | 21 |
| 46 | 316 | 844 | 11 | 56 | 11 | 10 | 19 | 13 | 30 | 19 |
| 47 | 61 | 3 | 41 | 29 | 31 | 15 | 45 | 23 | 30 | 39 |
| 48 | 569 | 217 | 48 | 26 | 84 | 45 | 12 | 61 | 57 | 12 |
| 49 | 449 | 113 | 2 | 13 | 54 | 58 | 54 | 69 | 73 | 12 |
| 50 | 264 | 366 | 48 | 10 | 18 | 37 | 31 | 34 | 94 | 62 |
| 51 | 174 | 460 | 46 | 79 | 67 | 91 | 29 | 29 | 59 | 37 |
| 52 | 443 | 361 | 19 | 49 | 11 | 28 | 13 | 5 | 48 | 11 |
| Mean demand | 241 | 252 | 237 | 260 | 212 | 228 | 239 | 218 | 233 | 240 |

Table C.5 The demand sets data for the variable demand – Pro6

| Period | Demand Sets for Pro6 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 0 | 11 | 23 | 3 | 15 | 37 | 30 | 4 | 13 | 25 |
| 2 | 0 | 61 | 38 | 30 | 9 | 38 | 45 | 44 | 29 | 42 |
| 3 | 1 | 65 | 23 | 41 | 21 | 7 | 12 | 3 | 3 | 81 |
| 4 | 5 | 0 | 10 | 41 | 34 | 4 | 59 | 77 | 69 | 7 |
| 5 | 75 | 33 | 4 | 23 | 38 | 51 | 27 | 19 | 19 | 0 |
| 6 | 0 | 8 | 78 | 6 | 4 | 17 | 6 | 4 | 40 | 0 |
| 7 | 36 | 0 | 42 | 18 | 30 | 22 | 65 | 5 | 75 | 10 |
| 8 | 46 | 0 | 27 | 35 | 7 | 15 | 19 | 0 | 11 | 59 |
| 9 | 20 | 0 | 20 | 60 | 7 | 6 | 40 | 2 | 20 | 13 |
| 10 | 3 | 0 | 2 | 22 | 39 | 0 | 17 | 50 | 40 | 3 |
| 11 | 21 | 1 | 7 | 18 | 39 | 3 | 25 | 65 | 47 | 77 |
| 12 | 52 | 3 | 31 | 48 | 8 | 35 | 0 | 23 | 7 | 12 |
| 13 | 13 | 0 | 8 | 19 | 37 | 38 | 30 | 17 | 37 | 62 |
| 14 | 28 | 18 | 14 | 0 | 0 | 9 | 16 | 17 | 12 | 27 |
| 15 | 4 | 8 | 57 | 7 | 65 | 9 | 0 | 29 | 34 | 0 |
| 16 | 12 | 2 | 14 | 27 | 0 | 2 | 0 | 31 | 23 | 0 |
| 17 | 2 | 33 | 14 | 0 | 1 | 4 | 0 | 75 | 30 | 35 |
| 18 | 32 | 0 | 19 | 17 | 20 | 41 | 75 | 20 | 2 | 45 |
| 19 | 27 | 1 | 1 | 1 | 16 | 16 | 74 | 51 | 3 | 18 |
| 20 | 34 | 69 | 15 | 45 | 4 | 29 | 7 | 12 | 73 | 58 |
| 21 | 44 | 47 | 32 | 10 | 53 | 0 | 0 | 15 | 27 | 3 |
| 22 | 66 | 2 | 2 | 30 | 8 | 58 | 65 | 4 | 44 | 35 |
| 23 | 53 | 24 | 11 | 52 | 1 | 41 | 32 | 65 | 27 | 48 |
| 24 | 35 | 3 | 12 | 9 | 73 | 9 | 59 | 0 | 12 | 48 |
| 25 | 37 | 43 | 20 | 7 | 18 | 7 | 1 | 33 | 5 | 26 |
| 26 | 32 | 1 | 0 | 26 | 43 | 4 | 0 | 8 | 90 | 8 |
| 27 | 16 | 4 | 12 | 43 | 10 | 31 | 12 | 0 | 49 | 21 |
| 28 | 20 | 29 | 52 | 0 | 37 | 20 | 1 | 0 | 32 | 40 |
| 29 | 45 | 5 | 8 | 12 | 76 | 0 | 37 | 0 | 23 | 69 |
| 30 | 10 | 10 | 72 | 4 | 70 | 12 | 21 | 0 | 2 | 26 |
| 31 | 16 | 22 | 46 | 39 | 24 | 84 | 66 | 1 | 8 | 21 |
| 32 | 30 | 56 | 4 | 8 | 70 | 49 | 46 | 3 | 36 | 55 |
| 33 | 43 | 11 | 21 | 3 | 9 | 4 | 7 | 0 | 9 | 22 |
| 34 | 69 | 25 | 36 | 29 | 31 | 2 | 7 | 18 | 16 | 0 |
| 35 | 67 | 2 | 70 | 16 | 10 | 10 | 29 | 8 | 67 | 9 |
| 36 | 31 | 59 | 6 | 56 | 30 | 37 | 36 | 2 | 16 | 32 |
| 37 | 11 | 16 | 0 | 33 | 37 | 36 | 14 | 33 | 17 | 0 |
| 38 | 30 | 35 | 0 | 51 | 72 | 17 | 5 | 0 | 22 | 20 |
| 39 | 15 | 65 | 9 | 48 | 1 | 75 | 55 | 1 | 1 | 1 |
| 40 | 32 | 10 | 51 | 22 | 3 | 50 | 25 | 69 | 17 | 52 |
| 41 | 10 | 17 | 11 | 18 | 24 | 42 | 28 | 47 | 38 | 12 |
| 42 | 31 | 34 | 3 | 29 | 37 | 30 | 1 | 2 | 3 | 35 |
| 43 | 22 | 40 | 66 | 46 | 7 | 22 | 33 | 24 | 13 | 60 |
| 44 | 22 | 6 | 10 | 85 | 7 | 2 | 53 | 3 | 14 | 11 |
| 45 | 27 | 32 | 53 | 45 | 13 | 9 | 15 | 43 | 23 | 8 |
| 46 | 9 | 11 | 23 | 31 | 3 | 0 | 75 | 1 | 0 | 31 |
| 47 | 10 | 29 | 0 | 29 | 31 | 55 | 40 | 4 | 14 | 50 |
| 48 | 86 | 20 | 0 | 42 | 7 | 6 | 1 | 29 | 61 | 0 |
| 49 | 3 | 26 | 30 | 34 | 11 | 1 | 37 | 5 | 9 | 14 |
| 50 | 1 | 1 | 39 | 3 | 11 | 5 | 11 | 10 | 84 | 4 |
| 51 | 0 | 3 | 16 | 0 | 2 | 17 | 34 | 22 | 54 | 45 |
| 52 | 7 | 63 | 50 | 78 | 40 | 35 | 50 | 56 | 4 | 9 |
| Mean demand | 26 | 20 | 25 | 36 | 26 | 32 | 24 | 19 | 22 | 16 |

Table C.6 The demand sets data for the seasonal demand – Pro8

| Period | Demand Sets for Pro8 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 0 | 25 | 13 | 12 | 22 | 12 | 0 | 15 | 7 | 18 |
| 2 | 26 | 2 | 0 | 26 | 32 | 0 | 4 | 18 | 12 | 9 |
| 3 | 2 | 0 | 17 | 1 | 11 | 8 | 45 | 0 | 47 | 35 |
| 4 | 28 | 0 | 39 | 34 | 0 | 19 | 5 | 31 | 0 | 30 |
| 5 | 11 | 21 | 24 | 25 | 7 | 29 | 7 | 28 | 10 | 21 |
| 6 | 12 | 13 | 15 | 25 | 23 | 20 | 27 | 20 | 21 | 0 |
| 7 | 16 | 5 | 26 | 37 | 15 | 12 | 21 | 0 | 42 | 17 |
| 8 | 36 | 19 | 14 | 23 | 33 | 15 | 7 | 10 | 34 | 37 |
| 9 | 37 | 89 | 0 | 50 | 17 | 37 | 49 | 32 | 14 | 38 |
| 10 | 23 | 34 | 35 | 19 | 49 | 3 | 33 | 44 | 26 | 32 |
| 11 | 34 | 46 | 39 | 45 | 29 | 38 | 25 | 29 | 38 | 67 |
| 12 | 26 | 23 | 64 | 76 | 39 | 20 | 29 | 28 | 43 | 36 |
| 13 | 63 | 13 | 4 | 55 | 17 | 49 | 60 | 43 | 27 | 42 |
| 14 | 61 | 51 | 78 | 75 | 65 | 79 | 71 | 47 | 74 | 58 |
| 15 | 88 | 63 | 93 | 85 | 75 | 82 | 77 | 81 | 11 | 82 |
| 16 | 64 | 63 | 81 | 68 | 42 | 83 | 65 | 82 | 60 | 88 |
| 17 | 99 | 87 | 90 | 69 | 78 | 10 | 89 | 82 | 10 | 73 |
| 18 | 49 | 85 | 57 | 94 | 40 | 71 | 59 | 19 | 64 | 21 |
| 19 | 18 | 36 | 47 | 25 | 65 | 58 | 0 | 60 | 77 | 76 |
| 20 | 71 | 42 | 0 | 54 | 31 | 23 | 70 | 49 | 79 | 59 |
| 21 | 84 | 56 | 40 | 53 | 38 | 62 | 87 | 54 | 29 | 27 |
| 22 | 87 | 88 | 54 | 68 | 67 | 64 | 57 | 80 | 10 | 53 |
| 23 | 57 | 59 | 50 | 56 | 46 | 69 | 64 | 59 | 50 | 73 |
| 24 | 79 | 80 | 59 | 55 | 74 | 94 | 75 | 59 | 63 | 82 |
| 25 | 49 | 69 | 54 | 51 | 54 | 60 | 73 | 52 | 68 | 85 |
| 26 | 90 | 76 | 10 | 93 | 74 | 65 | 73 | 11 | 10 | 63 |
| 27 | 75 | 79 | 95 | 57 | 10 | 10 | 91 | 10 | 10 | 62 |
| 28 | 116 | 93 | 69 | 11 | 10 | 12 | 96 | 95 | 11 | 84 |
| 29 | 104 | 97 | 10 | 11 | 73 | 10 | 49 | 95 | 49 | 70 |
| 30 | 210 | 417 | 33 | 0 | 22 | 17 | 35 | 15 | 63 | 66 |
| 31 | 210 | 182 | 21 | 0 | 22 | 18 | 14 | 15 | 34 | 34 |
| 32 | 308 | 134 | 35 | 21 | 18 | 26 | 31 | 20 | 31 | 25 |
| 33 | 114 | 223 | 21 | 27 | 20 | 12 | 17 | 39 | 31 | 34 |
| 34 | 0 | 225 | 16 | 17 | 24 | 27 | 24 | 21 | 69 | 14 |
| 35 | 10 | 17 | 2 | 9 | 13 | 11 | 9 | 6 | 6 | 6 |
| 36 | 16 | 1 | 6 | 0 | 7 | 0 | 8 | 10 | 9 | 8 |
| 37 | 0 | 14 | 0 | 9 | 3 | 3 | 0 | 1 | 12 | 0 |
| 38 | 0 | 4 | 4 | 17 | 17 | 10 | 11 | 11 | 0 | 1 |
| 39 | 2 | 0 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 0 |
| 40 | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 1 | 0 | 2 |
| 41 | 0 | 0 | 1 | 0 | 2 | 1 | 0 | 1 | 1 | 0 |
| 42 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 43 | 0 | 0 | 0 | 0 | 0 | 20 | 40 | 14 | 24 | 11 |
| 44 | 0 | 0 | 0 | 13 | 13 | 49 | 69 | 0 | 39 | 0 |
| 45 | 99 | 76 | 15 | 95 | 0 | 67 | 0 | 50 | 0 | 84 |
| 46 | 0 | 0 | 12 | 24 | 8 | 0 | 0 | 0 | 33 | 26 |
| 47 | 1 | 0 | 25 | 5 | 47 | 25 | 53 | 3 | 20 | 61 |
| 48 | 0 | 0 | 0 | 10 | 13 | 0 | 19 | 0 | 0 | 12 |
| 49 | 0 | 2 | 4 | 1 | 11 | 0 | 6 | 23 | 0 | 14 |
| 50 | 0 | 10 | 0 | 11 | 11 | 0 | 10 | 0 | 0 | 19 |
| 51 | 0 | 9 | 19 | 0 | 0 | 6 | 0 | 0 | 11 | 0 |
| 52 | 24 | 6 | 10 | 7 | 23 | 15 | 15 | 0 | 19 | 7 |
| Mean demand | 48 | 53 | 55 | 46 | 49 | 53 | 56 | 51 | 55 | 55 |

Table C.7 The demand sets data for the variable demand – Pro9

| Period | Demand Sets for Pro9 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 7 | 0 | 40 | 62 | 38 | 25 | 73 | 76 | 44 | 26 |
| 2 | 26 | 57 | 61 | 3 | 33 | 28 | 74 | 48 | 76 | 78 |
| 3 | 0 | 9 | 87 | 33 | 46 | 65 | 49 | 14 | 10 | 85 |
| 4 | 0 | 78 | 68 | 29 | 21 | 39 | 35 | 91 | 83 | 16 |
| 5 | 0 | 57 | 10 | 41 | 35 | 13 | 11 | 40 | 75 | 43 |
| 6 | 25 | 48 | 10 | 0 | 28 | 52 | 82 | 37 | 32 | 44 |
| 7 | 5 | 31 | 28 | 61 | 75 | 70 | 25 | 31 | 48 | 25 |
| 8 | 5 | 15 | 28 | 59 | 60 | 97 | 35 | 7 | 49 | 55 |
| 9 | 16 | 62 | 41 | 10 | 82 | 53 | 7 | 40 | 88 | 46 |
| 10 | 39 | 46 | 39 | 38 | 50 | 53 | 69 | 44 | 84 | 11 |
| 11 | 4 | 26 | 0 | 37 | 60 | 5 | 53 | 0 | 16 | 14 |
| 12 | 33 | 56 | 5 | 35 | 96 | 67 | 64 | 80 | 44 | 63 |
| 13 | 13 | 38 | 51 | 40 | 80 | 65 | 77 | 39 | 25 | 14 |
| 14 | 10 | 0 | 63 | 0 | 33 | 34 | 67 | 12 | 51 | 19 |
| 15 | 29 | 25 | 56 | 41 | 49 | 65 | 82 | 35 | 55 | 10 |
| 16 | 52 | 94 | 11 | 0 | 74 | 80 | 72 | 51 | 46 | 36 |
| 17 | 38 | 49 | 85 | 45 | 12 | 57 | 0 | 45 | 38 | 27 |
| 18 | 45 | 4 | 0 | 0 | 8 | 17 | 0 | 32 | 0 | 60 |
| 19 | 105 | 0 | 20 | 41 | 72 | 15 | 27 | 79 | 26 | 27 |
| 20 | 34 | 9 | 84 | 92 | 66 | 67 | 75 | 0 | 72 | 2 |
| 21 | 22 | 65 | 15 | 18 | 4 | 72 | 77 | 0 | 69 | 84 |
| 22 | 38 | 73 | 7 | 49 | 27 | 35 | 10 | 0 | 72 | 29 |
| 23 | 24 | 42 | 8 | 45 | 10 | 46 | 13 | 66 | 45 | 91 |
| 24 | 25 | 81 | 60 | 88 | 39 | 20 | 0 | 14 | 44 | 10 |
| 25 | 71 | 22 | 67 | 59 | 0 | 86 | 42 | 0 | 55 | 0 |
| 26 | 79 | 15 | 0 | 53 | 22 | 85 | 29 | 34 | 22 | 20 |
| 27 | 22 | 46 | 69 | 68 | 72 | 10 | 10 | 34 | 39 | 92 |
| 28 | 23 | 18 | 0 | 48 | 66 | 39 | 65 | 54 | 35 | 54 |
| 29 | 109 | 37 | 31 | 85 | 42 | 40 | 19 | 95 | 24 | 0 |
| 30 | 75 | 34 | 18 | 69 | 12 | 20 | 40 | 45 | 0 | 93 |
| 31 | 53 | 22 | 29 | 65 | 0 | 18 | 61 | 64 | 44 | 79 |
| 32 | 35 | 54 | 22 | 23 | 85 | 3 | 78 | 72 | 71 | 50 |
| 33 | 40 | 101 | 22 | 29 | 61 | 13 | 13 | 35 | 14 | 59 |
| 34 | 133 | 25 | 17 | 50 | 9 | 10 | 40 | 55 | 25 | 49 |
| 35 | 80 | 0 | 2 | 0 | 24 | 36 | 60 | 0 | 31 | 15 |
| 36 | 62 | 34 | 55 | 65 | 37 | 55 | 8 | 37 | 51 | 0 |
| 37 | 72 | 48 | 43 | 60 | 80 | 65 | 0 | 31 | 44 | 76 |
| 38 | 58 | 91 | 28 | 31 | 38 | 6 | 4 | 0 | 43 | 9 |
| 39 | 39 | 78 | 54 | 79 | 27 | 98 | 83 | 58 | 60 | 48 |
| 40 | 52 | 60 | 61 | 45 | 49 | 60 | 63 | 72 | 14 | 67 |
| 41 | 57 | 23 | 58 | 37 | 0 | 28 | 55 | 86 | 50 | 67 |
| 42 | 49 | 11 | 39 | 61 | 80 | 19 | 90 | 51 | 65 | 79 |
| 43 | 27 | 72 | 61 | 4 | 32 | 32 | 74 | 67 | 45 | 60 |
| 44 | 24 | 29 | 40 | 42 | 62 | 56 | 16 | 0 | 62 | 19 |
| 45 | 116 | 38 | 80 | 20 | 49 | 35 | 72 | 73 | 42 | 80 |
| 46 | 66 | 32 | 0 | 11 | 19 | 59 | 53 | 22 | 0 | 0 |
| 47 | 18 | 39 | 27 | 99 | 19 | 63 | 67 | 52 | 10 | 89 |
| 48 | 57 | 0 | 82 | 48 | 81 | 6 | 72 | 22 | 59 | 38 |
| 49 | 44 | 40 | 90 | 78 | 49 | 60 | 39 | 47 | 66 | 44 |
| 50 | 34 | 35 | 0 | 59 | 66 | 18 | 26 | 52 | 33 | 32 |
| 51 | 56 | 57 | 69 | 38 | 9 | 86 | 56 | 37 | 45 | 0 |
| 52 | 43 | 41 | 65 | 71 | 30 | 77 | 0 | 82 | 8 | 45 |
| Mean demand | 42 | 40 | 44 | 47 | 47 | 47 | 49 | 44 | 43 | 48 |

# APPENDIX D

# RESULTS FOR DIFFERENT TYPES OF PROBLEMS

Table D.1 The lower and upper bounds for (Q r) decision variables for %95 CSL

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 |
|---|---|---|---|---|---|---|---|---|---|
| lower_$Q_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| upper_$Q_i$ | 380 | 5000 | 5000 | 5000 | 5000 | 5000 | 20 | 1080 | 1120 |
| lower_$r_i$ | 0 | 228 | 558 | 0 | 269 | 35 | 0 | 102 | 50 |
| upper_$r_i$ | 100 | 672 | 1512 | 120 | 920 | 100 | 8 | 296 | 164 |

Table D.2 The lower and upper bounds for (Q, r) decision variables for %99 CSL

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 |
|---|---|---|---|---|---|---|---|---|---|
| lower_$Q_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| upper_$Q_i$ | 380 | 5000 | 5000 | 5000 | 5000 | 5000 | 20 | 1080 | 1120 |
| lower_$r_i$ | 0 | 322 | 787 | 0 | 380 | 50 | 0 | 144 | 71 |
| upper_$r_i$ | 100 | 672 | 1512 | 120 | 920 | 100 | 8 | 296 | 164 |

Table D.3 The lower and upper bounds for (Q, r) decision variables for %99.99 CSL

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 |
|---|---|---|---|---|---|---|---|---|---|
| lower_$Q_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| upper_$Q_i$ | 380 | 5000 | 5000 | 5000 | 5000 | 5000 | 20 | 1080 | 1120 |
| lower_$r_i$ | 0 | 515 | 1257 | 0 | 607 | 80 | 0 | 230 | 113 |
| upper_$r_i$ | 100 | 672 | 1512 | 120 | 920 | 100 | 8 | 296 | 164 |

Table D.4 Results for the scenario A

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 97 | 251 | 250 | 250 | 100 | 19 | 2 | 73 | 46 | |
| $r^*$ | 343 | 297 | 464 | 182 | 275 | 65 | 3 | 22 | 133 | |
| Expected Total Sales Revenue (ETSR) ($) | 332170.80 | 120272.31 | 355070.39 | 23125.44 | 136788.12 | 20158.17 | 130801.88 | 6532.94 | 6567.03 | 1131487.08 |
| Total Earning from Payment Term TPFS ($) | 10479.21 | 4402.27 | 4979.56 | 51.77 | 679.58 | 713.78 | 5473.09 | 276.40 | 140.49 | 27196.15 |
| Total Purchasing Cost (TPC) ($) | 301126.80 | 94876.49 | 171709.00 | 5950.00 | 58584.40 | 15383.16 | 125818.16 | 5956.80 | 6055.81 | 785460.62 |
| Total Losing from Payment Term (TLPFC) ($) | 7706.36 | 4185.48 | 10297.04 | 201.19 | 1586.74 | 701.50 | 5689.88 | 227.35 | 76.18 | 30671.72 |
| Total Holding Cost (THC) ($) | 835.38 | 946.13 | 6204.24 | 283.64 | 556.76 | 84.52 | 0.00 | 14.51 | 30.83 | 8955.99 |
| Total Ordering Cost (TOC) ($) | 520.00 | 219.00 | 124.00 | 70.00 | 427.00 | 468.00 | 520.00 | 240.00 | 484.00 | 3072.00 |
| Expected Total Shortage Cost (ETSC) ($) | 0.00 | 900.86 | 4969.50 | 0.00 | 3346.53 | 269.51 | 3726.80 | 293.20 | 29.78 | 13536.19 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 0.00 | 652.80 | 1280.80 | 0.00 | 3042.30 | 195.30 | 154.00 | 945.80 | 124.10 | 6395.10 |
| Expected Total Profit (ETPR) ($) | 32461.47 | 23546.61 | 166746.17 | 16672.37 | 72966.27 | 3965.25 | 520.13 | 77.48 | 30.93 | 316986.69 |

Table D.5 Results for the scenario B

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 200 | 155 | 513 | 500 | 100 | 0 | 1 | 93 | 175 | |
| $r^*$ | 18 | 173 | 914 | 37 | 594 | 0 | 4 | 406 | 26 | |
| Expected Total Sales Revenue (ETSR) ($) | 329360.12 | 104839.59 | 417508.49 | 22632.54 | 144489.32 | 485.78 | 66634.92 | 8935.17 | 6407.82 | 1101293.76 |
| Total Earning from Payment Term TPFS ($) | 10387.80 | 3984.70 | 6345.09 | 44.37 | 720.96 | 0.00 | 2736.54 | 438.68 | 136.94 | 24795.08 |
| Total Purchasing Cost (TPC) ($) | 298500.00 | 85877.13 | 218796.04 | 5100.00 | 62151.60 | 0.00 | 62909.08 | 9454.38 | 5902.40 | 748690.63 |
| Total Losing from Payment Term (TLPFC) ($) | 7641.15 | 3648.42 | 12107.75 | 196.90 | 1676.08 | 16.91 | 2898.62 | 310.94 | 74.33 | 28571.10 |
| Total Holding Cost (THC) ($) | 536.76 | 403.71 | 12984.50 | 471.07 | 1373.03 | 1.12 | 0.00 | 181.37 | 38.67 | 15990.24 |
| Total Ordering Cost (TOC) ($) | 250.00 | 321.00 | 77.00 | 30.00 | 453.00 | 0.00 | 520.00 | 299.00 | 124.00 | 2074.00 |
| Expected Total Shortage Cost (ETSC) ($) | 183.92 | 2043.37 | 881.92 | 41.23 | 2774.53 | 1722.52 | 4985.20 | 92.26 | 42.67 | 12767.61 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 44.00 | 1480.70 | 227.30 | 133.00 | 2522.30 | 1248.20 | 206.00 | 297.60 | 177.80 | 6336.90 |
| Expected Total Profit (ETPR) ($) | 32636.09 | 16530.66 | 179006.37 | 16837.71 | 76782.04 | -1254.76 | -1941.43 | -964.10 | 362.68 | 317995.26 |

Table D.6 Results for the scenario D

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 422 | 1317 | 500 | 52 | 100 | 65 | 8 | 796 | 109 | |
| $r^*$ | 110 | 345 | 781 | 90 | 610 | 11 | 4 | 356 | 33 | |
| Expected Total Sales Revenue (ETSR) ($) | 332170,80 | 131412.05 | 412897.50 | 22795.61 | 145081.72 | 20546.79 | 308495.00 | 10038.07 | 6393.00 | 1389830.54 |
| Total Earning from Payment Term TPFS ($) | 10520.76 | 4429.90 | 6103.98 | 64.60 | 724.14 | 756.56 | 13051.21 | 502.31 | 135.50 | 36288.97 |
| Total Purchasing Cost (TPC) ($) | 302320.80 | 95471.96 | 210482.00 | 7425.60 | 62426.00 | 16305.25 | 300027.92 | 10825.60 | 5840.66 | 1011125.79 |
| Total Losing from Payment Term (TLPFC) ($) | 7706.36 | 4573.14 | 11974.03 | 198.32 | 1682.95 | 715.03 | 13419.53 | 349.32 | 74.16 | 40692.84 |
| Total Holding Cost (THC) ($) | 2364.60 | 3763.29 | 11762.24 | 29.07 | 1430.40 | 99.02 | 624.95 | 340.84 | 25.17 | 20439.58 |
| Total Ordering Cost (TOC) ($) | 120.00 | 42.00 | 76.00 | 420.00 | 455.00 | 145.00 | 310.00 | 40.00 | 197.00 | 1805.00 |
| Expected Total Shortage Cost (ETSC) ($) | 0.00 | 76.18 | 1183.79 | 27.59 | 2730.53 | 240.81 | 242.00 | 0.00 | 43.87 | 4544.77 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 0.00 | 55.20 | 305.10 | 89.00 | 2482.30 | 174.50 | 10.00 | 0.00 | 182.80 | 3298.90 |
| Expected Total Profit (ETPR) ($) | 30179.80 | 31915.38 | 183523.42 | 14759.63 | 77080.99 | 3798.25 | 6921.81 | -1015.38 | 347.64 | 347511.53 |

Table D.7 Results for the scenario E

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 98 | 474 | 1500 | 259 | 133 | 26 | 19 | 718 | 521 | |
| $r^*$ | 79 | 670 | 775 | 119 | 918 | 102 | 3 | 445 | 10 | |
| Expected Total Sales Revenue (ETSR) ($) | 329232.37 | 116094.90 | 488540.35 | 23125.44 | 169885.51 | 24905.77 | 307261.02 | 9261.29 | 6306.13 | 1474612.79 |
| Total Earning from Payment Term TPFS ($) | 10383.64 | 4555.31 | 7228.40 | 45.97 | 973.69 | 918.31 | 12998.59 | 453.09 | 131.51 | 37688.51 |
| Total Purchasing Cost (TPC) ($) | 298380.60 | 98174.88 | 249255.00 | 5283.60 | 83938.96 | 19791.20 | 298818.13 | 9764.80 | 5668.48 | 1069075.65 |
| Total Losing from Payment Term (TLPFC) ($) | 7638.19 | 4040.10 | 14167.67 | 201.19 | 1970.67 | 866.72 | 13365.85 | 322.29 | 73.15 | 42645.85 |
| Total Holding Cost (THC) ($) | 170.52 | 2479.20 | 20420.60 | 259.84 | 2350.95 | 204.66 | 1555.91 | 321.14 | 114.63 | 27877.45 |
| Total Ordering Cost (TOC) ($) | 510.00 | 120.00 | 30.00 | 60.00 | 460.00 | 440.00 | 130.00 | 40.00 | 40.00 | 1830.00 |
| Expected Total Shortage Cost (ETSC) ($) | 192.28 | 0.00 | 0.00 | 0.00 | 1193.50 | 11.04 | 266.20 | 0.00 | 14.88 | 1677.90 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 46.00 | 0.00 | 0.00 | 0.00 | 1085.00 | 8.00 | 11.00 | 0.00 | 62.00 | 1212.00 |
| Expected Total Profit (ETPR) ($) | 32724.42 | 15836.03 | 211895.48 | 17366.78 | 80945.12 | 4510.46 | 6123.51 | -733.85 | 526.50 | 369194.45 |

Table D.8 Results for the scenario F

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 29 | 250 | 250 | 64 | 102 | 26 | 2 | 164 | 28 | |
| $r^*$ | 97 | 252 | 1023 | 14 | 712 | 71 | 4 | 187 | 200 | |
| Expected Total Sales Revenue (ETSR) ($) | 332170.80 | 125635.26 | 409015.49 | 22929.02 | 158129.33 | 23708.13 | 320834.80 | 9723.80 | 6723.57 | 1408870.21 |
| Total Earning from Payment Term TPFS ($) | 10479.21 | 4770.67 | 6662.17 | 92.29 | 872.84 | 862.20 | 13577.47 | 441.59 | 142.72 | 37901.16 |
| Total Purchasing Cost (TPC) ($) | 301126.80 | 102816.09 | 229730.03 | 10608.00 | 75244.60 | 18581.93 | 312125.82 | 9516.94 | 6151.82 | 1065902.03 |
| Total Losing from Payment Term (TLPFC) ($) | 7706.36 | 4372.11 | 11861.45 | 199.48 | 1834.30 | 825.04 | 13956.31 | 338.39 | 77.99 | 41171.44 |
| Total Holding Cost (THC) ($) | 11.76 | 943.07 | 11779.62 | 93.07 | 1785.97 | 180.14 | 560.30 | 110.60 | 24.15 | 15488.67 |
| Total Ordering Cost (TOC) ($) | 510.00 | 183.00 | 125.00 | 130.00 | 457.00 | 354.00 | 260.00 | 153.00 | 517.00 | 2689.00 |
| Expected Total Shortage Cost (ETSC) ($) | 0.00 | 503.84 | 1437.93 | 16.43 | 1761.43 | 7.31 | 0.00 | 26.29 | 17.11 | 3770.34 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 0.00 | 365.10 | 370.60 | 53.00 | 1601.30 | 5.30 | 0.00 | 84.80 | 71.30 | 2551.40 |
| Expected Total Profit (ETPR) ($) | 33295.09 | 21587.82 | 160743.64 | 11974.33 | 77918.88 | 4621.90 | 7509.84 | 20.17 | 78.22 | 317749.89 |

Table D.9 Results for the scenario G

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 98 | 14 | 253 | 250 | 100 | 47 | 43 | 1362 | 41 | |
| $r^*$ | 138 | 872 | 1209 | 185 | 810 | 50 | 5 | 198 | 71 | |
| Expected Total Sales Revenue (ETSR) ($) | 329232.36 | 121552.92 | 415896.42 | 23125.44 | 171804.88 | 23448.42 | 320834.8 | 10038.07 | 6853.13 | 1422786.47 |
| Total Earning from Payment Term TPFS ($) | 10383.64 | 4680.80 | 6793.24 | 51.765 | 1044.132 | 889.49 | 13577.47 | 575.08 | 146.61 | 38142.26 |
| Total Purchasing Cost (TPC) ($) | 298380.6 | 100879.52 | 234249.84 | 5950 | 90011.43 | 19170.13 | 312125.82 | 12394.02 | 6319.64 | 1079481.02 |
| Total Losing from Payment Term (TLPFC) ($) | 7638.19 | 4230.04 | 12060.99 | 201.19 | 1992.93 | 816.00 | 13956.31 | 349.32 | 79.49 | 41324.49 |
| Total Holding Cost (THC) ($) | 170.52 | 707.904 | 13986.64 | 283.64 | 2244.93 | 196.71 | 4667.73 | 457.96 | 30.67 | 22746.72 |
| Total Ordering Cost (TOC) ($) | 510 | 520 | 114 | 70 | 461 | 160 | 60 | 26 | 322 | 2243 |
| Expected Total Shortage Cost (ETSC) ($) | 192.28 | 806.058 | 987.46 | 0 | 745.69 | 26.49 | 0 | 0 | 6.62 | 2764.60 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 46 | 584.1 | 254.5 | 0 | 677.9 | 19.2 | 0 | 0 | 27.6 | 1609.3 |
| Expected Total Profit (ETPR) ($) | 32724.41 | 19090.21 | 161290.72 | 16672.37 | 77393.02 | 3968.56 | 3602.40 | -2614.15 | 241.30 | 312368.88 |

Table D.10 Results for the scenario H

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 98 | 50 | 250 | 262 | 10 | 96 | 6 | 131 | 236 | |
| $r^*$ | 105 | 652 | 1331 | 137 | 996 | 157 | 5 | 265 | 144 | |
| Expected Total Sales Revenue (ETSR) ($) | 329232.37 | 129218.03 | 422848.48 | 23125.44 | 173337.72 | 23807.15 | 320834.80 | 10019.17 | 6934.96 | 1439358.12 |
| Total Earning from Payment Term TPFS ($) | 10383.64 | 5142.99 | 7479.62 | 46.50 | 1274.65 | 914.14 | 13577.47 | 468.69 | 155.60 | 39443.30 |
| Total Purchasing Cost (TPC) ($) | 298380.60 | 110840.27 | 257918.00 | 5344.80 | 109883.48 | 19701.24 | 312125.82 | 10101.06 | 6706.70 | 1131001.97 |
| Total Losing from Payment Term (TLPFC) ($) | 7638.19 | 4496.79 | 12262.61 | 201.19 | 2010.72 | 828.49 | 13956.31 | 348.67 | 80.45 | 41823.41 |
| Total Holding Cost (THC) ($) | 170.52 | 1452.98 | 18018.60 | 265.24 | 1829.96 | 591.94 | 568.92 | 148.20 | 115.63 | 23161.99 |
| Total Ordering Cost (TOC) ($) | 510.00 | 488.00 | 73.00 | 60.00 | 520.00 | 113.00 | 430.00 | 168.00 | 98.00 | 2460.00 |
| Expected Total Shortage Cost (ETSC) ($) | 192.28 | 238.60 | 532.34 | 0.00 | 631.84 | 0.00 | 0.00 | 1.58 | 0.00 | 1596.64 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 46.00 | 172.90 | 137.20 | 0.00 | 574.40 | 0.00 | 0.00 | 5.10 | 0.00 | 935.60 |
| Expected Total Profit (ETPR) ($) | 32724.42 | 16844.38 | 141523.56 | 17300.71 | 59736.37 | 3486.62 | 7331.22 | -279.65 | 89.78 | 278757.41 |

Table D.11 Results for the scenario I

| Products | Pro1 | Pro2 | Pro3 | Pro4 | Pro5 | Pro6 | Pro7 | Pro8 | Pro9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q^*$ | 98 | 214 | 77 | 116 | 500 | 55 | 19 | 174 | 69 | |
| $r^*$ | 138 | 827 | 202 | 189 | 494 | 35 | 4 | 153 | 46 | |
| Expected Total Sales Revenue (ETSR) ($) | 329232.37 | 124619.34 | 189323.46 | 23047.61 | 173217.76 | 22252.64 | 307261.02 | 9251.29 | 6562.29 | 1184767.78 |
| Total Earning from Payment Term TPFS ($) | 10383.64 | 5244.38 | 4675.33 | 178.43 | 1798.42 | 949.22 | 12998.59 | 419.99 | 138.90 | 36786.88 |
| Total Purchasing Cost (TPC) ($) | 298380.60 | 113025.38 | 161218.13 | 20508.80 | 155036.00 | 20457.25 | 298818.13 | 9051.48 | 5986.99 | 1082482.77 |
| Total Losing from Payment Term (TLPFC) ($) | 7638.19 | 4336.75 | 5490.38 | 200.51 | 2009.33 | 774.39 | 13365.85 | 321.94 | 76.12 | 34213.48 |
| Total Holding Cost (THC) ($) | 170.52 | 2304.33 | 1314.06 | 42.78 | 1986.77 | 119.83 | 1555.91 | 96.05 | 20.39 | 7610.65 |
| Total Ordering Cost (TOC) ($) | 510.00 | 306.00 | 378.00 | 520.00 | 226.00 | 215.00 | 130.00 | 153.00 | 319.00 | 2757.00 |
| Expected Total Shortage Cost (ETSC) ($) | 192.28 | 579.05 | 15820.31 | 6.51 | 640.75 | 114.82 | 266.20 | 65.81 | 30.17 | 17715.90 |
| Expected Total Shortage Quantity (ETSQ) (Unit) | 46.00 | 419.60 | 4077.40 | 21.00 | 582.50 | 83.20 | 11.00 | 212.30 | 125.70 | 5578.70 |
| Expected Total Profit (ETPR) ($) | 32724.42 | 9312.20 | 9777.90 | 1947.44 | 15117.33 | 1520.57 | 6123.51 | -17.01 | 268.51 | 76774.88 |