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NATURAL CONVECTION-DRIVEN PHASE CHANGE INSIDE CYLINDRICAL ANNULUS

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ABSTRACT. The convective heat transfer inside latent heat thermal energy storage systems governs the spatial and temporal variations and significantly depends on the tendency of density variation. For most fluids in nature, the density variation has a linear tendency, so the density reduces with increasing temperature. In contrast, water has a nonlinear behavior, and density variation has an inversion point near the phase change temperature. This peculiar behavior makes water's melting and solidification processes have a unique nature. The current study aims to develop an in-house code to analyze the natural convection-driven melting process within an annular gap and simulate melting behaviours of phase change materials with linear, paraffin, and nonlinear, water, density variations.

INTRODUCTION

Natural convection, the transfer of heat without any external force, has an important role in thermal processes, thermal energy storage systems, and especially phase-changing problems. In the thermal systems/processes, the changing density depending on the temperature occurs fluid movement by nature. For this reason, natural convection is a key effect that needs to be examined in designing more efficient and effective thermal systems/processes. In this direction, the natural convection phenomenon is examined in detail as experimentally and/or numerically in the heat transfer analysis literature. Kuehn and Goldstein [1976] carried out an experimental and theoretical-numerical study on the natural convection of air and water inside the cylindrical annulus and obtained the local heat transfer coefficients, velocity, and temperature distributions.

In thermal energy storage systems, thermal energy is stored as sensible heat, latent heat, chemical reaction heat, or the sum of these, depending on the change in the internal energy of the storage material. In cases where a large amount of energy is required to be stored, a huge storage volume/mass is required when sensible thermal energy storage systems are used. Besides, when a latent thermal energy storage system is used by choosing a phase-change material as the energy storage material under the required thermal conditions, there is a higher energy storage potential compared to the same volume of sensible energy storage. For this reason, the material to be used as a phase change material in latent thermal energy storage systems should have a suitable phase change temperature for the temperature range of the operating conditions [Dincer and Rosen 2021]. In this study, water and paraffin (*n*-octadecane) were chosen to be used in low- and high-temperature energy storage applications, respectively. However, water has a peculiar behavior called density inversion. The density inversion is the difference of density variations as a function of temperature, and 4°C is the density inversion temperature point of the water. The density of the water liquid increases up to 4°C, and beyond this point, density reduces as the temperature increases. Therefore, the effect of density inversion was considered when water is used as a phase change material in low-temperature storage applications in the literature [Seki et al. 1978; Braga and Viskanta 1992; Banaszek et al. 1999; Moraga and Vega 2004; Ezan and Kalfa 2017]. On the other hand, phase change materials such as n-octadecane are widely preferred in the literature, especially in

thermal energy storage applications at higher temperatures than the melting/solidification temperature of water [Ng et al. 1998; Darzi et al. 2012].

In thermal energy storage applications, besides the storage material, various features such as geometric dimensions and configurations of the storage tank are also important factors regarding cyclic thermal efficiency, ease of implementation, and costs for operation and maintenance. Although various configurations are still being designed and researched, thermal energy storage tanks which are commercially available generally have rectangular and cylindrical geometries. In order to achieve higher efficiency and lower cost, the configurations of TES tanks need to be optimized [Dincer and Ezan 2018]. This study develops a numerical code in 2D cylindrical coordinates to analyze the natural convection-driven phase-change mechanism of water and *n*-octadecane inside a shell-tube thermal energy storage unit.

MATERIAL AND METHODS

Definition of the Problem and Solution Method In this study, the melting process of the PCMs (water and paraffin) between two concentric cylinders is investigated. The schematic representation of the problem is illustrated in Figure 1.

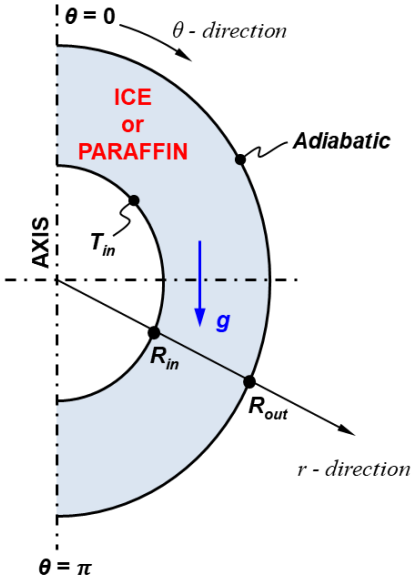


Figure 1. Schematic representation of the solution domain for the cylindrical annulus

The two-dimensional conservation equations in cylindrical coordinates are as follows:

Mass:

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial(u_\theta)}{\partial \theta}=0 \tag{1}$$

r-momentum:

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial r}-\frac{u_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\vartheta\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)-\frac{u_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right\}+\frac{\rho-\rho_{ref}}{\rho}g_r \tag{2}$$

θ-momentum:

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial \theta}+\vartheta\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right\}+\frac{\rho-\rho_{ref}}{\rho}g_\theta \quad (3)$$

Energy:

$$\frac{\partial(H)}{\partial t}+u_r\frac{\partial(H)}{\partial r}+\frac{u_\theta}{r}\frac{\partial(H)}{\partial \theta}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) \quad (4)$$

The temperature transforming method proposed by Cao and Faghri (1990) is implemented to carry out the phase change process. In this method, the relation between enthalpy and temperature is defined as;

$$H=CT+S \quad (5)$$

where C represents the volumetric heat capacity ($=\rho c$), and S is the source term. So the energy balance equation becomes as

$$\frac{\partial(CT)}{\partial t}+u_r\frac{\partial(CT)}{\partial r}+\frac{u_\theta}{r}\frac{\partial(CT)}{\partial \theta}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)-\frac{\partial S}{\partial t} \quad (6)$$

In the temperature transforming method, the phase change process takes place in a temperature range, mushy region, not at a specific temperature. The volumetric heat capacity and source term are defined separately for solid, mushy, and liquid regions as follows;

$$C=\begin{cases}\left(\rho c_p\right)_s & T\le T_m-\delta T_m\\ \left(\rho c_p\right)_m+\rho_l\Delta H/\left(2\delta T_m\right) & T_m-\delta T_m<T<T_m+\delta T_m\\ \left(\rho c_p\right)_l & T\ge T_m+\delta T_m\end{cases} \quad (7a)$$

$$S=\begin{cases}\left(\rho c_p\right)_s\left(\delta T_m-T_m\right) & T\le T_m-\delta T_m\\ \left(\rho c_p\right)_m\left(\delta T_m-T_m\right)+\dfrac{\rho_l\Delta H}{2}-\dfrac{\rho_l\Delta H T_m}{2\delta T_m} & T_m-\delta T_m<T<T_m+\delta T_m\\ \left(\rho c_p\right)_s\delta T_m-\left(\rho c_p\right)_l T_m+\rho_l\Delta H & T\ge T_m+\delta T_m\end{cases} \quad (7b)$$

where $T_m - \delta T_m$ and $T_m + \delta T_m$ stand for the solidus and liquidus temperatures, respectively. The thermophysical properties of the materials are given in Table 1. The temperature-dependent density of the water in the liquid phase is defined as [Ezan et al. 2011]

$$\rho_{liquid,water}=-0.007085T^2+3.925T+456.49 \quad (T \text{ in K}) \quad (8)$$

The temperature-dependent density of the paraffin liquid is defined in terms of the thermal expansion coeff.,

$$\rho_{liquid,paraffin}=\rho_{ref}\left(1-\beta\left(T-T_m\right)\right) \quad (9)$$

The Consistent Update Technique (CUT) algorithm is used to predict the transient heat transfer inside the annular gap during the melting process. The governing equations, i.e., mass, momentum, and energy, are reduced into algebraic sets of equations using the finite volume method of Patankar [1980]. Convection terms in the momentum and energy equations are discretized using the Power-Law scheme of Patankar [1980]. A uniform mesh structure is preferred along with the r- and θ-directions. After testing various alternative time-step sizes and mesh combinations, the total number of control volumes is decided to be 14400 (80×180), and the time-step is defined as 0.1 [s]. The iterative implicit solution algorithm is coded in MATLAB, and the convergence criterion is defined as 1E-5.

Table 1
Thermophysical Properties of Water and Paraffin

Property	Water [a]	Paraffin [b, c, d]
Density of solid, ρ_s (kg/m^3)	916.7	800
Density of liquid, ρ_{liq} (kg/m^3)	(Eq. 8)	(Eq. 9)
Thermal conductivity of solid, k_s (W/m·K)	2.26	0.358
Thermal conductivity of liquid, k_{liq} (W/m·K)	0.56	0.148
Specific heat of solid, c_s (J/kg·K)	2040	1912
Specific heat of liquid, c_{liq} (J/kg·K)	4205	2160
Melting temperature, T_m (°C)	0.0	27.5
Latent heat of fusion, ΔH (J/kg)	332400	243500
Thermal expansion coefficient, β (1/K)	-	9×10^{-4}
Dynamic viscosity, μ (Pa·s)	1.519×10^{-3}	3.09×10^{-3}

[a][Bergman et al. 2011], [b][Lacroix 1993], [c][Balikowski et al. 2007], and [d][Fukai et al. 2000]

Boundary Conditions It is assumed that the PCMs are initially subcooled by 1°C, and the inner wall is suddenly increased through a prescribed temperature and kept constant throughout the process. The outer wall of the annular space is adiabatic. The relevant conditions are defined below,

Initial condition

$$t = 0 \;\rightarrow\; T(r,\theta) = T_m - 1 \tag{10}$$

Boundary conditions

$$t>0 \;\rightarrow\; -k\left.\frac{dT}{dr}\right|_{r=R_{out}} = 0, \qquad T\left(r=R_{in},\theta,t\right) = T_{in} \tag{11}$$

RESULTS AND DISCUSSION

Validation A four-step validation process is implemented in order to ensure the validity of the developed code. At first, the problem of Kuehn and Goldstein (1976) is reproduced. Kuehn and Goldstein investigated the steady-state natural convection process of air inside the cylindrical annulus. The comparisons for the variations of the dimensionless temperatures along radial positions at various angular positions are provided in Figure 2.

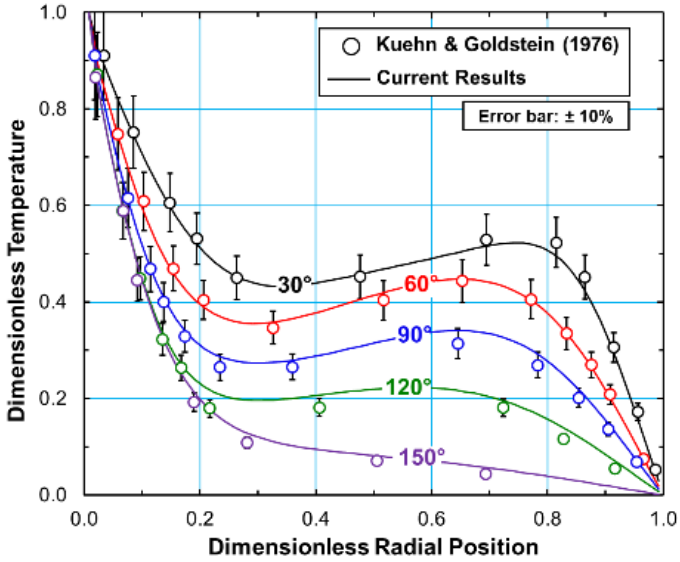


Figure 2. The comparison of the radial dimensionless temperature variations at various angular positions

As a second phase, the work of Li et al. (2011) reproduced, in which the natural convection process of water in the cylindrical annulus, including the density inversion region, is numerically resolved. The comparisons of the Nusselt number variations for several geometric aspect ratios and surface temperature differences are illustrated in Figure 3(a). At the next step, the problem of melting ice around cylinder studied by White et al. (1986) is reproduced, and time-wise variations of dimensionless liquid volume at different surface temperatures are compared in Figure 3(b). At the last stage of the validation process, the problem of Ng et al. (1998), the melting process of paraffin inside a cylindrical annulus, is examined, and comparative results of the streamlines and temperature contours distributions are given in Figure 4. According to this four-step validation process, it can be deduced that the predicted results of the developed mathematical model are in good harmony with the literature.

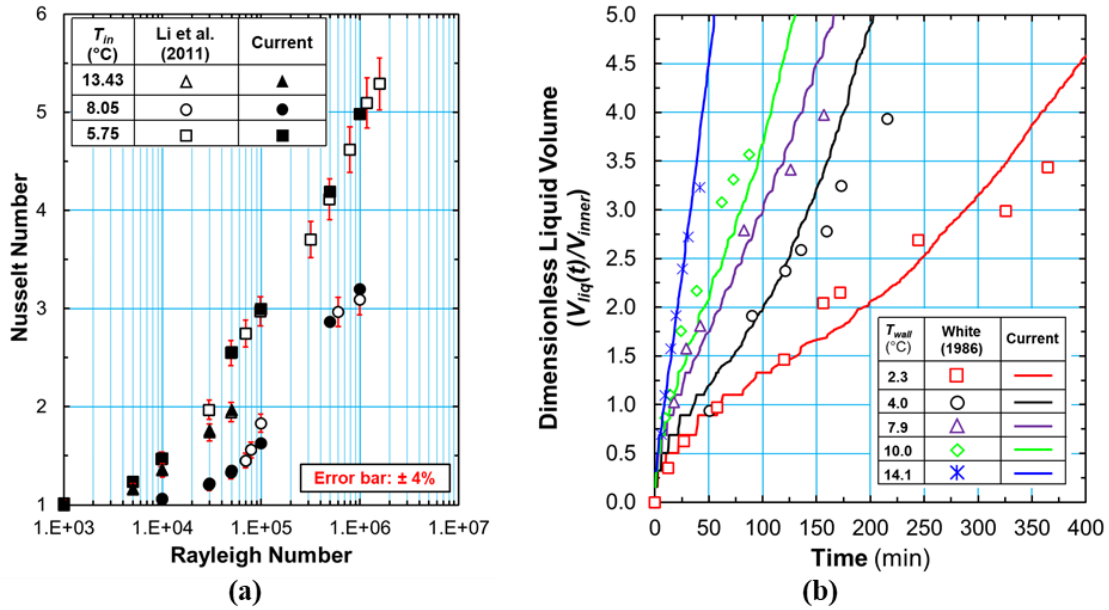


Figure 3. The comparison of Nusselt Number variation for various Rayleigh Number (a) and the comparison of time-wise variation of dimensionless liquid volume for several surface temperatures (b)

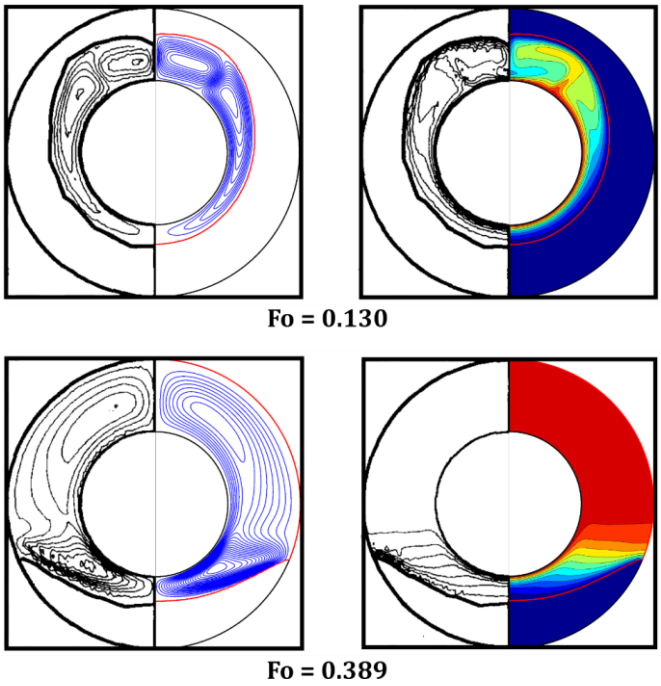


Figure 4. The comparison of the streamlines and temperature contours for different dimensionless times [Left: Streamlines (literature-current study) – Right: temperature contours (literature-current study)]

Parametric Studies Parametric analyzes were carried out using the validated numerical model in which natural convection phenomena were observed for both water and paraffin with selected studies from the literature. Three different inner surface temperature boundary conditions, depending on the melting temperature of both water and paraffin, were applied, and the time-dependent average PCM temperature was obtained as in Figure 5. In terms of average PCM temperature and melting rate, the results showed that the temperature of the paraffin material increased faster than the water, and accordingly, faster melting occurred. Due to the high computational cost, the results are given in the full-text for limited durations.

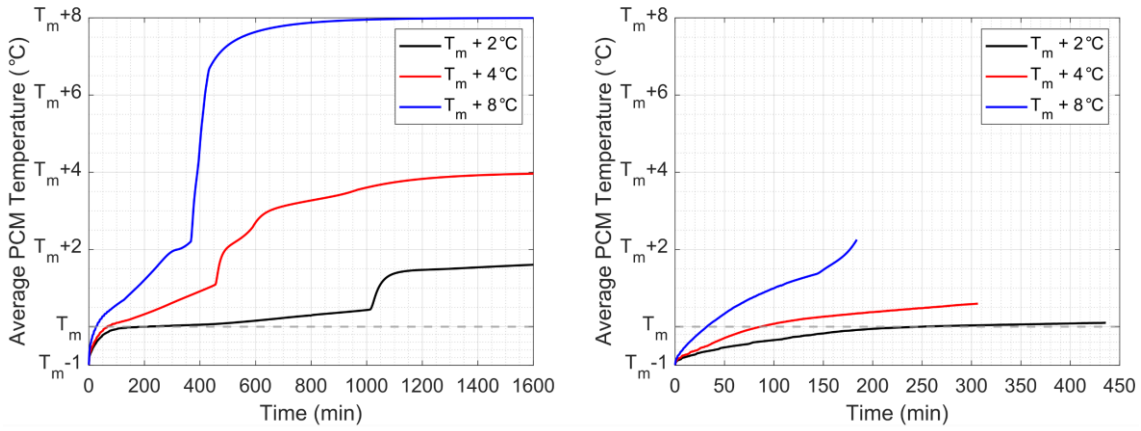


Figure 5. Time-wise variation of average PCM temperature for various surface temperatures using water (a) and paraffin (b)

In addition to the time-dependent variations, temperature contours (isotherms) and streamlines for three selected times were presented in Figure 6. For water, at t = 100 min, even though two discrete circulation cells appear, the dominant heat transfer mechanism is conduction, and the isotherms are almost uniform

around the cylinder. Beyond this time, the intensity of the natural convection increases, and the accelerated melting rate increases both average PCM temperature and liquid fraction. As the water temperature increases, the high-temperature water moves downwards and improves the melting rate at the lower part of the cylinder. A single clockwise circulation cell is observed when the water is completely melted, *i.e.*, liquid fraction reaches unity. Thus, the temperature stratification disappears, and distribution becomes uniform. Besides, for paraffin, the results at t = 20 min show that the dominant heat transfer mechanism is convection, and multiple circulation cells at the upper region of the cylinder improve the melting rate significantly. The interface moves upwards in advancing time, multiple circulations cells unite, and two counter currents are observed. Notice that at t = 100 min, the liquid fraction is around 0.315 for paraffin; besides, for water, the liquid fraction is around 0.190 for the same moment. The high Rayleigh number and lower latent heat of fusion of paraffin improve the rate of melting.

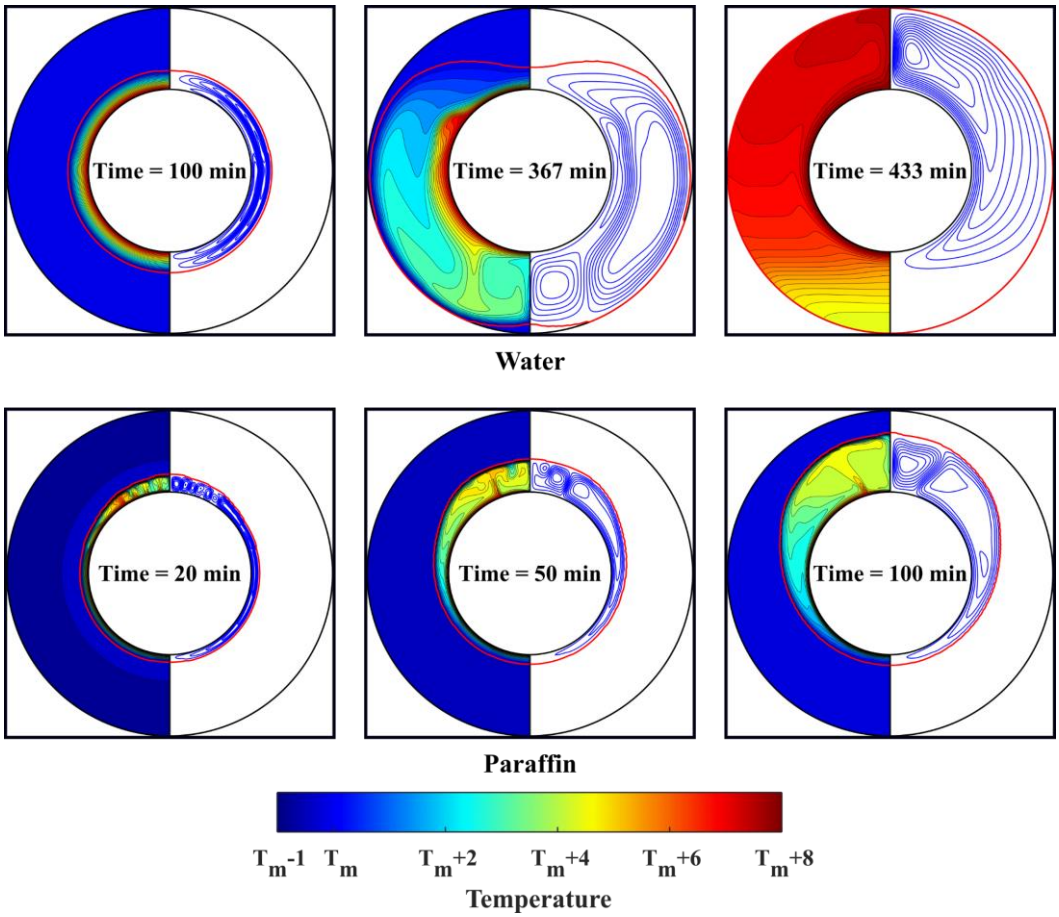


Figure 6. Isotherms (left side) and streamlines (right side) using water and paraffin at selected times in the cylindrical annulus

CONCLUSION

In the current study, an in-house finite-volume solver is developed in the MATLAB environment to predict the transient melting problem of PCMs inside the annular gap. The validity of the code is proved by repeating various numerical/experimental studies from the literature. A parametric set of simulations are carried out for water and paraffin, which have nonlinear and linear density variations, respectively, to highlight the importance of natural convection behavior on the melting rate. The results show that the selected paraffin has a high melting rate than water at the initial period. At high inner cylinder temperature, the direction of the dominant natural convection current differs and leads to a complete melting at the end of the process.

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