Dynamical response of the Ising model to the amplitude modulated time dependent magnetic field

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Abstract

The dynamical Ising model under the effect of the amplitude modulated time dependent periodic magnetic field has been solved by using EFT with Glauber type of stochastic process. Several cases with amplitude modulation have been investigated. It has been shown that, amplitude modulation could display dynamical phase transition on the magnetic system.

Keywords: **Dynamic Ising model; amplitude modulated field, dynamical phase transition**

1 Introduction

Dynamical response of the spin system to the time dependent magnetic field has attract attention both theoretically and experimentally. Relation between the relaxation time of the spin system and period of the driving periodic external magnetic field determines the dynamic phase of the system. Dynamic phase transition (DPT) in these systems come from the competition between these two time scales [1].

Theoretically DPT in spin systems first observed within the mean field approximation (MFA) [2] for the spin-1/2 Ising model. After that time, DPT in the spin-1/2 Ising model has been widely studied within the several techniques such as MFA [3], Monte Carlo simulation (MC) [4], effective field theory (EFT) [5]. Dynamical character was mostly included within the Glauber-type stochastic process [6] to these techniques.

Experimentally, DPT can be observed in several systems such as, ultrathin Co films [7], Fe/Au(001) films [8], epitaxial Fe/GaAs(001) thin films [9], fcc Co(001), and fcc NiFe/Cu/Co(001) layers [10] and Fe/InAs(001) ultrathin films [11].

Theoretical efforts continuing with different type of spin systems. Higher spin Ising models such as spin-1 Blume-Capel model [12] and spin-3/2 Ising model [13] have been studied as well as spin systems with quenched disorder; random fields [14], random crystal fields [15], bond dilution [16] and site dilution [17]. Besides dynamical properties of the Ising model on different geometries have been studied. Nanotube [18], nanowire [19], thin film [20] and Bethe lattice [21] geometries are among them. Although not as common as Ising model, dynamical properties of the other models do exist. Heisenberg model [22] and Ashkin-Teller [23] model have been studied by means of MC and EFT with Glauber type of stochastic process, respectively.

On the other hand, special attention was paid on the different types of external time dependent magnetic fields. These different types of driving fields are promising diverse dynamical phase transition and hysteresis properties. Ising ferromagnets under the effect of the pulsed [24, 25] and randomly varied external time dependent fields [26] have been studied within the MC. For the system with pulsed field, it has been shown that, ratio of time width of the response (i.e. time dependent magnetization) of the system to the time width of the pulse field diverges at the critical temperature of the static system [25]. Besides, ratio between the response magnetization peak height and the pulse height gives peak near this temperature [25]. For the randomly varied field it has been shown that, if the interval of the distribution of which random field chosen large enough, dynamically disordered phase can be created [26]. These results have been obtained within the MC simulation and by solving MFA equations created with Glauber type of stochastic process. Randomly varied (fast switching) external field problem was also studied with slightly different approach [27, 28]. Besides, effect of the polarized electromagnetic wave [29] and field which have gradient [30] inspected by the same methods. Other examples of this category are, standing wave [31] and propagating wave type of fields [32, 33].

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Indeed dynamical spin system under the different type of fields deserves more attention, due to the rich dynamical behavior and possible experimental applications. For this aim this work is devoted to the exploration of the dynamical properties of the Ising model which drives with external periodic magnetic field with amplitude modulation. Amplitude modulation is mostly used in communication systems and it is not a difficult task to obtain modulated signal. The formulation is EFT with Glauber type of stochastic process [6]. The paper is organized as follows: In Sec. 2 we briefly present the model and formulation. The results and discussions are presented in Sec. 3, and finally Sec. 4 contains our conclusions.

2 Model and Formulation

Let the periodic magnetic field with frequency ω $H_0 \cos\omega t$ acts on the Ising spin system. Amplitude modulated magnetic field can be defined by

$$H(t) = [H_0 + r(t)] \cos(\omega t), \tag{1}$$

where $r(t)$ is the time dependent field. Here, the term $H_0 + r(t)$ is called as modulating field, while $H(t)$ given by (1) as modulated field. Choosing modulating field as sinusoidal form with frequency ω_m is named as tone modulation in communication literature[34]. Then the total magnetic field is given by

$$H(t) = [H_0 + H_1 \cos(\omega_m t)] \cos(\omega t). \tag{2}$$

The (Fourier) spectrum of this field consists of frequencies $\pm\omega, \omega \pm \omega_m, -\omega \pm \omega_m$.

Although, the period of the magnetic field is not necessary for finding the time series of the magnetization, in order to calculate the dynamical order parameter we need the period of the magnetic field. In general, let $f(x)$ and $g(x)$ be the periodic functions with periods p and q, respectively. The period of product (or sum) of these two functions is defined by $r = ap = bq$ (the smallest common multiple), where a and b are the positive integers. Note that r need not to be the smallest period, but this could not change the value of the dynamical order parameter.

The Hamiltonian of the dynamical Ising model is given by

$$\mathcal{H} = -J \sum_{<i,j>} S_i S_j - H(t) \sum_i S_i, \tag{3}$$

where the first summation is over the nearest neighbors of the lattice, while the second one is over all the lattice sites. Here, S_i is the z component of the spin variables at a site i, J is the exchange interaction and $H(t)$ is the external magnetic field which is given by Eq. (2).

Glauber-type stochastic process [6] can be used for investigating dynamic properties of the considered system. In general, in the Glauber type of stochastic process (as done in Ref. [2] for the mean field approximation) the thermal average (denoted with $\langle\rangle$) of a spin variable S_i, which can take values ± 1 can be given as

$$\theta \frac{d\langle S_i \rangle}{dt} = -\langle S_i \rangle + \left\langle \frac{Tr_i S_i \exp(-\beta \mathcal{H}_i)}{Tr_i \exp(-\beta \mathcal{H}_i)} \right\rangle. \tag{4}$$

Here, θ is the single spin flip rate per unit time, $\beta = 1/(k_B T)$, k_B and T denote the Boltzmann constant and temperature, respectively. Tr_i stands for the trace operation over the site i. Also $\mathcal{H}_i$ denotes the part of the Hamiltonian of the system related to the site i, which is given by,

$$\mathcal{H}_i = -S_i \left(J \sum_{j=1}^{z} S_j + H(t) \right) = -S_i (h_i + H(t)) \tag{5}$$

where z is the number of nearest neighbor sites and h_i is the local field that represents the nearest neighbor interactions of a site i. Inserting Eq. (5) into Eq. (4) yields,

$$\theta \frac{d\langle S_i \rangle}{dt} = -\langle S_i \rangle + \left\langle \frac{Tr_i S_i \exp(\beta S_i (h_i + H(t)))}{Tr_i \exp(\beta S_i (h_i + H(t)))} \right\rangle. \tag{6}$$

Performing trace operation over degrees of freedom $S_i = \pm 1$

$$\theta \frac{d\left\langle S_i \right\rangle}{dt} = -\left\langle S_i \right\rangle + \left\langle \tanh\left(\beta\left(h_i + H(t)\right)\right)\right\rangle \tag{7}$$

can be obtained. Since the effect of the exponential differential operator on any function $F(x)$ is defined by

$$\exp\left(a\nabla\right) F(x) = F(x+a), \tag{8}$$

for any constant a and $\nabla = d/dx$, Eq. (7) can be written by within differential operator technique [35] as

$$\theta \frac{d\left\langle S_i \right\rangle}{dt} = -\left\langle S_i \right\rangle + \left\langle \exp\left(h_i \nabla\right)\right\rangle f(x + H(t))|_{x=0}, \tag{9}$$

where

$$f(x) = \tanh\left(\beta x\right). \tag{10}$$

The last term in Eq. (9) can be evaluated within the decoupling approximation and this will yield

$$\theta \frac{dm}{dt} = -m + \sum_{n=0}^{z} A_n m^n, \tag{11}$$

where $m = \langle S_i \rangle$ and

$$A_n = \frac{1}{2^z} \left(\begin{array}{c} z \\ n \end{array} \right) \sum_{r=1}^{z-n} \sum_{s=0}^{n} \left(\begin{array}{c} z-n \\ r \end{array} \right) \left(\begin{array}{c} n \\ s \end{array} \right) (-1)^s f\left[(z - 2r - 2s) J\right] \tag{12}$$

This first order differential equation can be solved by standard methods such as the Runge-Kutta method [36].

Dynamical order parameter of the system can be defined as integration of the time dependent magnetization over one period (P) of the magnetic field,

$$Q = \frac{1}{P} \oint m\left(t\right) dt. \tag{13}$$

Since the determination of the period of the modulated magnetic field (defined in Eq. (2)) will yield multiples of the period (nP, where n is integer), integral in Eq. (13) will be taken over the time nP. In this case P will be replaced by nP in Eq. (13).

3 Results and Discussion

The dimensionless quantities given by

$$\tau = \frac{k_B T}{J}, h_0 = \frac{H_0}{J}, h_1 = \frac{H_1}{J}, h(t) = \frac{H(t)}{J}. \tag{14}$$

will be used throughout the work. Our investigation will be focused on square ($z = 4$) lattice. We set $\theta = 1$ throughout our numerical calculations.

3.1 Unmodulated case

In order to construct the foundation of our discussion about the amplitude modulation, let us review unmodulated case briefly, i.e. $H_1 = 0.0$ in Eq. (2). We can see several time series of the magnetization for different values of the Hamiltonian parameters and temperature in Fig. 1. On each figure the values of these parameters have been denoted. Time series are obtained by Runge-Kutta method which was used for the solution of Eq. (11). In Fig. 1 (a) the system is in a dynamically disordered state, the value of the dynamical order parameter $Q = 0.0$, magnetization oscillates around the zero value. When the value of the frequency increases, the system becomes

unable to follow the magnetic field, then the system becomes ordered. This situation can be seen in Fig. 1 (b). Also, we can see by comparing Fig. 1 (a) with (c) that, decreasing amplitude of the field has similar effect. Decreasing amplitude means, decreasing energy supplied to the system by magnetic field, thus the spin-spin interaction overcome this effect. Lastly, decreasing temperature, due to the decreasing thermal fluctuations may result in ordered phase. This last observation can be seen by comparing Fig. 1 (a) with (d). These are very well known results in the literature.

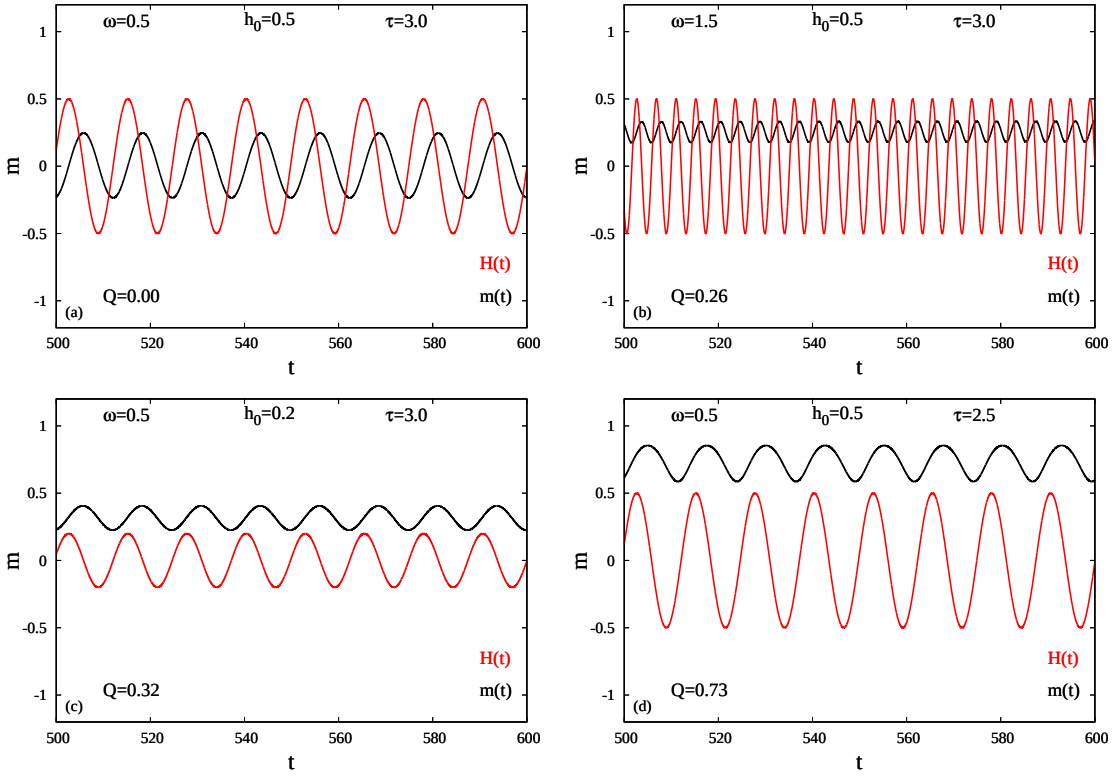


Figure 1: Time series of the magnetization for the unmodulated field ($h_1 = 0.0$) with selected values of frequency, amplitude and temperature.

3.2 Modulated case with $h_0 = 0.0$

Simpler case of modulation given by Eq. (2), is the case with $H_0 = 0.0$, named as multiplier modulation. The (Fourier) spectrum of this field consists of frequencies $\omega \pm \omega_m$, $-\omega \pm \omega_m$. Since the aim is to determine the effect of the modulation, we have fixed the frequency of the unmodulated field as $\omega = 1.0$ and $h_1 = 0.5$. Note that, due to the fact that field consist of product of two cosines, it will be enough to inspect the modulation frequency range $0.0 < \omega_m < 1.0$. The effect of the modulation can be seen in Fig. 2, where are time series of the field and magnetization for several values of the modulation frequency ω_m has been plotted. The value of the dynamical order parameter Q is given in each case. As seen in Fig. 2, changing the frequency of the modulation changes the behavior of the magnetization drastically. For lower values of the ω_m, both of field and magnetization have wave packet like behavior (see Fig. 2 (a) and (b)). System is in an ordered state with the value of dynamical order parameter $Q = 0.77$. When the value of the modulation frequency rises, the view of both field and magnetization changes while the value of the order parameter is the same (see Fig. 2 (c)). If the value of the ω_m still increases, the value of the order parameter starts to decrease (compare Fig. 2 (c) with (d)) and drop to zero (see Fig. 2 (f)).The modulation could create dynamical phase transition for fixed values of amplitude and temperature.

For a closer look at the variation of the dynamical order parameter with the modulation fre-

quency we depict it for several values of temperature and amplitude, which can be seen in Fig. 3. At first sight symmetry about the $\omega_m = 1.0$ takes attention. In general this symmetry is about the $\omega_m = \omega$ (remember that we have fixed the value of the modulation frequency as $\omega_m = 1.0$).

As seen in Fig. 3 abrupt change of value of dynamical order parameter to the value of zero, leave place to smooth change when the temperature rises (compare curves related to the $\tau = 1.5$ with $\tau = 1.8$ in Fig. 3 (a) or curves related to the $\tau = 1.0$ with $\tau = 1.5$ in Fig. 3 (b)). Second effect of the rising temperature is to widen the plateau of $Q = 0.0$.

3.3 Modulated case with $h_0 > 0.0$

This case includes fields that satisfy $H_0 > 0.0$. We can see from (2) that, the field term $h_0 \cos(\omega t)$ survives, regardless of the choice of ω_m and h_1.

In order to determine the behavior of the magnetization with modulation frequency we depict the time series of the applied modulated field and magnetization for selected values of modulation frequency, as can be seen in Fig. 4. Other parameter values are fixed as $h_0 = 0.25, h_1 = 0.25, \omega = 1.00$ and $\tau = 2.50$. As seen in Fig. 4, rising modulation frequency drastically change the behavior of magnetization in time, as in the case of modulation with $h_0 = 0.0$. Again, for lower frequencies, wave packet like behavior takes attention. Rising frequency changes this behavior. In contrast to the case $h_0 = 0.0$, modulation with $h_0 > 0.0$ could not create dynamically disordered phase (compare Fig. 2 (e), (f) with Fig. 4 (e), (f)). This is due to surviving term $h_0 \cos(\omega t)$ regardless of the value of modulation frequency.

4 Conclusion

The dynamical Ising model under the effect of the amplitude modulated time dependent periodic magnetic field has been solved by using EFT with Glauber type of stochastic process. Since DPT in time dependent periodic magnetic field is well known, values of some Hamiltonian parameters fixed, such as frequency and temperature. Detailed investigation on time series constructed with several amplitude modulation case has been carried out. It has been shown that in the multiplier amplitude modulation case, rising modulation frequency could create DPT. The modulation frequency and frequency of the magnetic field has inverse effect on dynamical phases of the system. While rising modulation frequency could create dynamically disordered phase (as shown in this work), rising frequency drag the system to the ordered phase. This is because of the fact that, since the frequency gets higher, magnetic system could not follow the driving field. Thus we can say that when the modulation frequency gets higher, magnetic system has the chance to follow the driving modulated magnetic field.

We hope that the results obtained in this work may be beneficial form both theoretical and experimental point of view.

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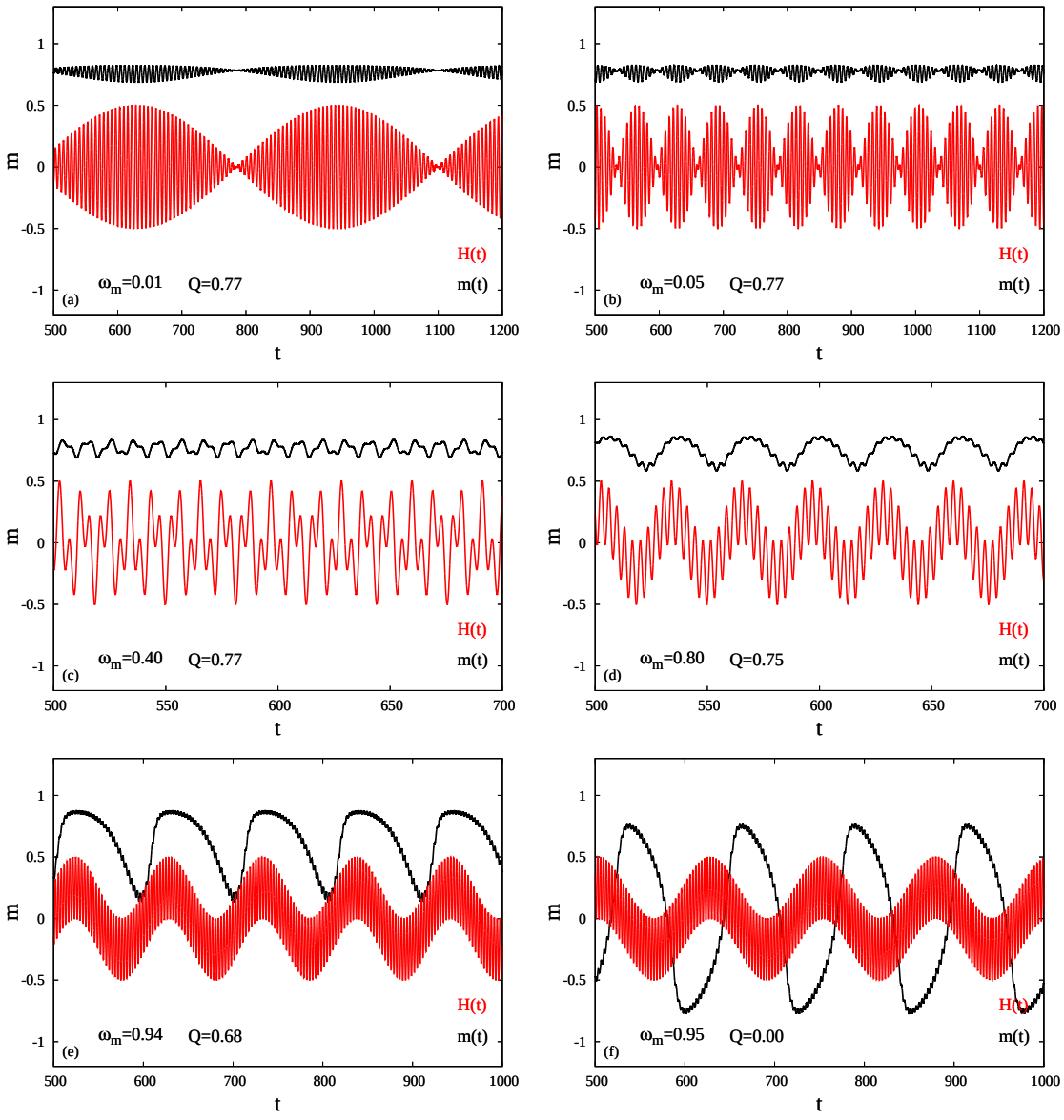


Figure 2: Time series of the magnetization for the modulated field with selected values of frequency ω_m. Other parameter values fixed as $h_0 = 0.00, h_1 = 0.50$, $\omega = 1.00$ and $\tau = 2.50$

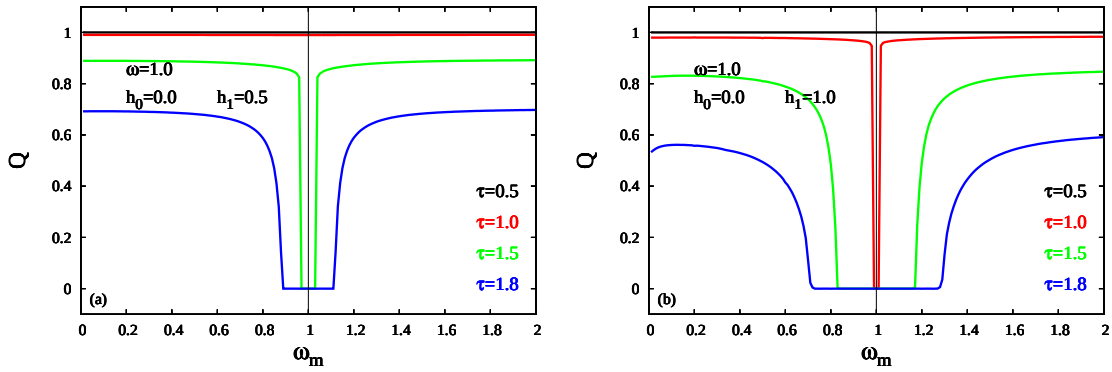


Figure 3: Variation of the dynamical order parameter with the modulation frequency for the parameter values of $\omega = 1.0$, $h_0 = 0.0$ and for chosen values of the temperature for amplitudes (a) $h_1 = 0.5$ and (b) $h_1 = 1.0$

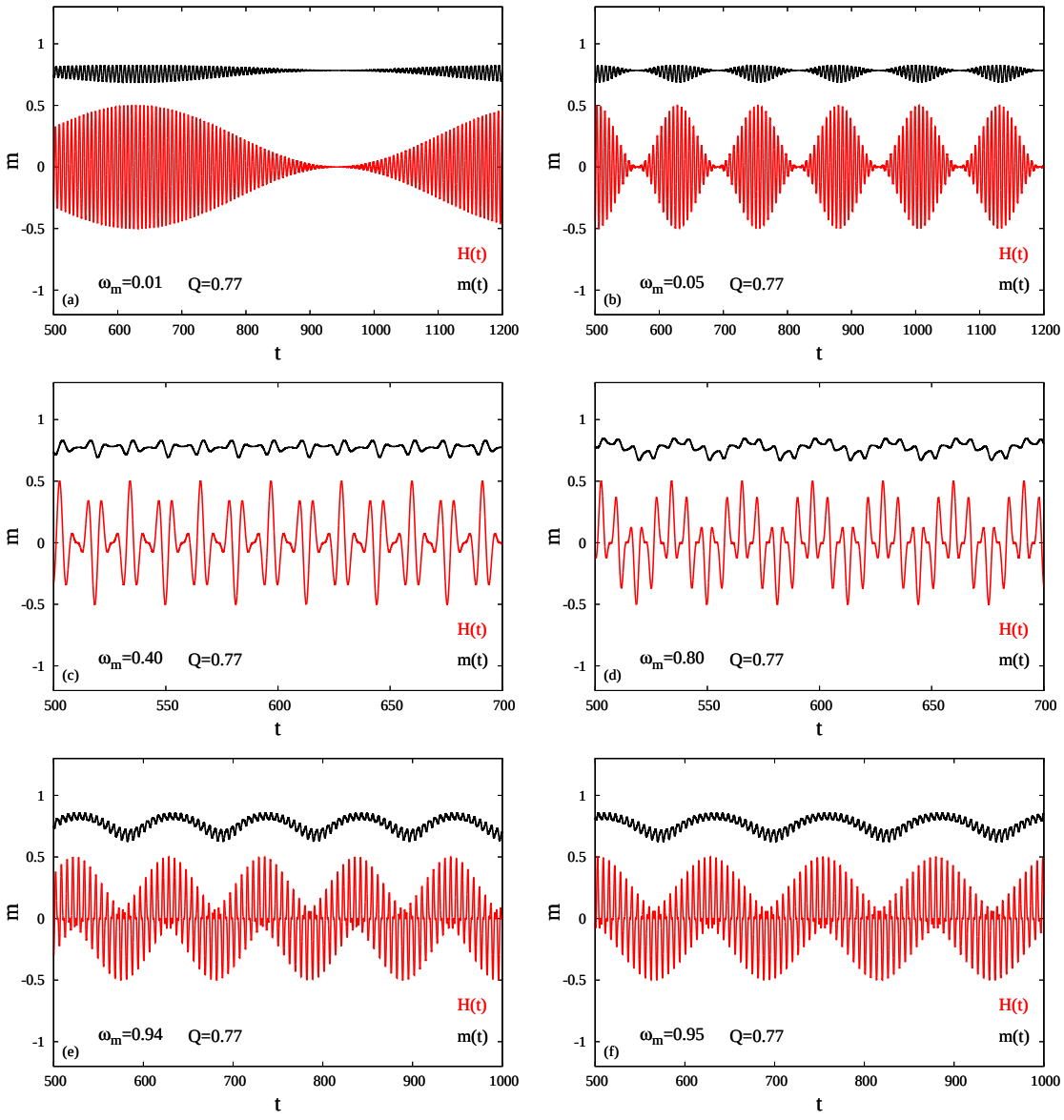


Figure 4: Time series of the magnetization for the modulated field with selected values of frequency ω_m. Other parameter values fixed as $h_0 = 0.25, h_1 = 0.25$, $\omega = 1.00$ and $\tau = 2.50$