Journal of Mathematics Teacher Education
https://doi.org/10.1007/s10857-026-09745-2

ORIGINAL PAPER

Exploring a mathematics teacher educator's knowledge and learning in designing and implementing a teacher training course on digital resource design

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Received: 26 March 2025 / Revised: 9 January 2026 / Accepted: 21 January 2026
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Abstract
This self-study examines a mathematics teacher educator's (MTE) knowledge and learning while designing and implementing a teacher training (TT) course on the design and integration of digital resources for exploring mathematical concepts with technology-added value. The study provides insights into the MTE's objectives, course design, and the challenges encountered by both the MTE and her students. It focuses on the enacted pedagogical content knowledge (PCK) of the MTE, its development, and its relationship to technological aspects, particularly in the context of digital resource design and integration. Grounded in the mathematics teacher educators' pedagogical content knowledge (MTEPCK) framework, the study draws primarily on the MTE's self-reflections and organizational notes, supplemented by students' work. Data analysis reveals the MTE's diverse teaching strategies, knowledge of examples and resources, and understanding of student thinking, while underscoring the specific role of technological knowledge. Key challenges include integrating a conceptual framework for digital task design, guiding students toward designing exploratory instead of practice-based digital resources, and managing the broad scope of the course. The findings emphasize the complexity of designing and implementing a technology-based TT course, which requires the MTE's comprehensive PCK while adding new layers of complexity to MTE knowledge. The study suggests extending the MTEPCK framework to include categories specific to technological knowledge and underscores the value of self-study and reflective practice for MTE professional development.

Keywords Mathematics teacher educator · Teacher training · Technology integration · Digital resources · Digital task design

Introduction

Mathematics teacher educators (MTEs) are crucial in shaping the pre-service mathematics teachers' (PMTs) practices, particularly in integrating technology into mathematics teaching (Abboud et al., 2020; Thurm et al., 2024). With an increasing demand for tech-

Extended author information available on the last page of the article

Published online: 17 March 2026

nology-enhanced pedagogies, designing and implementing courses that support PMTs in creating effective, technology-based instructional tasks have become increasingly significant (Clark-Wilson et al., 2023). Central to this effort is the multifaceted expertise of MTEs, including a robust foundation in technology, content, and pedagogy, along with hands-on experience, reflective practice, and ongoing professional development. Understanding how MTEs develop this expertise, particularly regarding technology integration, is crucial but remains under-researched compared to other facets of teacher education (Beswick & Goos, 2018). Within the context of teacher training (TT), earlier studies on MTEs have primarily focused on PMTs' practice development rather than MTEs' own learning processes (Chapman, 2008a; Jaworski, 2008). Although research on MTEs is expanding, it is still relatively recent and underdeveloped; focused examination of MTEs—especially regarding their professional growth and adaptation to evolving technologies—requires greater attention (Chorney et al., 2025; Krainer et al., 2021). A recent review on TT with technology further highlights that, while extensive research exists on teachers' learning, limited attention has been given to the learning of MTEs as facilitators (Thurm et al., 2024). Addressing this gap could provide critical insights to support and advance the work of MTEs in their role as educators of educators.

Building on these findings, our study examines how an MTE plans and implements a TT course on technology integration for PMTs, addressing the need for MTEs to enhance teachers' professional knowledge and practice in this field (Bozkurt & Yiğit Koyunkaya, 2022, 2023; Bowers & Stephens, 2011; Niess, 2005, 2012; Thurm et al., 2023). Specifically, it explores what professional knowledge of an MTE (the first author - Edith) is activated and the potential professional learning that occurs through this process, particularly as the MTE supports PMTs in designing and implementing digital resources[1]. This self-study investigates the MTE's considerations, actions, experiences, and reflections during course design and implementation. Conducted at an Austrian teacher education university, it examines the MTE's pedagogical strategies and approaches for guiding PMTs in designing digital resources for specific mathematical concepts.

The central research question is: How does an MTE apply and develop pedagogical content knowledge (PCK) while implementing a TT course aimed at supporting PMTs in designing digital resources that (a) focus on mathematical exploration and (b) provide technology-added value? In particular, the study investigates which elements of the MTE's PCK become salient in her practice during the course implementation, as well as how the MTE's professional learning may emerge through the process of supporting PMTs in designing these resources, with particular focus on the role of technology in the course. To explore this, we examine the teaching strategies employed by the MTE, how the MTE considered and supported PMTs' knowledge and thinking about digital resources, the types of resources used, and their integration into the course.

Despite growing recognition of the importance of technology integration in mathematics teacher education, limited research has examined how MTEs themselves apply and develop their professional knowledge while designing and enacting technology-based TT courses. This gap is particularly consequential, as MTEs are expected not only to model effective uses of technology, but also to support PMTs in developing approaches to designing digital resources that add mathematical value rather than serving merely as technical add-ons. By

[1] In this paper, a *digital resource* denotes a GeoGebra-based interactive material accompanied by tasks, whereas *(digital) tasks* refer to prompts or questions designed to guide pupils' engagement with the material.

closely examining an MTE's planning, enactment, and reflection during a technology-based TT course, this study addresses this gap and provides fine-grained insight into the professional knowledge enacted and developed in practice.

Mathematics teacher educators' knowledge and professional development.

Over the past two decades, research on MTEs' self-learning and professional development, particularly regarding their knowledge and teaching practices, has grown significantly (Beswick & Goos, 2018; Even et al., 2020). These studies highlight how MTEs reflect on what, whom, and how they teach, while also considering their own professional identities in interactions with mathematics teachers (Beswick & Goos, 2018; Helliwell & Ebbelind, 2024; Suazo-Flores et al., 2025). MTEs' knowledge both encompasses and extends that of mathematics teachers, integrating content knowledge with insights into how teachers engage with and convey mathematics in their practice (Goos & Beswick, 2021; Zaslavsky & Leiken, 2004). This broader perspective has been defined as a form of meta-knowledge: "*knowledge for teaching knowledge for teaching mathematics*" (Goos & Beswick, 2021, p. 3).

An early illustration of this perspective is Nicol's (1997) self-study, in which she analyzed her knowledge and practice, revealing the complexities, uncertainties, and contradictions of teaching and learning about teaching mathematics. She argued that such challenges provide valuable learning opportunities while underscoring the necessity of ongoing reflection on one's own teaching practice. Her study highlighted the importance of MTEs engaging in reflection, considering PMTs' experiences, refining instructional strategies, fostering meaningful dialogue, and incorporating diverse perspectives to strengthen the quality and effectiveness of teacher education (Nicol, 1997). Similarly, Tzur (2001) emphasized that reflection enables MTEs to develop awareness of the underlying perspectives that guide their practice. He stated that by reflecting on what it means to teach mathematics, how teachers learn to teach it, and how their actions influence others' learning of mathematics teaching, MTEs can intentionally shape how others develop their understanding and practice of teaching mathematics. In his later study, Tzur (2008) described how MTEs can benefit from recognizing the paradoxical nature of learning—situations where lessons may appear confusing or counterintuitive at first, yet ultimately support deeper understanding. This perspective highlights not only the importance of noticing such moments and adjusting instruction to guide PMTs through them, but also of situating reflection as an epistemologically grounded process that reinforces its central role in MTEs' professional identity and development. Chapman (2008b) also emphasized the role of reflection, describing it as a process of "examining, framing, and attempting to solve the dilemmas of classroom practice; and being aware of and questioning the assumptions and values he or she brings to teaching'' (p. 121).

A recurring theme across these and recent studies (e.g., Goos & Beswick, 2021; Kastberg et al., 2025; Suazo-Flores et al., 2025) is the central role of reflection on knowledge and teaching practices in supporting the development of MTEs' PCK (Beswick & Goos, 2018; Chick & Beswick, 2018; Zaslavsky & Leiken, 2004). Research has further sought to specify distinctive dimensions of MTEs' PCK. For instance, Escudero-Ávila et al. (2021) identified unique components—including knowledge of mathematics teacher professional development, program content, and standards—which can be developed through reflective practice. They highlighted its more holistic nature compared to mathematics teachers' PCK,

noting that it integrates perspectives on mathematics learning, the use of teaching and learning theories to empower teachers, understanding pre-service teachers' learning processes and conceptual challenges, knowledge of teacher education standards, effective strategies for training future educators, and methods for evaluating program outcomes. To facilitate MTEs' PCK development within a constructivist teaching approach, promote teachers' reflective practice, and support professional community building, Zaslavsky and Leiken (2004) developed a conceptual framework. This framework examines MTEs' professional growth through practice through a three-layer model: challenging content for mathematics teachers, sensitivity to mathematics teachers, and management of mathematics teachers' learning. It has influenced subsequent studies by Chick (2007), Chick et al. (2006) and Chick and Beswick (2013, 2018), shaping their focus on mathematics teachers' PCK for teaching school mathematics and MTEs' PCK for teaching the PCK for teaching mathematics (Chick & Beswick, 2013, 2018), which serves as theoretical foundation for this study. Together, these studies underscore the centrality of reflection in MTEs' professional learning, highlighting it as a mechanism for navigating challenges, confronting dilemmas, and making sense of paradoxes in teaching. Building on this perspective, we adopt reflection as the primary lens for examining an MTE's learning and, following Beswick and Goos (2018), "conceive MTE learning as reflective practice" (p. 421). In this sense, reflection is framed as a process through which the MTE's PCK not only informs but also evolves in response to the challenges of supporting PMTs in a TT course on digital technology integration. Reflection thus serves both as a tool for professional growth and as a driver of ongoing refinement and expansion of the MTE's PCK.

A growing body of research examines MTEs' knowledge and professional development regarding the design and implementation of mathematics methods or content courses (e.g., Li & Superfine, 2018; Appova & Taylor, 2019; Masingila et al., 2018). While these studies provide important insights into MTEs' PCK, research specifically addressing their knowledge, teaching practices, and professional development related to digital tools—particularly in the context of designing technology-based TT courses—remains limited, despite some studies focusing on MTEs' professional development with digital tools. For instance, Psycharis and Klogeria (2018) designed a professional development program in Greece, using a cyclical sequence of observation, reflection, design, and implementation to prepare MTEs for integrating digital tools into mathematics teaching. Their findings revealed that the program's structured design facilitated MTEs' professional growth. Abboud et al. (2020) examined a French program that bridged research and practice by adapting didactical research for MTEs' ongoing development. This two-year program emphasized generalist skills in the first year and technology in the second, requiring MTEs to collaborate by sharing experiences, practices, and research outcomes. Both studies examine MTEs' professional development through structured programs that integrate theory, practice, and reflection—elements also embedded in our TT course. However, they emphasize MTEs' professional growth in explicitly designed development programs, rather than how MTEs learn *through* designing and conducting technology-based TT courses. Other studies provide complementary insights into MTEs' technological readiness and development. Mukuka and Alex (2024) investigated Zambian MTEs' readiness to integrate digital tools by evaluating their perceived technological proficiency, familiarity, usefulness, and ease of use. Their survey study found that although technological proficiency and ease of use were limited, MTEs valued digital technology and were committed to preparing PMTs for its effective integra-

tion. Thurm et al. (2023) introduced the Content-Activity-Technology-Model, encompassing content goals, mathematical activities, and technology use, to conceptualize educators' [facilitators'] and teachers' noticing when utilizing multi-representational tools. They also developed a video-case-based strategy to support MTEs in fostering teachers' recognition of student learning through technology. While these studies address aspects of MTE professional development, they focus on specific activities or contexts rather than the design and implementation of a technology-based TT course. Consequently, insights into how MTEs apply and develop their knowledge through the implementation of such courses remain scarce. In their recent study, Thurm et al. (2024) further highlighted the limited research on MTEs' technological professional development and called for additional studies.

Taken together, these studies highlight important elements of MTEs' professional development related to technology, such as structured design, reflection, collaborative learning, and engagement with technology-focused tasks. However, much of this work examines MTE learning within explicitly designed professional development programs or through isolated activities, rather than through the ongoing work of designing and enacting technology-based TT courses. As a result, little is known about how MTEs' PCK is activated, challenged, and potentially developed through the everyday practice of supporting PMTs in designing digital resources for mathematics teaching. Addressing this gap, this study examines how an MTE applies and potentially develops her PCK while designing and implementing a technology-based TT course, with particular attention to how she supports PMTs in creating digital resources that promote mathematical exploration and offer technology-added value. By focusing on the MTE's reflections alongside her instructional decisions, the study provides insight into professional learning processes that are often implicit yet central to effective mathematics teacher education.

Conceptual framework

To examine the MTE's professional knowledge, we drew on an established framework that specifies the knowledge domains unique to teaching PMTs. We employed Chick and Beswick's (2018) mathematics teacher educators' pedagogical content knowledge (MTEPCK) framework described as "*PCK for teaching the PCK for teaching mathematics*" (Chick & Beswick, 2018, p. 476). This framework was developed to support the analysis of the knowledge and skills required for effectively preparing PMTs to teach mathematics. Its origins lie in Chick and Beswick's (2013) *School Mathematics Teacher PCK (SMTPCK)* framework, itself informed by Shulman's (1986, 1987) notion of PCK. In their 2018 paper, Chick and Beswick revised and extended the SMTPCK framework by reorganizing components and elaborating categories specifically for MTEs. Since our focus is on how the MTE supported PMTs in designing and implementing digital resources, the MTEPCK framework is particularly well-suited. It offers a structured way to examine the PCK that MTEs draw on when preparing PMTs to develop their own PCK while highlighting the pedagogical and content-specific considerations central to this work. Using the MTEPCK framework in this context allows us not only to describe aspects of MTE practice, but also to examine how technology-related PCK for teacher education is constructed and refined through reflective engagement with course design and enactment.

The framework comprises three interrelated domains: (1) *Clearly PCK*, (2) *Content knowledge in a pedagogical context*, and (3) *Pedagogical knowledge in a content context*, with each domain further specified into components. This study focuses on the *Clearly PCK* domain, as it captures the MTE's strategies, reflections, and decisions most directly related to supporting PMTs during the design and integration of digital resources. Table 1 outlines its components, drawing directly from Chick and Beswick (2018, pp. 479–482), which we used both to frame our observations and to code the MTE's reflections during analysis. While Chick and Beswick (2018) originally formulated these categories to capture *observable* aspects of MTE practice, we refined the wording to encompass the MTE's own reflections and reasoning. This adaptation aligns the framework with our self-study methodology, which draws on both actions and self-reports as data sources. Accordingly, we interpreted the categories not only as external indicators (e.g., "the educator discusses…"), but also as internal processes visible in the educator's reflections (e.g., "the educator recognized…" or "the educator questioned…").

In this study, we used MTEPCK as a conceptual lens and analytical tool. Conceptually, it framed our understanding of the knowledge demands of teaching PCK to PMTs in a technology-based mathematics education context. Analytically, it structured our coding of data by helping us trace the MTE's pedagogical strategies, her attention to PMTs' thinking, and her integration of curriculum and resources. For example, when the MTE integrated a technology-specific framework into her teaching that should support PMTs' digital resource design, we coded this under "knowledge of resources". When her reflection highlighted the varying complexity of designing a digital resource for exploration depending on the mathematical topic (e.g., multiplying fractions, properties of a parallelogram), we coded this under "cognitive demand of task".

Table 1 Clearly PCK components of the MTEPCK framework (adapted from Chick & Beswick, 2018, pp. 479–482, refined for self-study methodology)

PCK category	Evident when the MTE …
Clearly PCK	
Teaching strategies	Selects, uses, or reflects on strategies for teaching an SMTPCK concept
Student thinking	Engages with or reflects on PMTs' ways of thinking about an SMTPCK concept
Student thinking - misconceptions	Identifies, addresses, or reflects on PMTs' misconceptions about an SMTPCK concept
Student affect (in relation to content)	Considers or reflects on PMTs' affective responses to SMTPCK ideas
Cognitive demand of task	Analyzes or reflects on aspects of (MTE) tasks that influence complexity
Representations of concepts	Uses or reflects on ways to illustrate an SMTPCK concept
Explanations	Develops, analyzes, or reflects on explanations of an SMTPCK concept or procedure
Knowledge of examples	Chooses or reflects on examples that highlight an SMTPCK concept
Knowledge of resources	Uses or reflects on resources that support SMTPCK teaching
Curriculum knowledge	Connects or reflects on links between SMTPCK topics and the teacher education curriculum
Purpose of content knowledge	Reflects on reasons for including SMTPCK content in the curriculum

Applying the framework in this way provides a structured basis for examining how the MTE applied and developed PCK during the design and enactment of the course. By extending the coding categories to include reflective reasoning, the analysis enables us to systematically trace the MTE's strategies, decisions, and reflections, highlighting instances of PCK in action and providing a foundation for exploring potential knowledge development throughout the course. In doing so, it contributes to mathematics teacher education research by advancing understanding of how MTEPCK related to technology emerges through course design and implementation, and by offering empirically grounded insights that can inform the design of future teacher education programs.

Methods

To explore the complexities of MTEs' knowledge for teaching PMTs and their role in supporting PMTs' learning and change, mathematics education researchers emphasize self-understanding and self-methodologies to analyze their experiences to gain a deeper understanding of their knowledge and practice to enhance it (Chapman, 2008a; Chapman et al., 2020; Goos & Beswick, 2021). Among these, self-study aims to enhance one's own practice, focusing on examining oneself as a teacher within a specific context (LaBoskey, 2004). Chapman (2008a) connects self-study to the "notions of beliefs as a basis of what teachers know and do and why they do it." (p. 1). She highlights its dual role as both a valuable teaching tool for educators and a learning method for PMTs, considering beliefs as a basis of knowledge, disposition, action, change, and learning.

In this study, we adopted self-study to examine the first author's professional knowledge, particularly PCK, and her potential professional learning through reflection as she supported PMTs' professional development of technology integration through a TT course, thereby enhancing our understanding of essential MTE knowledge. Like Kastberg's self-study (Chapman et al., 2020), Edith considered two knowledge categories: (1) "knowing in relation to self" (p. 165), and (2) "knowing in relation to prospective teachers to describe how awareness of each of these ways of knowing supported [her] own learning to teach about teaching" (p. 165).

The participant

The first author, Edith, is the participant under examination. She has taught at an Austrian university college for education for 11 years, holds a master's degree in mathematics and physics education for secondary level and a PhD in mathematics education (focused on digital resources for functional thinking). Before transitioning to teacher education, she taught for eight years, including one year using a CAS calculator and three years in laptop-equipped classes with GeoGebra, a Dynamic Mathematics Software. As a teacher educator, she has presented digital resources for visualizing mathematical concepts and used spreadsheets for modeling but has limited experience in teaching *with* technology. However, she has researched technology use for eight years and participates in a national project for developing digital resources for lower-secondary mathematics, collaborating with technology-experienced teachers and teacher educators.

The context of this study is a course on mathematics education, attended by 18 PMTs in a master's program in secondary mathematics education.

The course design

The course focused on developing digital resources for lower-secondary education (grades 5–8; ages 10–14), supporting PMTs in assessing and designing digital resources, implementing design ideas with technology, and designing lesson plans integrating them. The 13-week course (90-min sessions/week) had the following objectives:

1. Analyzing mathematical knowledge for teaching (Ball et al., 2008) on a secondary school topic (grades 5–8) and assessing tasks and resources in the context of digitalization.
2. Designing two digital resources using GeoGebra.
3. Developing and micro-teaching a teaching sequence integrating the developed digital resource(s).

For the design of the digital resources, the MTE articulated two central goals: to focus on *exploring* mathematical concepts rather than practicing skills (see Drijvers, 2018) and to provide *technology-added value* compared to traditional paper-and-pencil tasks. Drawing on ongoing research (Lindenbauer et al., 2022), the course incorporates four key design elements: (i) providing condensed information on mathematics education and technology along with a checklist to guide the digital resource design, (ii) requiring intensive literature review on mathematical knowledge for teaching, (iii) offering technical support, and (iv) encouraging peer support and discussions with experienced teachers and digital experts. Table 2 details the objectives, structure, and requirements of the course.

The course consists of three phases: Preparing, designing and redesigning, and implementation (see Fig. 1). Edith conducted the course in the winter term 2022/23. As the MTE, she was responsible for its implementation and for systematically reflecting on her teaching practice and the course content based on PMT inputs, exploration, and reflections.

In phase one (weeks 1–5, weekly 90-min lectures), the PMTs were informed about course organization and objectives and paired (a team) to select a topic from lower-secondary curriculum from a topic list. Two or three teams formed a workgroup around related topics. This phase included lectures and tasks on mathematical knowledge for teaching (mainly a review of bachelor course contents), technology in mathematics education (e.g., design principles, added value of technology), a GeoGebra workshop, and the Trocki and Hollebrands' (2018) Dynamic Geometric Task Analysis (DGTA) framework, adapted to general mathematics topics. Each team had to write an analysis of mathematical knowledge for teaching the chosen topic based on mathematics education literature, curriculum, and schoolbooks. To support the PMTs, Edith provided a summary document covering relevant aspects of mathematical knowledge for teaching and technology in mathematics education. During one 90-min lesson, Edith introduced the Digital Task Analysis (DTA) model (Lindenbauer et al., 2023; see Table 3), an adaptation of Trocki and Hollebrands' (2018) DGTA framework for non-geometric topics. Like the DGTA framework, the DTA model should support teachers in (re-)designing tasks that engage pupils in mathematically meaningful activities and assessing digital resources based on these activities. It retains two dimensions

Table 2 Summary of course

Objectives	● Develop two digital resources per team ● Develop and test a teaching sequence using digital resources ● Create cognitively activating, "mathematically rich" tasks to promote conceptual understanding ● Reflect on development and learning process
Structure	● Teams of 2 persons ● Workgroup combined 2–3 teams with similar topics ● Meetings, discussions, and reflections with: ○ Workgroup colleagues ○ Experienced teachers ○ MTE
Requirements	● Developing two digital resources ● Conducting specified meetings and micro-teaching ● Timely submission of the following documents: ○ Analysis of mathematical knowledge for teaching on chosen topic ○ Documentation of the digital resource development process, including accompanying tasks and meeting records ○ Documentation of lesson planning ○ Homework ○ Learning diary

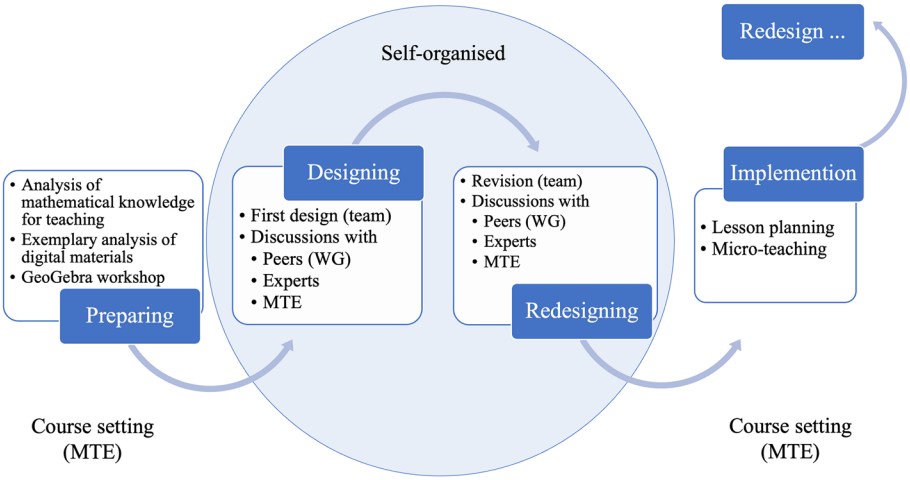


Fig. 1 Course design

(mathematical depth, technological actions) but streamlines the technological dimension by merging related categories to better capture technological actions relevant to non-geometric topics. Thus, the DTA model allows teachers to evaluate instructions within a digital resource along these two dimensions to create cognitively activating and higher-quality digital tasks (Lindenbauer et al., 2023; Trocki & Hollebrands, 2018).

Table 3 Digital task analysis (DTA) model (from Lindenbauer et al. (2023), pp. 489–490)

Allowance for mathematical depth	
Levels	*Description: Prompt requires …*
N/A	… a technology task with no focus on mathematics.
0	The digital resource does not have mathematical fidelity.
1	… to recall a mathematical fact, rule, formula, or definition. The cognitive demand of the prompt corresponds to *memorization.*
2	… to report information from the digital resource or to use a procedure without connections to concepts or meaning. The student is not expected to provide an explanation.
3	… to consider the mathematical concepts, procedures, or relationships in the digital resource.
4	… to explain mathematical concepts, procedures, or relationships in the digital resource.
5	… to go beyond the current digital resource and generalize mathematical concepts, processes, or relationships, or to make and test assumptions. The cognitive demand of the prompt corresponds to *doing mathematics*.
Types of technological action	
Code	*Description: Prompt requires*
N/A	… no action within the digital resource.
A	… a technological action whose result could also be achieved without technology. (*Note: imitation of working with paper & pencil*)
B	… dragging or using other dynamic aspects of the digital resource.
C	… a manipulation within the digital resource that enables the recognition of emergent invariant relationship(s) or pattern(s) among or within mathematical objects. (*Note: continuous experimentation*, *hypothesize: find out 'something' (by chance)*)
D	… a manipulation within the digital resource(s) that may surprise or cause one to refine thinking based on themes within the surprise when exploring the relationships represented. (*Note: continuous exploration*, *test hypotheses: examine purposefully*)

In phase 2 (weeks 6–9), PMTs self-organized their workload, completing two cycles of digital resource design (see Fig. 1). Each cycle included a design and/or creation phase and a review phase. Teams discussed and reflected their ideas within the workgroup and with an expert teacher experienced in teaching lower-secondary mathematics. Edith also met with each workgroup to discuss their work and provide detailed feedback. In phase three (weeks 10–13), PMTs integrated their digital resources into lesson plans, presented in the course as micro-teaching sessions. Although micro-teaching sessions were planned, time constraints and the fact that they unfolded as presentations rather than as actual micro-teaching limited their relevance for this study. Consequently, they do not contribute substantially to the findings and are not addressed further in this paper. The final meeting summarized and discussed relevant design criteria and the overall course.

Data collection and analysis

The primary data for this study consisted of Edith's self-reflection and organizational notes, which documented her evolving course design, underlying rationale, and decision-making. This self-reflection protocol included notes on course content, learning objectives, requirements for PMTs, course structure, a table summarizing the content and focus of each lesson, additional comments, task-related notes, and reflections on Edith's rationale for course planning. Post-lesson reflections addressed her impressions, what worked well, and areas for improvement. Together, these reflections served as the main data for investigating Edith's PCK and potential professional learning processes; through these data, the analysis aimed to study Edith as a teacher in context, identify specific challenges, and examine the development of her professional knowledge. To contextualize and triangulate these reflections, we incorporated selected supplementary data from PMTs' learning diaries, digital resources, and video recordings of meetings. These data did not serve as independent sources of analysis but acted as mirrors of Edith's practice, demonstrating how her course design and instructional decisions unfolded in practice and how PMTs' responses were considered in her subsequent reflections. For example, when PMTs' digital resources or learning diaries highlighted challenges or successes, these outcomes informed Edith's reflections. One PMT, for instance, highlighted struggles in aligning tasks with the learning objectives, which provided an opportunity for Edith to examine the scaffolding provided by the DTA model and to consider adjustments for future course iterations. To preserve confidentiality, the participating PMTs are anonymized and identified alphabetically (e.g., Student A).

For data analysis, the first author translated the self-reflection protocol into English. The first author and two co-authors then analyzed the protocol separately using the document analysis method (Bowen, 2009). Data were coded deductively using a refined version of Chick and Beswick's (2018) MTEPCK framework, which provided a structured analytical lens to identify indicators of Edith's PCK and to analyze how this knowledge was enacted during the course design and implementation process. To capture technology-related knowledge aspects identified in the data, we coded these aspects inductively to provide a more comprehensive understanding of MTE knowledge in technology-based teaching. For example, when Edith explained a way of dynamically visualizing percentages to make the numerical concept more accessible and to leverage technology for dynamic exploration, this was coded as *dynamic representations of concepts*. Subsequently, the authors compared their codings, discussed and condensed the analyses until consensus was reached, and integrated the final coding into MAXQDA (Fig. 2 illustrates a final coding example). In the results section, each coded segment is identified by its paragraph number as "pos. xx". This phase identified the elements of Edith's PCK that became salient in her practice and provided a basis for investigating her potential professional learning through documented moments of challenge and reflection.

In the second phase of analysis, we applied data triangulation by incorporating selected PMT-related data following grounded theory guidelines (Charmaz, 2014) to validate and enrich insights from the self-reflection analysis. Consistent with theoretical sampling, we selected five learning diaries, three digital resource designs, transcriptions from two resource discussions, and one lesson plan. This selection was informed by an initial assessment of the digital resource designs, which identified distinct PMT groups. Specifically, we selected

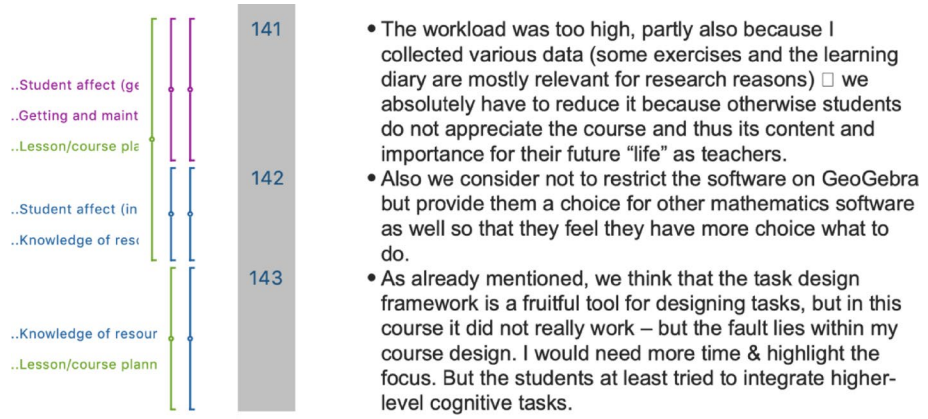


Fig. 2 Example of MAXQDA coding

three pairs with varying outcomes—exemplary (B&R), satisfactory (M&O), and below-average (E&I). To further triangulate findings from Edith's self-reflection, we included two resource discussions (B&R, M&O), along with team B&R's lesson plan and learning diaries for a complete dataset of one team. For a balanced perspective, we also incorporated learning diaries from two average PMTs (C&D) and one high-performing PMT (K). We selected these diverse data sources because contributions of PMTs with varying experiences—reflected in their learning diaries, digital resource designs, and resource discussions—offer a comprehensive perspective on Edith's actions and reflections on the different challenges encountered during the course. As with the self-reflection protocol, these data were translated into English and coded using the MTEPCK framework to identify aspects of Edith's enacted knowledge, along with moments of her practice that prompted self-reflection. These sources provided a perspective on how her course design and implementation unfolded, thus confirming, nuancing, or contradicting her own perceptions. Findings were then structured into episodes to examine what happened in practice, identify salient PCK elements, and investigate how specific tensions and challenges contributed to Edith's professional learning.

Collecting multiple data types from different participants enabled triangulation, strengthening the validity of this study. Constant data comparison and rich description should ensure validity (Creswell, 2013). To enhance reliability, we followed Creswell's (2013) recommendations by obtaining detailed field notes and using MAXQDA for data analysis. Framework of this study was employed as a codebook, with data analyzed separately before comparing and discussing results until reaching a consensus (Patton, 2014). This study adhered to the ethical standards of the American Educational Research Association (AERA). Voluntary informed consent was obtained from PMTs after full disclosure of the study's purpose, procedures, and their right to withdraw at any time. Data were anonymized and stored securely, ensuring confidentiality and preventing disclosure to third parties. Care was taken to avoid any harm, exploitation, or discrimination throughout the research, and to maintain transparency and integrity in all aspects of data collection, analysis, and dissemination (AERA, 2011).

Results

Edith's central objective was to prepare PMTs for integrating digital technology in mathematics education by enhancing their digital skills, enabling them to select, design, and effectively plan the use of digital resources. Guided by the MTEPCK framework (Chick & Beswick, 2018), the data analysis resulted in themes related to Edith's PCK, particularly her *teaching strategies*, *knowledge of resources and examples*, and understanding of *student thinking*[2]. The first two categories are especially comprehensive, reflecting her emphasis on informed course design.

This section begins with an overview of Edith's *teaching strategies* as an entry point to her MTEPCK, without delving into detailed analysis. Subsequently, the results focus on two key aspects of the course: her adoption of the DTA model and the digital resource discussions. These two aspects demonstrate Edith's *knowledge of resources and examples* and her perspectives on *student thinking*. Both are closely tied to her two primary goals for digital resource design: designing digital resources that enable exploratory learning and exploit the technology-added value. Through these key aspects, we analyzed her MTEPCK in action, illuminating the challenges and tensions that evoked her ongoing professional learning while focusing on the role of technology in the course.

Organizing the course: overview of teaching strategies

The course consisted of three phases: preparation, design and redesign, and implementation (see Sect. "The course design"). Within these phases, data analysis revealed the following *teaching strategies* that Edith implemented, aligned with the weekly planned course content and goals (see Fig. 3).

Edith's main objective in the preparation phase was to activate and enhance PMTs' knowledge of digital resource design and integration, fostering a shared understanding and knowledge base. She employed the TPACK framework (Mishra & Koehler, 2006) to structure relevant knowledge, which integrates PMTs' PCK, technological knowledge, knowledge about integrating technology into mathematics teaching (e.g., design principles, exemplary digital resources), and knowledge about digital resource design. In this phase, Edith delivered lectures incorporating open-ended questions, chosen for their time-efficiency in reviewing foundational knowledge, to stimulate classroom discussions, and to activate PMTs' prior knowledge. Furthermore, she included an assignment to enhance PMTs' PCK on a specific mathematical topic and a GeoGebra workshop to promote technological knowledge. The design and redesign phases incorporated project-based, cooperative, and collaborative learning to engage participants actively through hands-on design tasks. The final phase, lesson planning, aimed to activate and enhance PMTs' knowledge for integrating digital resources into mathematics teaching and simulated implementation via micro-teaching.

As an overarching *teaching strategy* across all three course phases, Edith employed framework-based teaching, integrating theoretical models from practice-oriented mathematics education and research-based frameworks. These encompassed a range of top-

[2] To maintain readability, categories from the MTEPCK framework (Chick & Beswick, 2018) are indicated in *italics* in the results without repeated citation.

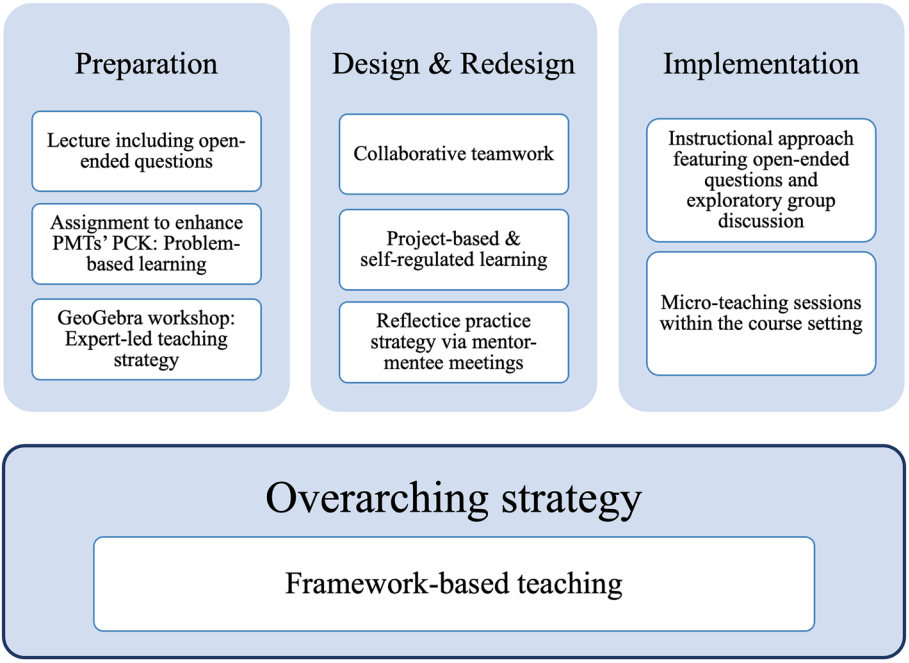


Fig. 3 Implemented teaching strategies

ics, from structuring mathematical knowledge in the preparation phase of the course to aspects relevant during digital resource design, such as the DTA model, and to models that addressed lesson planning in the implementation phase. This strategy aimed to support PMTs in making well-founded decisions about selecting and designing digital resources, formulating tasks, choosing teaching methods, and planning lessons while enhancing their teaching quality and professionalism. In this way, the use of frameworks reflects Edith's expertise in MTEPCK and technology integration, particularly her *knowledge of resources and examples*.

The DTA model is presented as a specific case of framework-based teaching, which is central to the results for several reasons. First, it serves as the central model because it addresses two course objectives (see Table 2): developing digital resources, including creating cognitively activating tasks. Additionally, its integration illustrates Edith's knowledge and learning during the preparation and design/redesign phases. Most importantly, the DTA model directly relates to her two goals for digital resource design through its two dimensions. The first dimension, mathematical depth, encourages PMTs to design tasks that engage pupils more deeply with mathematical content, thereby enhancing the exploratory potential of digital resources. The second dimension, technological action, supports PMTs in leveraging the dynamic affordances of GeoGebra to achieve technology-added value compared to traditional paper-and-pencil tasks.

DTA model as knowledge resource: adoption and challenges in framework-based teaching

This section examines Edith's adoption of the DTA model within the TT course. Drawing on the MTEPCK framework (Chick & Beswick, 2018), this theoretical model selected by Edith as part of her framework-based teaching approach illustrates her knowledge of *teaching strategies* and represents an aspect of her *knowledge of resources*. The section focuses specifically on the DTA model, analyzing its integration into the course, its role in supporting PMTs' task designs, and the challenges that emerged in integrating the model fruitfully and that prompted Edith's reflection and learning.

Discovery and adaptation

During course preparation, Edith as an MTE recognized a gap in her PCK in the sense that she needed to develop structured guidance and strategies to support PMTs in designing tasks that engage pupils with digital resources in mathematically meaningful ways. Drawing on her experiences working with PMTs in a project on digital resource design, as well as discussions with colleagues, she realized that PMTs often struggled to design tasks that accompanied digital resources in ways that fostered pupils' engagement, promoted mathematical thinking, and supported pupils in systematizing emerging mathematical knowledge. Owing to her dual role as both MTE and mathematics education researcher, she actively engaged with colleagues and research literature to find structured guidance for this challenge. This effort enabled her to identify suitable frameworks, such as Smith and Stein's (1998) classification[3] of tasks by cognitive demand and Trocki and Hollebrands' (2018) DGTA framework, which builds on and extends Smith and Stein's work with a specific focus on technology use. The DGTA framework aims to support teachers in designing cognitively activating tasks that leverage the added value of technology. She thought that employing the structure and ideas from these frameworks in her course would be particularly suitable since they align closely with the two specific goals for the digital resource design in this course.

When preparing the TT course, Edith considered which mathematical topics to include for the PMTs. Since the GeoGebra software originated as a dynamic geometry system, many existing resources already focused on geometry, and the software particularly supports exploration of geometric objects. Therefore, she prioritized non-geometrical topics (e.g., fractions, percentages, whole numbers) to broaden the PMTs' scope in designing digital resources. She believed that resources on non-geometrical topics "are more difficult to design … and wanted the PMTs to really consider how technology can support pupils", and she further reflected: "I believe that if a task is a bit more difficult, PMTs can learn more because they have to think more deeply … and be more creative" (self-reflection, pos. 72). However, because the DGTA framework focuses solely on geometrical tasks, Edith adapted it to address a broader range of mathematical topics, resulting in the DTA model (Lindenbauer et al., 2023; see Table 3). For this adaptation, the categories for mathematical depth remain unchanged, apart from the addition of Smith and Stein's (1998) formulations to better support PMTs. However, the technological dimension is streamlined by conceptualizing actions in terms of whether technology replaces, amplifies, or transforms paper-and-pencil activities (Hughes et al., 2006). For instance, the DTA model integrates the three geometric-specific prompts of "drawing", "measuring", and "constructing within a sketch" under the

[3] Tasks are classified as memorization, procedures without meaning, conceptually connected procedures, and doing mathematics; the first two reflect tasks with lower-level, the latter two with higher-level demands.

more general category of "a technological action whose results could also be achieved without technology".

Edith planned to include the DTA model in the course to "provide a framework for formulating … cognitively activating tasks [for pupils]" (self-reflection, pos. 39) to support PMTs in analyzing, evaluating, and designing digital resources. She noted:

> I believe this framework is very helpful and supportive for PMTs when designing digital resources … because it can serve as a well-founded guideline (a kind of toolbox) on which to rely when considering how to formulate tasks that engage pupils deeply with the resource. (self-reflection, pos. 92).

In essence, this episode illustrates both a challenge Edith encountered in designing the TT course and the learning that occurred through her reflection on course design, collaboration with colleagues, and engagement with research. Viewed through the analytical lens of the MTEPCK framework, it reveals that she recognized the *cognitive demand* placed on PMTs in designing accompanying tasks for pupils without structured guidance. By adapting the DGTA framework into the DTA model, she extended her *knowledge of resource*s for achieving her specific goals in teaching digital resource design. In her reflections, she explained why the DTA model should be included in the TT course, thereby clarifying her *purpose of the content knowledge* in her decision to adopt the DTA model. Furthermore, this episode underscores the need for specialized MTEPCK in technology-focused TT courses.

Introduction and observed difficulties

Given the broad range of aspects Edith intended to address during the first course phase, she designated one weekly session to the topic of accompanying task design. Since this session also included general considerations regarding the wording of tasks, approximately 75 min remained for introducing the DTA model. Edith began with a lecture on Smith and Stein's (1998) considerations regarding cognitively activating tasks, drawing on one of their examples that had served as an initial motivation for the course. She chose this starting point to connect with the PMTs' prior knowledge of Smith and Stein's ideas, to focus initially only on the dimension of mathematical depth, and to align with the DGTA framework, which builds upon these ideas. Before discussing the DTA model, she outlined the foundational DGTA framework to extend to the second dimension of technological action, both to guide the PMTs based on her own learning process and because it is a well-established framework with thoroughly elaborated examples on which she wanted to rely. Although she selected a lecture format for its time efficiency, she acknowledged that this format alone would not sufficiently support PMTs in developing the skills required to productively adopt the DTA model, which demonstrates her awareness of the *cognitive demands* involved in this process. To promote student-centered engagement, she selected two elaborated and authentic parallelogram tasks from Trocki (2014, p. 702). Drawn from a professional development program, the parallelogram tasks were designed to support mathematical exploration through a dynamic geometry system. Each task was based on a pre-designed parallelogram sketch in which pupils were prompted to measure lengths and angles, construct related geometric elements, drag vertices, and formulate and justify conjectures about parallelogram properties. The two tasks differed in the level of cognitive demand and the integration of technological actions with mathematical depth. The first task, created by a teacher with limited experience in designing dynamic geometry tasks, included lower-level prompts that focused on using

specific technological actions and investigating invariant properties of parallelogram. The second task was developed by a researcher using the DGTA framework and incorporated higher-level prompts that more explicitly linked dragging, construction, and measurement to explanation and justification, thereby fostering deeper mathematical reasoning (Trocki, 2014; Trocki & Hollebrands, 2018). Initially, the PMTs analyzed the first task in small groups by applying the framework, followed by a whole-class discussion of their considerations and questions regarding their categorization of prompts. They then worked with the second parallelogram task in small groups again to gain additional practice in applying the framework. She incorporated these learner-centered, exploratory group discussions to promote engagement with authentic tasks while encouraging connections to PMTs' own experiences and reflections on their learning processes. The session concluded with another whole-class discussion of their work, after which she introduced the adapted DTA model, which the PMTs were expected to apply in the subsequent course phase of individual digital resource design. This approach illustrates her MTEPCK in selecting *teaching strategies* that balance time-efficiency with active participation in a learner-centered format.

After the course session, Edith questioned her approach: "I had the impression that the time [used for introducing the DTA model] was too short and that the PMTs did not really grasp the meaning and supportive function of the framework." (self-reflection, pos. 92). PMTs' reflections on the session varied: Student B found the group discussions helpful, stating they "made the theory more tangible" (learning diary, pos. 32), while Student C would have preferred to "first reflect alone and then discuss in the group" (learning diary, pos. 56). Consideration of these reflections requires a closer examination of how Edith's approach unfolded in the PMTs' digital resource designs. The following paragraphs provide a more detailed account of these developments. We analyze the effectiveness of Edith's *teaching strategy* by examining the digital resource designs of two teams, followed by a collective reflection on her knowledge and professional learning by drawing on PMTs' comments and designs.

One team, Students M and O, developed a digital resource for grade 6 to visualize the multiplication of fractions (see Fig. 4). Pupils use sliders to vary two fractions, each represented by a square, and a "superimpose" slider to overlap them. They can also display the numerical solution to explore its relation to the visual representation.

Students M and O formulated tasks accompanying the digital resource and analyzed them using the DTA model. For instance, Task 1, "Set numerator1 to 2 and denomina-

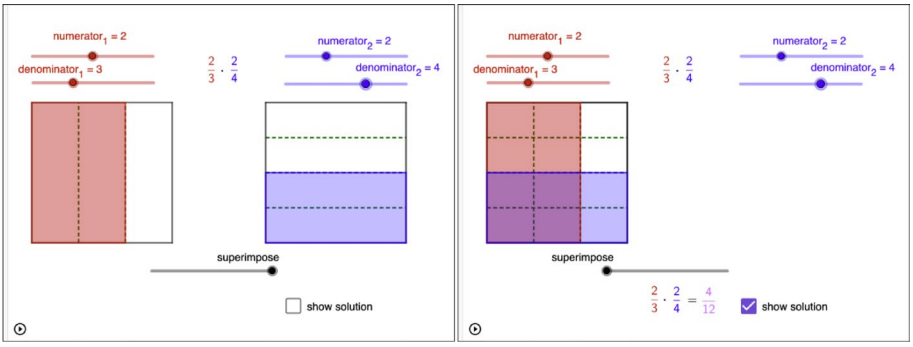


Fig. 4 Digital resource: multiplying fractions (translated) (https://www.geogebra.org/m/gsmgvqtu)

tor1 to 4. How many pieces are formed?" was categorized with (i) mathematical depth level 2, as it involves "reproducing information from the digital resource" (Students M&O, design 4, pos. 55), and (ii) technological action B, as pupils "use sliders to independently set examples and observe the grid" (Students M&O, design 4, pos. 63). This categorization accurately reflects both mathematical depth and technological action.

In Task 3, pupils should set the numerator2 to 3 and the denominator2 to 5, overlap two squares using a slider, and determine the total number of pieces. Again, the PMTs correctly categorized this as mathematical depth level 3 (considering mathematical concepts, procedures, or relationships) and technological action B (dragging) and C (manipulating the resource to recognize emergent invariant relationship(s) or pattern(s)) because this task requires recognizing the relationship between representations and the meaning of fractions and their product.

The DTA model seemed to encourage these PMTs to formulate higher-level tasks as seen in Task 7: "Display the solution and explain in your own words how to multiply two fractions." Although it lacked a direct prompt for technological action (which was addressed earlier), PMTs aimed for at least mathematical depth level 4. They reasoned levels 4 and 5: "Pupils must use the visualized content to generalize a procedure. They should explain how the procedure works and formulate a hypothesis for the rule." (Students M&O, design 4, pos. 61).

Another team, Students E and I, designed a digital resource for grade 7 visualizing whole-number addition on a number line (see Fig. 5) and formulated seven related tasks. Instead of analyzing each task separately along the DTA model, they summarized them into two overarching questions: (1) How can a whole number be added to a natural number? and (2) How does natural-number addition change when the second addend becomes a whole number?

They subsumed both questions in their analysis but examined them separately in terms of mathematical depth and technological action. For example, they categorized mathematical depth as level 2–3, stating that while pupils are not expected to explain the interaction of plus and minus, "they should consider and connect the relationships and consequences of arithmetic and signs" (Students E&I, design 3, pos. 29). However, their questions lack

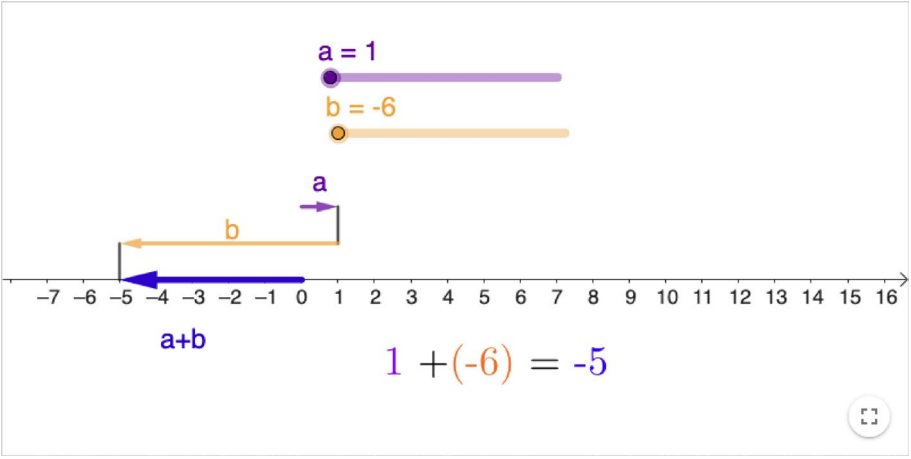


Fig. 5 Digital resource design of Students E and I

clear prompts for both categories, thus failing to align with the DTA model, and presenting imprecise requirements for pupils.

Compared to Students M and O, who competently applied the DTA model and integrated higher-level tasks, Students E and I struggled on multiple levels. They failed to formulate prompts that incorporated both technological actions and mathematical depth and inconsistently analyzed overarching questions. In sum, their task designs reveal a limited understanding of the model.

In essence, across the course the PMTs struggled to coordinate mathematical depth with technological actions, repeatedly categorizing prompts separately and, at times, grouping multiple prompts under a single category. Overall, "the PMTs' task designs revealed that they did not really grasp the meaning and supportive function of the model …. So, I must reconsider how to introduce the model [in the course]" (self-reflection, pos. 92).

Reflection, recognition, and refinement

The DTA model is intended to enhance PMTs' digital task design and thereby support their mathematics teaching. Data suggest that, despite their struggles, PMTs nonetheless attempted to incorporate higher levels of mathematical depth in final designs. Student C, for instance, found it supportive to learn "how to formulate questions better to encourage pupils to form assumptions and then test them" (learning diary, pos. 75). On this basis, Edith acknowledged the model's value as a resource for PMTs in fostering cognitive activation.

However, Edith's reflection on PMTs' work revealed that they had not integrated the DTA model as intended, leaving her dissatisfied with the outcome. PMTs' considerable challenges heightened her awareness of the *cognitive demand* for PMTs involved in understanding and applying the DTA model productively in digital resource design. In particular, the integration of technology increases this *demand*, as PMTs must not only focus on designing cognitively activating tasks that support exploratory learning (the mathematical depth) but also attend to employing technology in ways that leverage its potential, while simultaneously coordinating both dimensions.

In her reflection, Edith identified not only the difficulties PMTs faced but also her own limitations in translating her researcher-developed *knowledge of resources* into a form accessible to PMTs. The PMTs' documents prompted the following reflections: "The DTA model is a fruitful tool for designing tasks" (self-reflection, pos. 143). However,

> my way of introducing [the DTA model] did not work. … I must reconsider how to introduce the model. At least, I must take more time and include more practical exercises on how to assess and design tasks according to the model. (self-reflection, pos. 92)

Although she recognized the model's potential for preparing and evaluating digital task designs, she struggled to find an effective *teaching strategy* for introducing it, thereby exposing a gap in her MTEPCK regarding framework-based instruction. Edith's self-reflection attributed these difficulties to insufficient time devoted to exploring, discussing, and especially practicing the model. She concluded that although this model is a valuable tool, its implementation in the course was ineffective. Based on her experiences, she intended to retain the model in future iterations of the course but also recognized the need to adapt and refine her *teaching strategy* by combining instructive and student-centered approaches, providing clearer guidelines, and offering more structured support to PMTs. Her reflections

on PMTs' work illustrated her knowledge transformation based on "knowing in relation to prospective teachers", demonstrating that challenges in PMTs' learning stimulated her own professional learning.

Beyond the DTA model, data analysis revealed that Edith integrated a variety of resources and illustrative examples to enhance PMTs' knowledge throughout the course, demonstrating her *knowledge of resources and examples*. During the design and redesign phase, this aspect of her MTEPCK was visible in the digital resource discussions. Each discussion was distinct, depending on the PMTs' chosen mathematical topic and design ideas. To highlight her *knowledge of examples* and her perspectives on *student thinking*, we present excerpts from these resource discussions as representative cases. These episodes highlight challenges commonly encountered by most PMTs, illustrating the central problems they faced, and are closely tied to her two primary goals for digital resource design.

Digital resource discussions: lenses into MTE knowledge and learning

For the digital resource design process, Edith organized PMTs in teams of two. To guide the pairing process, she administered a brief questionnaire in which PMTs self-assessed their GeoGebra skills (0–10 scale). While PMTs were free to choose their partners, they were encouraged to form teams that included at least one more experienced GeoGebra user. This *strategy* reflected two considerations: fostering peer support and collaboration, a key design element of the course (see Sect. "The course design"), and ensuring that teams had sufficient technological expertise while also allowing PMTs to choose partners with whom they felt comfortable. Balancing technical proficiency with social compatibility was intended to promote meaningful discussion, enhance the quality of digital resource design, and support PMTs' learning. Building on this team structure, Edith aimed to guide PMTs' digital resource development by facilitating discussion, reflection, and feedback. For this purpose, she scheduled two mandatory digital resource discussions with each team, lasting 20–25 min each: one during the design and another during the redesign phase. This *teaching strategy* represents reflective practice via mentor-mentee meetings, and it allows PMTs to organize their workload freely while collaborating with peers, expert teachers, and Edith to reflect on their design considerations. As she explained: "We discussed their ideas, designs, and tasks… The underlying assumption is that [discussions with] experts can enhance PMTs' knowledge and awareness. Also, study data showed the importance of discussions with experts and peers for sharing ideas." (self-reflection, pos. 112–113). Edith adopted this strategy for two reasons: research on the course design emphasized the value of peer, teacher, and digital expert collaboration, and her own expertise in technology-based education suggested such discussions would enhance both the quality of digital resources and PMTs' professional knowledge. PMTs found these meetings supportive; for example, Student R noted the benefit of "addressing various aspects with [the MTE]" (learning diary, pos. 67), and Student B appreciated "receiving many valuable … tips" (learning diary, pos. 140) for the design.

The following episodes of interactions with PMTs illustrate the influence of this *teaching strategy* on their digital resource designs. They also show how Edith diagnosed challenges, operationalized her design goals, and scaffolded PMTs' thinking, while revealing how her knowledge manifested, was challenged, and evolved as she aligned their work with her two primary design goals.

Episode 1: Validating reflective discussions as a teaching strategy for PMTs' design development

Students B and R designed two digital resources on (calculating) percentages for grade 6 (typically ages 11–12). Their initial draft of the first resource featured two fictional pupils who explained, step by step, the algorithm for calculating shares (i.e., parts of the base value). In this version, the fictional pupils followed a concrete example—determining how many individuals correspond to 30% of 70 participants in a competition—through a sequence of nine steps, closely resembling teacher-led mathematics instruction. However, this instructional approach did not align with the goal of designing resources that promote the exploration of mathematical content. During their first meeting, Edith emphasized the importance of visualization:

> Edith: Try to make what happens understandable through visualizations … because then, technology can add value. (discussion 1, pos. 11)

To pursue this idea, she introduced the concept of visualizing a percentage band:

> Edith: What you could do is consider visualizations, and there are the so-called percentage bands. And you could transform the resources into ones including visualizations.
> Student R: Percentage band simply means ….
> Student B: Percentage strips.
> Student R: From 0 to 100, right?
> Edith: Exactly. I have a strip from 0 to 100, and above it [one] with the base value. I can move them together, and the ratios always remain the same … Then, we also have this dynamic aspect [included]. For example, you could … change the rate [percentage] and observe how the part [share, part of the base value] above changes. … So, I think that can help. Visualizations can always be supportive, and I think they fit very well here. And you always have, on the one hand, these percentages [rates] and, on the other hand, these parts of the base value. In this way, let them [the pupils] experiment and ask questions about it. (Students B&R, discussion 1, pos. 16–20)

She concluded by stressing: "Less [focus] on the algorithm … but more on … what are the relations, how can you understand them" (pos. 20).

In this conversation, Edith guides the PMTs to shift their focus toward dynamic exploration and visualization. To achieve this, she introduces the idea of a dynamic percentage rubber band that enables interactive exploration of the relationship between rate, base value, and part of the base value. This example clarifies expectations for the PMTs' work and illustrates how Edith scaffolds their learning by modeling the use of representations to deepen mathematical understanding. The conversation demonstrates both her *knowledge of examples* and her knowledge of *representations of concepts* to support learning of mathematical concepts, as she articulates what the representation via percentage bands can contribute to mathematics teaching. In particular, it underscores Edith's emphasis on enhancing pupils' conceptual knowledge rather than focusing solely on procedural knowledge. Furthermore, the discussion illustrates her effort to explicitly align PMTs' resource design with the pri-

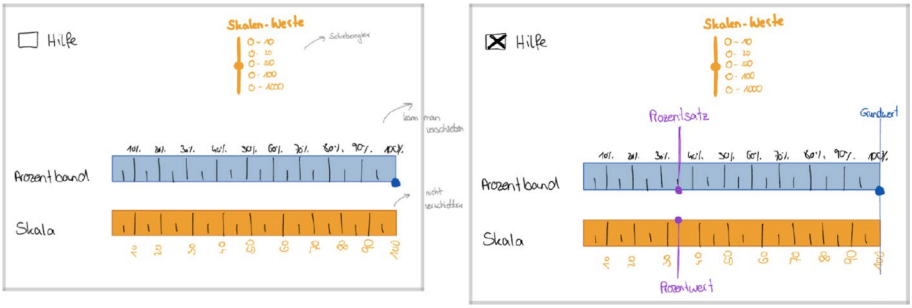


Fig. 6 Digital resource design of Students B and R

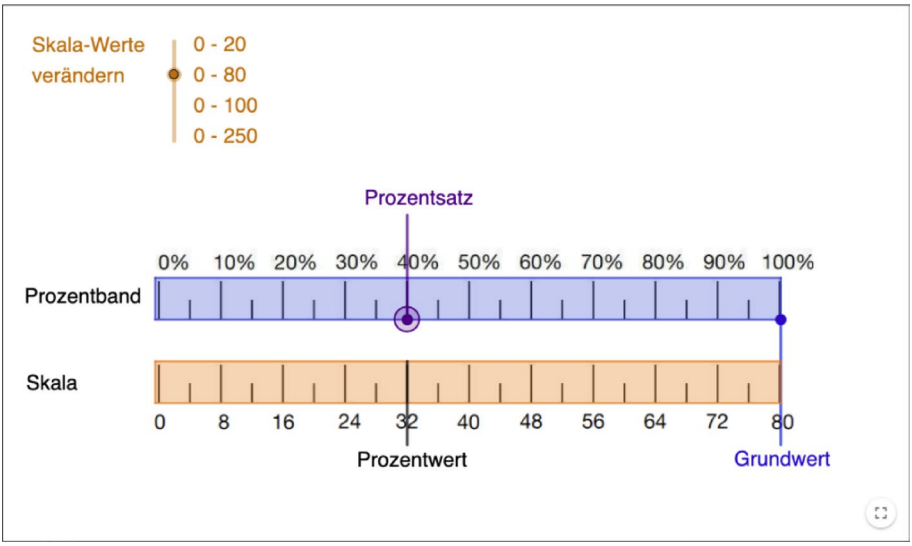


Fig. 7 Digital resource: discovering percentage calculation (https://www.geogebra.org/m/tdauxpvp)

mary design goals. Her understanding of the added value of technology is specifically connected to her knowledge of technology-enhanced learning, an aspect not represented in the MTEPCK framework. Consequently, this distinct scope warrants an extension of the framework by incorporating a new category of *technological knowledge in a pedagogical and/or content* context, with *added value of technology* as a subcategory.

Based on Edith's suggestions, Students B and R developed a preliminary design (see Fig. 6) that incorporated a dynamic percentage band and prioritized visualization for concept exploration over procedural calculation. Pupils can interact with the digital resource by dragging the blue point to adjust the percentage band (blue bar) and a vertical slider to modify the base value (orange bar). The help button displays the percentage (upper bar) and share of base value (lower bar). By dragging the lilac point, pupils can explore the relationships between percentage, share, and base value.

Figure 7 presents the finalized resource and illustrates the influence of Edith's feedback on the redesign of the resource.

Figure 7 demonstrates the influence of Edith's feedback on the redesign of the resource. Student R reflected on this process:

> During the feedback, I learned what was required; how simple a resource for discovering [a mathematical issue] can be, and the crucial role of good visualizations. Additionally, the "percentage rubber band" idea for implementation was very helpful and a new method for me. (learning diary, pos. 62)

Building on this suggestion, the redesigned resource integrates a "percentage rubber band" visualization with dynamically linked representations, enabling pupils to explore relationships between mathematical concepts through interactive technology, thereby aligning with the course goals.

The effectiveness of this *teaching strategy*, as illustrated in this episode, helped to shift the digital resource design from an instructional approach to one enabling exploratory learning. The synthesized PMT feedback, which validated the reflective practice of resource discussions, further reinforced Edith's intention to continue using this strategy in future courses, as she noted:

> The discussions were a good idea; I would keep them in any further course iteration. They provide meaningful support to the PMTs, as they create space to dive deeply into their specific designs. I can share concrete … ideas for refining or reshaping their approaches. This focused exchange … also expands their view of what can be achieved with digital resources and what considerations are important along the way. (self-reflection, pos. 114)

Through her self-reflection on the resource discussions, Edith recognized that providing concrete examples of digitally enhanced exploratory learning, as in this case, can scaffold PMTs' work and learning. This indicates that both her awareness of PMTs' resource developments and her self-reflection on the resource discussions enhanced her professional knowledge, simultaneously supporting her own learning.

Edith identified a particular challenge for PMTs in designing digital resources for exploration. Student R reflected: "While brainstorming and sketching our design, I realized that creating such [resources] involves many thought and revision processes." (learning diary, pos. 45). This statement encapsulates the struggles PMTs faced, particularly in distinguishing between resources for exploration and practice, as well as in creating designs that encourage exploration rather than providing step-by-step instructions. The following examples of a discussion between PMTs and Edith further illustrate this challenge.

Episode 2: Supporting PMTs' transition from instructional to technology-enhanced exploratory design

The second example (see Fig. 8) features a resource for grade 6 (typically ages 11–12) designed by Students M and O to illustrate the multiplication of fractions by a natural number, represented through repeated addition. The process is structured in a sequence of steps that pupils can navigate using "Next" and "Back" buttons. The resource begins with a scenario involving leftover pizza slices from a birthday party, which pupils are asked to add. Subsequent steps include a visualization, a numerical representation, an explanation about expressing a repeated addition as multiplication, the rule for multiplying fractions by natural numbers, and finally a prompt inviting pupils to calculate an example.

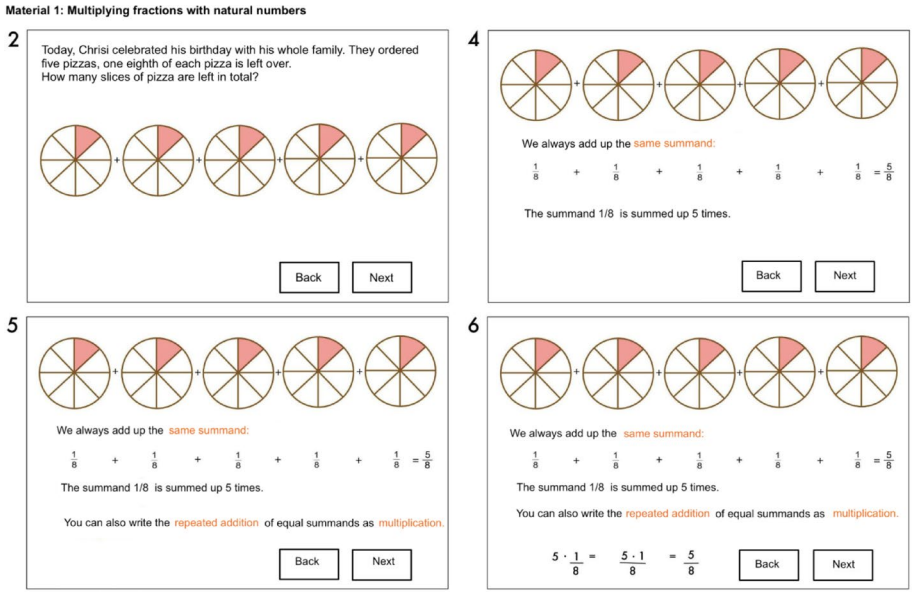


Fig. 8 Parts of digital resource design of Students M and O (translated)

The following excerpt from a discussion between PMTs and Edith highlights the challenge of integrating exploration into digital resource design.

Edith: I think you've already put a lot of work into it. But it doesn't quite align with the idea of exploration. … Let's start from the beginning: three aspects are always important [for digital resource design]: the added value of technology; what exactly should be discovered in the first place, and whether it can be explored in this way or if it's more of a demonstration; and this model [DTA]. … So, what exactly should be explored with this resource?

Student M: Well, we haven't developed the [accompanying] tasks yet … and those would be for the exploration. We'd like to focus more on the aspect of exploration … because right now it's just a presentation.

Edith: Exactly. It's a beautiful idea, but it rather leans toward showing how to do [something] and less toward [enabling exploration].

Student M: Exactly. That's why … we'd really like to bring in the aspect of exploration even more through the tasks.

Edith: I think it could work if you streamlined the whole thing. Right now, you have quite a lot of steps. … I don't want you to [completely] rebuild it, since that would mean a lot of extra work. But you could reduce this idea of "guiding [the pupil] through". … I think there is always a certain challenge when it comes to letting pupils explore something. ….

Student M: Maybe it's not that easy for this particular topic?

[…]

> Edith: But I think it's good that this happened, because I think it's generally difficult to let pupils explore something. What sometimes helps me when I think about how I would approach it [the chosen mathematical content], what if I were in the classroom and had no technology? What would I do? And then, is there any way to translate that? In such a way that you can get more out [added value] because you have technology at your disposal and can vary something dynamically, for example. Currently, the added value would be the guiding through [the resource] … But the question is, what would make it different from a video where you can press stop? … Maybe you can reduce [this aspect] and [include] instead something with sliders. (Students M&O, discussion 1, pos. 10–27)

This conversation between Edith and Student M highlights several aspects of her MTEPCK. She initiates the discussion by clarifying the objectives of resource design and the relevance of the DTA model for task development (see Sect. "DTA model as knowledge resource"), thereby illustrating her awareness of *goals for learning*. She then critically evaluates the draft, acknowledging its strengths (e.g., "beautiful idea") while identifying its presentational nature as a limitation, since it does not meet the course's learning objectives. As in Episode 1, PMTs proposed an instructional approach that they believed could be transformed into an exploratory resource through additional accompanying questions. Although not stated directly, Edith recognizes the limitations of this approach, as the digital resource itself must be designed in a way that inherently promotes exploratory learning.

Acknowledging the *cognitive demand* of designing such resources, Edith proposes a problem-solving approach: first considering non-technological teaching methods and then adapting them to leverage the added value of technology. Her comments demonstrate a reflective, methodical approach to technology integration and underscore her *knowledge of teaching strategies*. By providing constructive feedback and suggesting a practical strategy for improvement, she scaffolds *student thinking* and offers *methods of solutions* for PMTs' design problems, guiding them to design resources aligned with the course objectives. Moreover, Edith demonstrates a nuanced understanding of technology's added value in both exploratory and practice-oriented resources. Her knowledge of technology integration into mathematics education is evident as she identifies the distinctive potential of technology for fostering exploratory learning. Specifically, she emphasizes the integration of dynamic variations through sliders, which enable interactive experimentation and thereby provide added value compared to paper-based tasks or videos. Essentially, the discussion underscores Edith's *knowledge of technology-related examples*, *dynamic representations of concepts*, and *added value of technology.* These forms of MTE knowledge are specific to technology-based teaching and therefore justify extending Chick & Beswick's (2018) framework by introducing them as subcategories of *technological knowledge in a pedagogical and/or content context*, since the original categories do not fully capture the specialized knowledge required to integrate digital tools into mathematics teaching.

Upon reflection on PMTs' statements, Edith identified a *misconception in student thinking*: they attempted to address the *cognitive demand* of resource design by reducing its complexity—initially focusing on one dimension, technology-added value, and only later addressing the second, exploratory learning. She further recognized a recurring tendency among PMTs to explain mathematical content rather than design resources that facilitate exploration, revealing broader challenges in fostering pupils' engagement with mathemati-

cal concepts. Through this realization, Edith also became aware of how such design choices reflect deeper conceptions of teaching. Specifically, PMTs' tendency to explain what pupils should do suggested that they perceived teaching primarily as delivering instructions and explaining mathematics rather than facilitating exploration, a perspective that offered her valuable insights into their underlying attitudes toward teaching. Moreover, Edith's professional learning was stimulated by feedback through PMTs' design drafts, which illuminated the challenges they faced. Reflecting on her own teaching, she identified a scaffolding strategy of first considering introducing a mathematical idea without technology and only then incorporating technology fruitfully. This strategy was not deliberately planned; rather, it emerged during the discussion and became apparent through self-reflection. The experience made her aware of a practical strategy for aligning PMTs' design ideas with her instructional aims, enriching her *knowledge of resources* as a specific approach that supports PMTs' teaching. Episode 2 illustrates her expertise in technology integration, encompassing pedagogical (content) knowledge, an understanding of technology's role in mathematics education, and the ability to guide PMTs in aligning their designs with educational objectives. Summarized, Edith's professional development was prompted by reflection on both PMTs' work and her own teaching practice. This development, which includes "knowing in relation to self and knowing in relation to prospective teachers", supported her "learning on how to teach about teaching" with technology.

Through her interactions with PMTs and subsequent self-reflection on resource discussions and PMTs' digital designs, Edith identified a recurring pattern: PMTs tended either to explain mathematical content or to design practice-oriented resources, while struggling to create digital resources that facilitate mathematical exploration. She acknowledged the dual challenge of fostering exploratory learning and leveraging the added value of technology, yet her experiences reinforced these goals of digital resource design. Edith "wanted PMTs to think about how technology can support pupils in engaging with, learning, and understanding mathematical ideas" (self-reflection, pos. 72). Overall, her reflective practice, stimulated by PMTs' work and responses, contributed to her professional development.

Discussion and conclusion

In this section, we first discuss the findings by examining the MTE's professional learning through the design and enactment of a technology-based TT course, before turning to implications for framework-based teaching, professional learning, and research.

MTE professional learning through designing and enacting a technology-based TT course

This study examines Edith's applied knowledge and professional learning in designing and implementing a TT course on technology integration in mathematics education. Her approach aimed to prepare PMTs for their future profession by enhancing their skills and providing practical support in designing digital resources and integrating them into mathematics lesson planning. The course was structured in three phases—preparation, design and redesign, and implementation—and the digital resource design was guided by two learning objectives: the resources had to (1) offer technology-added value compared to

paper-and-pencil tasks and (2) support mathematical exploration. The findings highlight Edith's MTEPCK, technology-related knowledge, and professional learning through reflective practice. Reflection was evident in her use of teaching strategies, selection of resources and examples, and integration of theoretical frameworks into course design. Moreover, her understanding of student thinking became visible as she guided PMTs through the multifaceted process of digital resource design. Consistent with existing research on technology-related professional development (e.g., Beswick & Goos, 2018; Chapman, 2008b; Nicol, 1997; Psycharis & Klogeria, 2018; Thurm et al., 2022), the TT course incorporated structured design phases, reflection, collaborative learning, and technology-focused tasks. In addition, personalized feedback and individualized interactions with PMTs proved particularly valuable for supporting learning.

Edith's professional learning was shaped by the interplay between her prior knowledge, her reflective practices during course design and implementation, and her role as a mathematics education researcher. Her learning can be understood along two dimensions: (i) knowledge transformation through teaching, including self-reflection on course design ("knowing in relation to self") and feedback from PMTs ("knowing in relation to prospective teachers") (Chapman et al., 2020, p.165), and (ii) knowledge transformation through learning, facilitated by engagement with colleagues, researchers, and research-based literature. These dimensions align with Tzur's (2001) view that MTEs develop knowledge through the interplay of teaching and reflection on learning, gradually becoming aware of how instructional actions shape others' learning of mathematics teaching. Through this self-study, Edith uncovered implicit approaches and transformed them into explicit, actionable knowledge. This process illustrates how self-based research can sharpen pedagogical focus and strengthen intuitive teaching by embedding it within theoretical frameworks, thereby fostering a literature-informed, reflective practice (e.g., Beswick & Goos, 2021; Chapman et al., 2020). Methodologically, the study demonstrates the value of self-study approaches for developing MTEs' PCK in the context of technology integration, supporting ongoing professional learning through productive engagement with challenges.

Framework-based teaching as a productive tension in technology-based teacher education

Despite many effective design decisions, Edith encountered several challenges that became central drivers of her professional learning. A key challenge concerned her framework-based teaching strategy, particularly the integration of the DTA model intended to support PMTs in shifting from largely intuitive to more research-informed digital task design. Although PMTs struggled to fully understand and apply the model in their own designs, it nevertheless functioned as an important pedagogical anchor for structuring feedback, shaping instructional decisions, and encouraging engagement with higher-level task design. While previous studies have documented the value of framework-based approaches in teacher education (e.g., Gueudet et al., 2021; Psycharis & Klogeria, 2018), our findings empirically illustrate the micro-level work required of MTEs to enact such approaches in practice, including moments of breakdown, adaptation, and re-mediation.

More broadly, these findings highlight the role of the MTE as a mediator between theory and practice, illustrating that theoretical frameworks are not simply transmitted to PMTs but must be actively reworked, contextualized, and enacted through activity-oriented teaching. In this study, we conceptualize this work as *framework-based teaching*, referring to the

deliberate use of theoretical frameworks as pedagogical resources that structure instructional decisions, feedback, and task design in teacher education. Empirically, our analysis shows that framework-based teaching involves ongoing negotiation between theoretical intentions and practical constraints, as the MTE adapts and re-mediates frameworks in response to PMTs' interpretations and needs. Rather than resolving tensions between theory and practice, framework-based teaching makes these tensions productive, positioning them as sites for both PMT and MTE learning.

This challenge extended beyond the DTA model to the integration of multiple theoretical frameworks within the TT course. While Edith values framework-based teaching, its enactment proved complex, revealing a tension linked to her dual role as researcher and MTE. As a researcher, she routinely applies frameworks analytically; as an MTE, she must design learning environments that enable PMTs to appropriate these frameworks for practice. This tension aligns with research on professional identity development emerging through professional activity and institutional demands (Helliwell & Ebbelind, 2024), highlighting how MTEs' instructional work is shaped by the need to negotiate multiple professional roles.

From a theoretical perspective, the ways in which Edith navigated this tension can be interpreted through the documentational approach to didactics (Gueudet et al., 2014; Trouche et al., 2020). Frameworks such as the DTA model can be understood as meta-resources—conceptual tools explicitly designed to support teachers' planning and design of teaching activities (Gueudet et al., 2021, 2024). For PMTs, appropriating such frameworks involves developing utilization schemes that allow theoretical ideas to be meaningfully enacted in practice. Importantly, our findings show that this process of documentational genesis also applies to MTEs, who engage in ongoing documentational work as they adapt, refine, and re-mediate frameworks in response to learners' needs. Notably, the challenges involved in this work are not merely obstacles but can become productive moments where theory and practice meet, supporting the professional growth of both PMTs and MTEs.

A second challenge emerged during individual digital resource discussions with PMT teams. While these discussions demonstrated Edith's strong knowledge of digital resources and dynamic representations, they also revealed PMTs' difficulties in designing tasks for exploration rather than practice or instruction, pointing to deeper conceptions of teaching. These interactions raised Edith's awareness of limitations in PMTs' thinking and prompted further professional learning. Additional challenges stemmed from the breadth of course content and limited time, highlighting the need for clearer prioritization and deeper PCK for teaching about teaching with technology. Addressing such challenges requires continued professional development through reflection and engagement with the professional community (Tzur, 2001; Zaslavsky & Leiken, 2004).

Building on these findings, future research could examine how MTEs in different contexts enact and mediate theoretical frameworks, how such mediation supports PMTs' appropriation of research-based tools, and how framework-based teaching shapes MTEs' instructional decisions and professional identities over time.

Implications for research and professional learning in technology-based teacher education

This study contributes to the limited research on MTEs' knowledge and experiences in teaching about teaching mathematics with technology (e.g., Abboud et al., 2020; Psycharis &

Klogeria, 2018) by empirically documenting how an MTE bridges theory and practice through framework-based teaching in a technology-based TT course. Using Chick and Beswick's (2018) MTEPCK framework as an analytical lens, the study demonstrates that most Clearly PCK categories were consistently evident in the MTE's practice, confirming the framework's relevance for analyzing technology-based teacher education. At the same time, the findings show that technology integration increases the complexity of MTE knowledge, pointing to the need for more explicit attention to technology-based dimensions within MTEPCK.

Drawing parallels to Mishra and Koehler's (2006) extension of Shulman's PCK through the TPACK framework, we propose extending MTEPCK by incorporating technology-based categories within a new dimension—analogous to Clearly PCK—termed Technological knowledge in a pedagogical and/or content context. These categories encompass knowledge of technology-added value, dynamic representations, and technology-based resources that support digital resource design and technology-integrated lesson planning. Future research can build on these proposed categories by using them as analytical tools to examine MTEs' planning, enactment, and reflection across different technology-based teacher education contexts, or by testing their usefulness in the design and evaluation of professional learning programs for MTEs.

Beyond theoretical contributions, the findings offer concrete guidance for the design of professional learning opportunities for MTEs. TT courses that foreground digital resource design, structured reflection, and framework-based teaching can function not only as learning environments for PMTs but also as powerful professional learning contexts for MTEs themselves. By deliberately positioning MTEs in situations where they must mediate theoretical frameworks within activity-oriented teaching, such designs make visible the knowledge demands of bridging theory and practice and create structured opportunities for reflective professional growth. These design features can be taken up and adapted in future studies aiming to investigate MTE learning across contexts.

Consistent with existing research, this study reinforces the view that challenges and uncertainties in practice can function as productive resources for MTE professional learning (e.g., Nicol, 1997; Kastberg et al., 2025). While the findings stem from a self-study and are therefore not directly generalizable, they offer empirically grounded insights into processes—such as framework-based teaching, documentational work, and reflective engagement with tensions—that are likely to be relevant across institutional and cultural settings. Future research involving multiple MTEs, comparative designs, or longitudinal approaches could build on these processes to deepen understanding of MTE knowledge development in technology-based teacher education and to further refine the proposed extensions of the MTEPCK framework.

Author contributions All authors read and approved the final manuscript.

Funding Open access funding provided by Johannes Kepler University Linz. The authors did not receive funding for conducting this study.

Data availability The datasets generated and analysed during the current study are available from the corresponding author on request.

Declarations

Competing interests The authors have no competing interests to declare that are relevant to the content of this article.

Ethics approval In this study we followed the guidelines of the American Educational Research Association (AERA). All participants volunteered to participate in the data collection. To preserve confidentiality and anonymity, we ensured a secure data storage and changed names of the students to pseudonyms so that they cannot be identified in any publications.

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